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Life-Cycle Models: Lifetime Earnings and the Timing of Retirement

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Abstract

After dropping for a century, the average retirement age for U.S. males seems to have leveled off in recent decades. An important question is whether as future improvements in technology cause wages to rise, desired retirement ages will resume their downward trend, or not. This paper attempts to use HRS panel data to test how relatively high (or low) earnings affect male retirement ages. Our goal is to use cross—sectional earning differences to help anticipate likely time—series developments in coming decades. Our preliminary regression results show that higher earnings do lead to somewhat earlier retirement. Unless additional analysis changes the parameter estimates, the implication is that the downward trend in male retirement ages will ultimately return.

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Life-Cycle Models: Lifetime Earnings and the Timing of Retirement

1. Introduction

After dropping for a century, the average retirement age for U.S. males seems to have leveled off in recent decades. An important question is whether as future improvements in technology cause wages to rise, desired retirement ages will resume their downward trend, or not. This paper attempts to use HRS panel data to test how relatively high (or low) earnings affect male retirement ages. Our goal is to use cross–sectional earning differences to help anticipate likely time–series developments in coming decades.

Our theoretical framework is the life–cycle model of household behavior. This paper builds from a general first–order restriction for optimal household retirement based on a so–called "free endpoint" condition from optimal control theory (e.g., Kamien and Schwartz [1981]). This condition may be adapted to a wide variety of life–cycle models and implies that, when choosing its best retirement age, a household balances its loss of current earnings, converted to units of utility, against its utility gain from retirement. The free endpoint condition generates our regression equations.

At this stage our tentative conclusion is that higher earnings may lead to earlier retirement. We believe that we have not yet exhausted the potential of our analysis, however, and that additional steps, which we outline below, may attenuate this connection.

The organization of this paper is as follows. Section 2 presents our theoretical framework, which is based on the life–cycle model of household behavior. Section 3 uses a first–order condition for optimal retirement age to derive equations for estimation. Section 4 describes our data. Section 5 presents our estimation procedure and results.[1] Section 6 concludes.

2. Basic Framework

We begin with a basic life–cycle framework. We derive our basic optimality condition for retirement and use it to construct a regression equation. The regression equation is the basis for this paper's empirical analysis.

For expositional convenience, begin with a single member household. The household chooses (i) how much to consume at each age and (ii) the age at which it will retire. Assume that work options are discrete: employers require full–time work; to obtain reduced work hours, an individual must retire altogether. The household starts at age and date 0, lives to age T, and, when not retired, has earnings flow y_t. The household solves

$$\max_{c_t\,,\,R} \int_0^R e^{-\rho\cdot t}\cdot u(c_t\,,t\,,R)\,dt + \varphi(a_R\,,\,R) \tag{1}$$

[1] An earlier version of this paper, presented at the RRC Conference in Washington, 8/2007, employed linearizations and maximum likelihood estimation techniques. The present version eschews linearizations to obtain more precise results and turns to direct, nonlinear estimation of first–order conditions based on the method of moments.

$$\text{subject to:} \quad \dot{a}_t = r \cdot a_t + y_t - c_t \,,$$

$$a_0 = 0 \,.$$

The function $\varphi(a_R \,, R)$ gives post–retirement utility conditional on retirement age R :

$$\varphi(A \,, R) \equiv \max_{c_t} \int_R^T e^{-\rho \cdot t} \cdot u(c_t \,, t \,, R) \, dt \tag{2}$$

$$\text{subject to:} \quad \dot{a}_t = r \cdot a_t - c_t \,,$$

$$a_T \geq 0 \,,$$

$$a_R = A \,.$$

Formulation I. Our basic formulation of the life–cycle model assumes intratemporal additivity of consumption expenditure and leisure.[2] It assumes that for some $\gamma < 1$ and $\Gamma > 0$,

$$u(c \,, t \,, R) \equiv \begin{cases} \frac{1}{\gamma} \cdot [c]^\gamma \,, & \text{if } t < R \,, \\ \frac{1}{\gamma} \cdot [c]^\gamma + \Gamma \,, & \text{if } t \geq R \,. \end{cases} \tag{3}$$

A household thus enjoys an improved utility function after retirement. Indeed, it is the prospect of this improvement that causes an agent to retire in the first place.

Analysis. Under Formulation I, this paper's attention focuses on the magnitude of γ . As we shall see, it is γ that determines the likely correlation between the magnitude of earnings and optimal retirement age.

Maximizing in (1)-(2) with respect to consumption is a familiar problem. The solution is

$$c_t = c_o \cdot e^{\nu \cdot t} \tag{4}$$

where

$$\nu \equiv \frac{r - \rho}{1 - \gamma} \,, \quad c_0 = \frac{Y(R)}{\int_0^T e^{-r \cdot s} \cdot e^{\nu \cdot s} \, ds} \,, \quad Y(R) = \int_0^R e^{-r \cdot s} \cdot y_s \, ds \,.$$

[2] This formulation finds wide use in the existing literature — e.g., Bound et al. [1998], Rust and Phelan [1997], and Gustman and Steinmeier [1986, 2000].

Maximizing with respect to retirement age R is a somewhat less common procedure. We can maximize with respect to R taking (4) as given. This yields our so–called free endpoint condition (e.g., Kamien and Schwartz [1981], Laitner and Silverman [2005, 2007]):

$$\frac{y_R}{[c_R]^{1-\gamma}} = \Gamma \,. \tag{5}$$

According to this condition, at the moment of its optimal retirement a household's loss of earnings, y_R, converted to units of utility through multiplication by the marginal utility of expenditure, $[c_t]^{\gamma-1}$, exactly counterbalances the gain in flow utility Γ from retiring. If the left–hand side of (5) exceeds the right, the advantage of working longer overwhelms the advantage of immediate retirement; hence, if the left–hand side exceeds the right, the household should postpone its retirement beyond the age specified in (5).

Combining conditions (4)-(5), we have a basic equation: at optimal retirement age R, one has

$$\ln(y_R) - (1-\gamma)\cdot\ln\big(Y(R)\big) - (1-\gamma)\cdot\big[\nu\cdot R - \int_0^T e^{-r\cdot s}\cdot e^{\nu\cdot s}\,ds\big] = \ln(\Gamma)\,. \tag{6}$$

Suppose that earnings rise proportionately at every age for later cohorts. Changes in y_R and $Y(R)$ cancel one another. However, if $\gamma \in (0\,,\,1)$, the second left–hand side term leads to ever greater retirement ages. If $\gamma < 0$, the second term leads to ever earlier retirement ages. If $\gamma = 0$, desired retirement age remains the same.

Formulation II. A second possible formulation has non-separable utility. We specify it as follows: for some $\gamma < 1$ and $\lambda > 1$, household flow utility satisfies[3]

$$u(c\,,\,t\,,\,R) \equiv \begin{cases} \frac{1}{\gamma}\cdot[c]^{\gamma}\,, & \text{if } t < R\,, \\ \frac{1}{\gamma}\cdot[\lambda\cdot c]^{\gamma}\,, & \text{if } t \geq R\,. \end{cases} \tag{7}$$

A household thus enjoys an improved utility function after retirement as every level of consumption generates a higher level of utility.

The analysis is only slightly different in the context of this paper — in particular, given this paper's data set (see below). In the denominator of the expression for c_0 above we need (see, for example, Laitner and Silverman [2005, 2007])

$$\int_0^R e^{-r\cdot s}\cdot e^{\nu\cdot s}\,ds + \Lambda\cdot\int_R^T e^{-r\cdot s}\cdot e^{\nu\cdot s}\,ds\,, \quad \Lambda \equiv [\lambda]^{\frac{\gamma}{1-\gamma}}\,.$$

The free endpoint condition is

[3] This formulation also finds wide use in the literature — e.g., variants can be found in Auerbach and Kotlikoff [1987], Altig et al. [2001], French [2005], Laitner and Silverman [2005, 2007], Cooley and Prescott [1995]. See also Hurd and Rohwedder [2003], King et al. [1988], and Aguiar and Hurst [2005].

$$\frac{y_R}{c_R} = 1 + \lambda^{\frac{\gamma}{1-\gamma}} + \frac{1}{\gamma} \cdot [\lambda^{\frac{\gamma}{1-\gamma}} - 1] \,.$$

Inspection shows that this is observationally equivalent to (5) when $\gamma = 0$ in the latter (leaving the only difference between the two models in the denominator of the expression for c_0).

In one empirically plausible case, average male retirement age does not change over time. Then Formulation I leads us to estimate $\gamma = 0$. In respects beyond the scope of this paper — say, households' response to risk or to changing interest rates — $\gamma = 0$ can be very restrictive. Formulation II, on the other hand, implies a constant average retirement age over time for any value $\gamma < 1$, conceivably giving one latitude to pick values of γ to fit other aspects of a larger data set.

Given this paper's primary goal, it henceforth utilizes Formulation I, which is simpler. If we estimate $\gamma = 0$, then future work will turn to potentially richer, non-separable specifications.

Couples. Empirical prevalence leads us to focus on couples rather than single–adult households. We consider two cases. Our empirical work at this point, however, focuses on the first.

<u>No Retirement Complementarity.</u> In one specification, spouses vicariously benefit from each others utility but their household does not gain additional utility when both are retired together.

Returning to Formulation I, let the male's gain from retirement be Γ^m and the female's Γ^f. To take into account the idea that two adults may be able to live more cheaply than two singles, let a couple correspond to v_t "equivalent adults" (e.g., Tobin [1967]). Let $y^f(t\,,\, R^f)$ be the wife's earnings, which are 0 for $t \geq R^f$ (and, perhaps, at other ages). Similarly, let $y^m(t\,,\, R^m)$ be the husband's earnings at household age t. For concreteness, think of the household's age as the husband's age.

First, for a given R^f, think of the household as solving

$$\max_{c_t\,,\, R = R^m} \int_0^R e^{-\rho \cdot t} \cdot \frac{v_t}{2} \cdot \left[u(\frac{c_t}{v_t}\,,\, t, R) + u(\frac{c_t}{v_t}\,,\, t, R^f)\right] dt + \varphi(a_R\,,\, R) \tag{8}$$

$$\text{subject to:} \quad \dot{a}_t = r \cdot a_t + y^m(t\,,\, R) + y^f(t\,,\, R^f) - c_t\,,$$

$$a_0 = 0\,,$$

where

$$\varphi(A\,,\, R) \equiv \max_{c_t} \int_R^T e^{-\rho \cdot t} \cdot \frac{v_t}{2} \cdot \left[u(\frac{c_t}{v_t}\,,\, t, R) + u(\frac{c_t}{v_t}\,,\, t, R^f)\right] dt\,, \tag{9}$$

$$\text{subject to:} \quad \dot{a}_t = r \cdot a_t + y^m(t\,,\, R) + y^f(t\,,\, R^f) - c_t\,,$$

$$a_T \geq 0 \,,$$

$$a_R = A \,.$$

The solution has

$$c_t = \frac{e^{\nu \cdot t} \cdot v_t \cdot [Y^m(R^m) + Y^f(R^f)]}{\int_0^T v_s \cdot e^{-r \cdot s} \cdot e^{\nu \cdot s} \, ds} \,, \tag{10}$$

$$\frac{y_R^m}{[c_R]^{1-\gamma}} = \Gamma^m \quad \text{for} \quad R = R^m \,. \tag{11}$$

Solving a symmetric problem (8)-(9) for $R = R^f$, condition (10) remains the same but

$$\frac{y_R^f}{[c_R]^{1-\gamma}} = \Gamma^f \quad \text{for} \quad R = R^f \tag{12}$$

replace (11).

Given (10), condition (11) implies a "reaction function" $R^m = R^m(R^f)$ showing the husband's optimal retirement age conditional on his retiring at age R^f. Condition (12) implies a second reaction function $R^f = R^f(R^m)$ for the wife.

Figure 1A presents an example.[4] If the reaction–function graphs cross at (R^{m*}, R^{f*}), then we expect to see the husband retiring at age $R^m = R^{m*}$. Figure 1B illustrates an outcome in which $R^{f*} = 0$ and $R^{m*} > 0$ at the optimal–choice point. In the latter case, the wife chooses never to engage in market work.

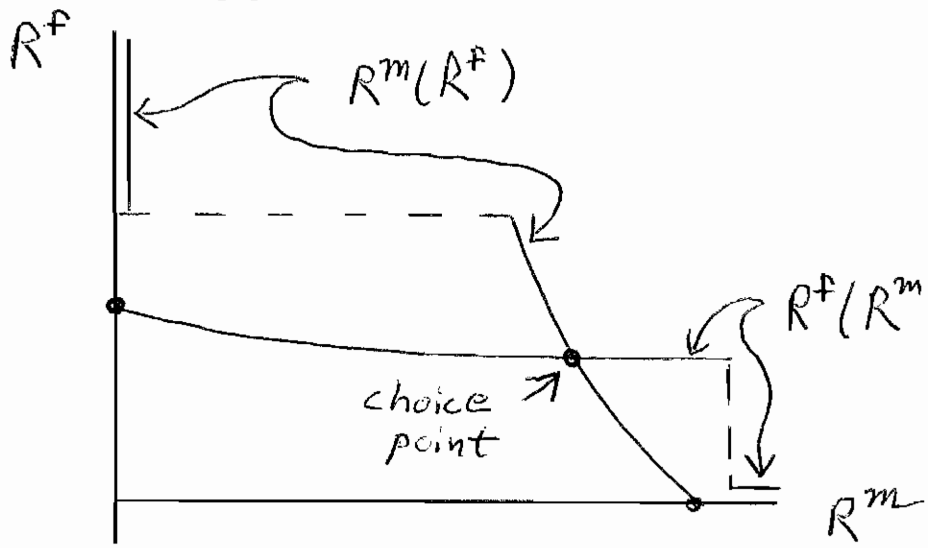

Figure 1A: Reaction functions $R^m(R^f)$ and $R^f(R^m)$

[4] Low earnings in youth may lead a spouse either to never work, or to work a fairly substantial number of years — creating discontinuities in the reaction function graphs, as shown.

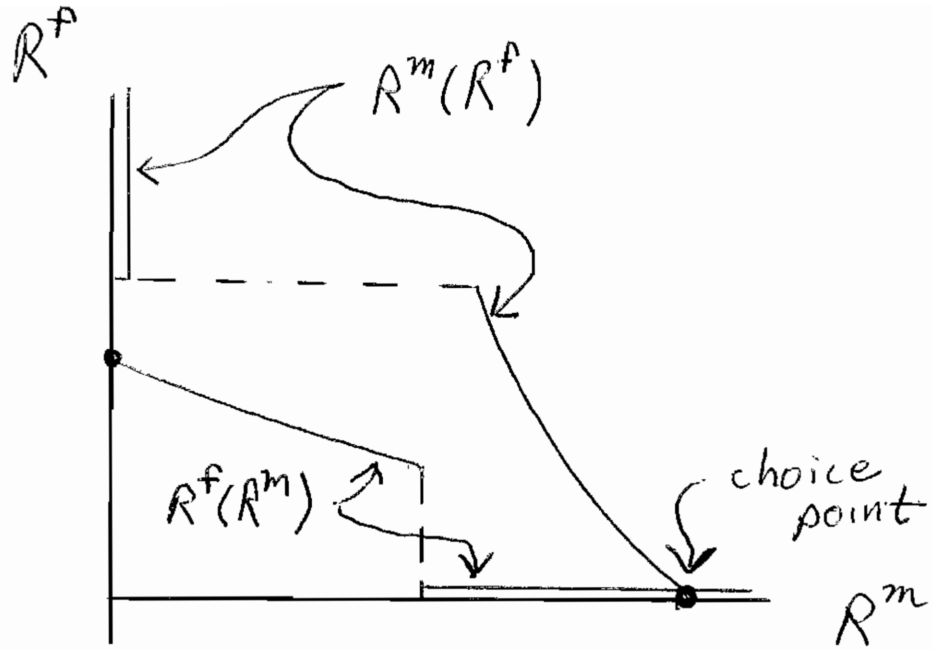

Figure 1B: Reaction functions with an intersection having $R^f = 0$

Retirement Complementarity. It seems possible that a household can benefit most from retirement when both partners have stopped working. For example, a fully retired household might be free to move to an area with a favorable climate or nearby golf course.[5] Our framework could incorporate this by including an addition to household utility, say, Γ^c, at the retirement of the second spouse. The consumption equation remains as above. The free endpoint conditions and reaction–function analysis, however, would become more elaborate.

This paper focuses on a specification without retirement complementarity; formulations with complementarity remain a topic for future research.

Children. As in Tobin [1967], we can extend the definition of v_t to include additions when children are members of their parents' household — say, when children are ages 0-20.[6]

3. Empirical Analysis

Combining (6) with (10)-(11), our basic equation for statistical analysis of $R = R^m$ is

$$\ln(y_R^m) - (1-\gamma) \cdot [\ln\big(Y^m(R) + Y^f(R^f)\big)] - \\ (1-\gamma) \cdot [\nu \cdot R + \ln(v_R) - \ln\Big(\int_0^T v_s \cdot e^{-r \cdot s} \cdot e^{\nu \cdot s}\, ds\Big)] - \ln(\Gamma^m) = \tilde{\eta}\,. \qquad (13)$$

[5] See, for example, Hamermesh [2005].

[6] See, for example, Laitner and Silverman [2005].

If all households have the same utility function, then differences in male age–earning profile shapes (which determine y_R^m), in male and female lifetime earnings, and in family composition profiles (which determine $v_s\,, 0 \leq s \leq T$) would determine interhousehold differences in $R = R^m$. We might set $\tilde{\eta} = 0$, or we might assume that $\tilde{\eta}$ is a random variable (with mean 0) that reflects measurement error on the left–hand side of (13). Alternatively, we might assume that heterogeneity of preferences sets the right–hand side of (13) — in the sense that household i derives pleasure $\Gamma^m \cdot e^{\eta_i}$ from male retirement, with η_i an *iid* sampling from a random variable $\tilde{\eta}$ with mean 0. In general, this paper adopts the latter assumption.

Our focus is γ. Accordingly, at this stage we calibrate $\nu = 0.0273$ on the basis of Laitner and Silverman [2007].[7] And, we treat the left–hand side terms in (13) not varying with retirement age or earnings as a constant α:

$$\alpha \equiv -\big[(1-\gamma)\cdot[\ln(v_R) - \ln\big(\int_0^T v_s \cdot e^{-r\cdot s}\cdot e^{\nu\cdot s}\,ds\big)] - \ln(\Gamma^m)\big]\,.$$

In practice, of course, different households have different numbers of children and different timing of marriage and fertility. Although our data set has the virtue of measuring such factors, we leave their inclusion for future work. Our version of (13) is

$$\ln(y_R^m) - (1-\gamma)\cdot[\ln\big(Y^m(R) + Y^f(R^f)\big)] - (1-\gamma)\cdot\nu\cdot R + \alpha = \tilde{\eta}\,. \tag{14}$$

Although one could think of (14) as implicitly determining $R = R^m$ and then attempt to apply nonlinear least squares techniques, we estimate first–order condition (14) directly, using method of moments estimation. Thinking of the left–hand side of (14) as $q_i(\alpha\,,\gamma)$ and of the sample size as I, we estimate $(\alpha\,,\gamma)$ from a set of moment equations

$$\frac{1}{I}\cdot\sum_{i=1}^{I} q_i(\alpha\,,\gamma)\cdot\vec{Z}_i = 0\,, \tag{15}$$

where $\vec{Z}_i$ is a vector of instruments. Since we assume $E[\tilde{\eta}] = 0$, the first component of $\vec{Z}_i$ can be 1 — in other words,

$$\frac{1}{I}\cdot\sum_{i} q_i(\alpha\,,\gamma)\cdot 1 = 0\,.$$

To estimate two parameters, however, we need at least one more instrument. Choosing $\ln(y_R^m)$ or $Y^m(R)$ would be unsatisfactory because both depend on R, which (implicitly) is the endogenous variable in our equation. Our choice for a second instruement below is an index e^{μ_i} of the earning ability of the male in household i. Although we must estimate μ_i, our data set has 20 or more observations of male earnings for most households. We use

$$\vec{Z}_i \equiv \begin{pmatrix} 1 \\ E[\mu_i] \end{pmatrix} \quad \text{or} \quad \vec{Z}_i \equiv \begin{pmatrix} 1 \\ E[e^{\mu_i}] \end{pmatrix}\,. \tag{16}$$

[7] The latter estimate comes from Consumer Expenditure Survey data on consumption.

Our second instrument is hardly unassailable. One might think, for instance, of each young household as having a vector of characteristics $(\eta_i \, , \, \mu_i \, , \, \eta_i^f \, , \, \mu_i^f \, , \, \kappa_i)$, where η_i reflects male taste for leisure, μ_i male earning ability, η_i^f female taste for leisure, μ_i^f female earning ability, and κ_i taste for children (i.e., desired number of children). There is a distribution of this vector in the population. We can assume the expected value of the vector in the population is 0 with little sacrifice of generality since our earning dynamics equation and (14) include unrestricted constants. The second moments of the vector are a different matter, however. If $e[\eta_i \cdot \mu_i] = 0$ (or $E[\eta_i \cdot e^{\mu_i}] = 0$) this paper's second instrument is valid. Otherwise, it is not. Future work will investigate alternative instruments.

4. Data

The data set that we use is the Health and Retirement Study (HRS).[8] In addition to demographic information, the HRS provides panel data on the retirement choices of older Americans. Further, it has lifetime earning records in the form of linked annual Social Security earning data for many of its men and women.

Male Retirement. The HRS asks men and women twice whether they are retired and what year they retired. We use the questions in sequence. If either says "retired," we set the individual's R to the minimum of the listed year and the current year. We also check annual market–work hours in each survey wave.If a retired male subsequently works more than 1500 hours in a year, we drop his household from our sample — assuming that he retired but then changed his mind and returned to work.[9]

Male Earnings. The linked Social Security annual earning figures have the virtue of comprehensiveness — annual records go all the way back to 1951 — and of administrative–record quality. Their disadvantages include right censoring at the Social Security earnings cap prior to 1981, and censoring at 100000, 250000, or 500000 thereafter (for reasons of confidentiality); lack of records for non–FICA jobs prior to 1981; and, lack of measurements of work hours. (After 1992, we have bi-annual HRS survey data, including both earnings and hours.) An additional potential problem, not peculiar to the HRS, is that earnings immediately prior to retirement might reflect shortened work hours or a period of disinvestment in human capital in anticipation of retirement.

We proceed as follows. We estimate an earnings dynamics equation for males:

$$\ln(y_{it}^m) = X_{it} \cdot \beta + \mu_i + \epsilon_{it} \, , \tag{17}$$

where X_{it} contains a quadratic in experience and time dummies (reflecting technological progress); the first error component, μ_i, is a random individual effect; ϵ_{it} is an annual, white–noise error that is independent of the individual effect; and, $(\mu_i \, , \, \epsilon_{it})$ is bivariate normal. We estimate (17) separately for high school and college graduates. We use maximum likelihood estimation (MLE). We know which observations are censored, and our estimation can take that into account as follows. Let

[8] This paper uses HRS surveys from 1992-2002.

[9] See, for example, Maestas [2004].

$$e_{it} = e_{it}(\beta) \equiv \ln(y^m_{it} - X_{it} \cdot \beta)\,,$$

let observations at times s be uncensored, and let observations at times t be censored. Then we solve

$$\min_{\beta} \{-\ln(\mathcal{L})\}\,,$$

$$\ln(\mathcal{L}) \equiv \sum_i \ln(P_i)\,,$$

$$P_i \equiv \int_{-\infty}^{\infty} p_i(\mu_i)\, d\mu_i\,,$$

$$p_i(\mu_i) \equiv [\prod_s \phi(e_{is} - \mu_i\,,\, h_\epsilon)] \cdot [\prod_t \int_{e_{it}-\mu_i}^{\infty} \phi(\xi\,,\, h_\epsilon)\, d\xi] \cdot \phi(\mu_i\,,\, h_\mu)\,, \tag{18}$$

where i refers to households and $\phi(.,h)$ to the normal density function with precision h. We want (18) to reflect full-time earnings, so we exclude observations with less than 4 quarters of Social Security earning credit for the year, with an earning amount less than 1500 annual hours times the minimum wage, or within two years of starting work or retiring.[10] In a second approach, we additionally exclude observations (other than the first or last) adjoining a blank — the worry being that a male might, for example, have taken a non–FICA job and started or finished that job in the middle of a year — and males with less than 10 earning observations. Table A1 in the Appendix presents details on our sample size; Table A2 presents our earning–dynamics regression results.

Our next step predicts male earnings at every working age from (17). This has the advantage of overcoming censoring, imputing earnings from non–FICA jobs, and avoiding potential understatement of earning ability immediately prior to retirement (see the warning above). It has the potential disadvantage of overstating earnings during periods of unemployment or part-time work. Our measure of $Y^m(R^m)$ is the present value, with interest rate $r = 0.04$, at age 50 of male earnings between the age of starting work and R^m.

In fact, by no means all males reach retirement in the survey data. We adapt our first–order condition to encompass this below. Furthermore, we write $Y^m(S)$ to mean the present value at age 50 of lifetime male earnings if the male reaches retirement in the survey, or earnings up to the maximum age the male attains in the survey data if the latter is short of his retirement age. Similarly for y^m_S and $Y^f(S)$.

The correction for censoring seems to make a substantial difference (although our "tighter" samples, somewhat surprisingly, do not). In the high school educated sample, out of 11,620 observations, 3564 are censored and our imputations are higher in 2663 cases.[11] The average increase for the prediction for the 3564 censored figures is about 15

[10] We assume a male with ED years of education starts work at age $= \min\{ED+6, 18\}$.

[11] Sampling variation means that not all of the predicted values are higher than the corresponding observation.

percent. Average male lifetime earnings up to retirement — or the last age observable in the survey — in present value at age 50, is $1,823,000. (We use HRS household weights for all averages.) The tighter sample generates average total earnings of $1,820,000. For college–educated males, there are 3039 censored observations among the 7709 total. Imputations are higher for 2722 cases. Comparing predictions to the original 3039 censored cases, we find almost a 39 percent increase. Average present value male earnings are $2,634,000 for this group. For the tighter sample of college males, the predicted average present value of $Y^m(S)$ is $2,718,000.

Women's Earnings. We do not filter female earnings below the minimum wage, with fewer than four Social Security quarters per year, or early or late in career. Thus, for the basic sample of high school educated males, we have 9551 spousal earnings observations — with positive female earnings for 408 households. For the college–male sample, we have 7152 spousal earning observations — with positive spousal earnings for 330 households. Censoring is much less prevalent: among spouses of high–school educated males, 272 observations are censored; among spouses of college–males, 333. In the former sample, the present value at age 50 (for the husband) of average spousal lifetime earnings is $305,000; in the second sample, the average is $352,000. These are substantial sums, but they are small relative to male totals. Evidently, women's earnings play a larger role for the households with high school educated males. We do not want to impute women's earnings for years with none reported, as many women in this generation did not work every year. In the end, we simply compute the present value (at husband's age 50) of lifetime female earnings using observed data points alone.

Distribution of Household Earnings. To estimate (14), we replace $\ln(y_R^m)$ with $E[\ln(y_{iS}^m)]$ where the role of S is described above. We use our earnings dynamics equation (17), disregarding the short–term shocks ϵ_{iS}. Then with the notation of (18),

$$E[\ln(y_{iS}^m)] = X_{iS} \cdot \beta^m + E[\mu_i] = X_{iS} \cdot \beta^m + \frac{\int_{-\infty}^{\infty} \mu_i \cdot p_i(\mu_i)\, d\mu_i}{P_i} . \tag{19}$$

We construct $\overline{Y}^m(S)$ from the present value at each age t between starting work and S of $X_{it} \cdot \beta^m$. As always, compute the present value as of male age 50. Then

$$E[Y^m(S)] = \overline{Y}^m(S) \cdot E[e^{\mu_i}] = \overline{Y}^m(S) \cdot \frac{\int_{-\infty}^{\infty} e^{\mu_i} \cdot p_i(\mu_i)\, d\mu_i}{P_i} . \tag{20}$$

Returning to (14), let $Y^{post}(S)$ be the present value at age 50 of male earnings after retirement. (If the male does not reach retirement in the survey, this is 0. Otherwise, we take the earnings figures directly from the data, not using the earnings dynamics equation in this case.) Then

$$E[\ln\big(Y^m(S) + Y^{post}(S) + Y^f(S)\big)] =$$

$$\frac{\int_{-\infty}^{\infty} \ln\big(\overline{Y}^m(S) \cdot e^{\mu_i} + Y^{post}(S) + Y^f(S)\big) \cdot p_i(\mu_i)\, d\mu_i}{P_i} . \tag{21}$$

Finally, our second instrument comes from

$$E[e^{\mu_i}] = \frac{\int_{-\infty}^{\infty} e^{\mu_i} \cdot p_i(\mu_i)\, d\mu_i}{P_i}\,. \tag{22}$$

Table A3 in the Appendix provides summary descriptions of key variables.

5. Estimation

We first describe the remainder of our estimation procedure. Then we present outcomes.

A complication in our estimation arises from the fact that not all males reach retirement within the sample time frame. In one instance, a man becomes disabled and leaves work at age prior to his optimal retirement age R^m as computed in our model.[12] In a second instance, a man dies before retiring. In a third, our last survey occurs before a man reaches retirement. We call these cases with $S < R^m$.[13] In the notation of (14)-(15), when $S < R^m$ for household i, we assume that $q_i = q_i(\alpha\,,\,\gamma)$ provides an upper bound for η_i. In other words, we assume that the male has less desire to retire than the level which would have induced his retirement prior to disability, death, or the last survey. Assuming $\tilde{\eta}$ is $N(0\,,\,\sigma^2)$, for households with $S < R^m$ we have

$$E[\eta_i] = \frac{\int_{-\infty}^{q_i} \eta \cdot \phi(\eta\,,\,h)\, d\eta}{\int_{-\infty}^{q_i} \phi(\eta\,,\,h)\, d\eta} \tag{23}$$

where $\phi(.,h)$ is the normal density function and h is the precision, $h = 1/\sigma$.

To implement (23), we need to estimate σ. For households with $S < R^m$, we have

$$E[\eta_i \cdot \eta_i] = \frac{\int_{-\infty}^{q_i} \eta^2 \cdot \phi(\eta\,,\,h)\, d\eta}{\int_{-\infty}^{q_i} \phi(\eta\,,\,h)\, d\eta}\,. \tag{24}$$

Our complete estimation is then as follows. Define $S \equiv \min\{R^m\,,\,\text{last sample age}\}$ and define

$$\begin{aligned} q_i(\alpha\,,\,\gamma\,,\,\sigma) \equiv\; & E[\ln(y^m_S)] - (1-\gamma)\cdot E[\ln(Y^m(S) + Y^{post}(S) + Y^f(S))] \\ & - (1-\gamma)\cdot \nu \cdot (S - 50) + \alpha\,. \end{aligned} \tag{25}$$

We derive our estimate $(\widehat{\alpha}\,,\,\widehat{\gamma}\,,\,\widehat{\sigma})$ by finding $(\alpha\,,\,\gamma\,,\,\sigma)$ that satisfies three moment equations. Let

[12] If a male classifies himself as "disabled," and if the year in which he became disabled preceded his retirement, we assume $S < R^m$.

[13] A buyout offer could induce a similar phenomena, and future drafts will make use of HRS data on such offers – see, for example, Brown [2002].

$$q_i^*(\alpha\,,\,\gamma\,,\,\sigma) \equiv \begin{cases} q_i(\alpha\,,\,\gamma\,,\,\sigma)\,, & \text{if } S \geq R_i^m\,, \\ E[\eta_i] \text{ as in } (23)\,, & \text{if } S < R_i^m\,. \end{cases}$$

Then the first two moment equations are

$$\frac{1}{I} \cdot \sum_i q_i^*(\alpha\,,\,\gamma\,,\,\sigma) \cdot \vec{Z}_i = 0\,, \tag{26}$$

where $\vec{Z}_i$ is as in (16). Let

$$q_i^{**}(\alpha\,,\,\gamma\,,\,\sigma) \equiv \begin{cases} q_i(\alpha\,,\,\gamma\,,\,\sigma) \cdot q_i(\alpha\,,\,\gamma\,,\,\sigma)\,, & \text{if } S \geq R_i^m\,, \\ E[\eta_i \cdot \eta_i] \text{ from } (24)\,, & \text{if } S < R_i^m\,. \end{cases}$$

Then the third moment equation is

$$\frac{1}{I} \cdot \sum_i [q_i^{**}(\alpha\,,\,\gamma\,,\,\sigma) - \sigma^2] \cdot 1 = 0\,. \tag{27}$$

Table 1 below presents results. For the standard and tight samples of households with both high school and college–graduate male heads, each $\widehat{\gamma}$ is less than 0, though greater than -0.5. This is true for either set of instruments in (16). The T–statistics are large.

The estimates suggest that higher earnings induce somewhat earlier retirement. The implied effect is stronger in the samples of households headed by high school graduates.

This is preliminary work, however. Further steps will add a number of additional covariates. Several reasons to anticipate that final estimates of γ might be even closer to 0 are: (i) correcting earnings for income taxes will tend to lower y_S^m by a household's marginal tax rate and $Y^m(S) + Y^{post}(S) + Y^f(S)$ by its average tax rate — with the marginal rate quite possibly tending to be higher relative to the average rate for higher earners; and, (ii) higher earning households may tend to have fewer children — which will affect (13) if we stop simplifying through (14). As noted, we need to investigate alternatives to the second instrument in $\vec{Z}_i$ as well.

6. Conclusion

We have set up a nonstochastic life–cycle model of household behavior and derived a first–order condition for optimal retirement. Using HRS panel data on older married couples, including lifetime Social Security earning records for both men and women, we have derived method of moments estimates of several key parameters. With an additively separable utility function, we can estimate the additional utility flow accruing to males after retirement. More important, if we assume that utility is isoelastic in consumption, the sign of the corresponding exponent parameter predicts whether households with higher earnings will tend to retire earlier or later. Zero is the borderline case — implying an optimal retirement age independent of earning level. At this point, all of our results point toward an isoelastic parameter near zero but negative. A negative parameter implies

Table 1. Estimated Coefficients for Equations (26)-(27): Health and Retirement Study Data 1992-2002[a]				
Parameter	High School Males		College Males	
	Basic Sample	Tight Sample	Basic Sample	Tight Sample
$\vec{Z}_i^T = (1, E[\mu_i])$				
α (S.E.) [T Stat.]	11.4824 (1.2168) [9.4365]	10.8708 (1.2526) [8.6785]	7.4765 (0.6460) [11.5733]	7.9573 (0.7232) [11.0023]
γ (S.E.) [T Stat.]	-0.4633 (0.08208) [-5.6446]	-0.4225 (0.08449) [-5.0008]	-0.2094 (0.04260) [-4.9165]	-0.2406 (0.04755) [-5.0603]
σ (S.E.) [T Stat.]	0.3196 (0.02527) [12.6479]	0.2975 (0.02643) [11.2561]	0.2718 (0.01938) [14.0226]	0.2796 (0.02114 [13.2260]
$\vec{Z}_i^T = (1, E[e^{\mu_i}])$				
α (S.E.) [T Stat.]	11.2557 (1.1452) [9.8286]	10.5679 (1.1610) [9.1095]	6.7144 (0.6138) [10.9389]	7.2246 (0.6893) [10.4803]
γ (S.E.) [T Stat.]	-0.4480 (0.07725) [-5.7995]	-0.4020 (0.07825) [-5.1378]	-0.1589 (0.04060) [-3.9147]	-0.1922 (0.04546) [-4.2280]
σ (S.E.) [T Stat.]	0.3160 (0.02408) [13.1223]	0.2927 (0.02481) [11.7952]	0.2592 (0.01665) [15.5650]	0.2680 (0.01843) [14.5401]
Sample Size				
DFE	406	377	328	289

a. See text.

higher earnings tend to lead to earlier optimal retirement age. There are, however, many interesting steps remaining for the future, and the text outlines how we anticipate that we will proceed.

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Appendix

Table A1. HRS Sample Size		
Criterion	Male ED Years 12	Male ED Years 16-17
Household married only once	2734	2734
Male meets ED criterion	878	670
Spousal age difference ≤ 10	848	649
Number kids ≤ 10	846	649
Male birth year 1926-43	831	634
Both spouses linked SSA records	566	447
Adequate male retirement data	565	447
Retired male never returns to work	454	361
Woman does not work prior 1950	438	353
Male retirement age not < 52 or > 71	412	333
Households with male earnings	411	330
Male earnings observations	11620	7706
Tighter Sample (drop male earnings adjacent to blank; 10 or more figures per male)[a]		
Households in sample	380	294
Male earnings observations	9978	6596

a. See text.

Table A2. HRS Earnings Regressions: Male ED Years 12		
Parameter	Regular Sample	Tighter Sample[a]
CONSTANT	9.1544 (0.02954)	9.1820 (0.03014)
EXP	0.04553 (0.004924)	0.03423 (0.004932)
EXP**2/100	-0.06467 (0.005255)	-0.05482 (0.005412)
DUMMY 1951-60	0.000144 (0.006187)	0.005393 (0.006365)
DUMMY 1961-65	0.03892 (0.006408)	0.04545 (0.006591)
DUMMY 1966-70	0.05017 (0.006672)	0.06410 (0.006921)
DUMMY 1971-75	-0.009391 (0.006235)	0.000102 (0.006313)
DUMMY 1976-80	-0.03517 (0.005755)	-0.02713 (0.005636)
DUMMY 1981-85	-0.005861 (0.005481)	0.002436 (0.005364)
DUMMY 1986-90	-0.02082 (0.005654)	-0.01667 (0.005489)
DUMMY 1991-95	0.008214 (0.007107)	0.005728 (0.006829)
DUMMY 1996+	0.03297 (0.01009)	0.03953 (0.009567)
PRECISION h_ϵ	2.7978 (0.02306)	2.9907 (0.02606)
PRECISION h_μ	2.8770 (0.1077)	3.1106 (0.1202)
Summary Statistics		
DFE	11606	9964
-ln(Likelihood)	5,576	4,376

a. See text and Table 1.

Table A2. HRS Earnings Regressions (cont.): Male ED Years 16-17		
Parameter	Regular Sample	Tighter Sample[a]
CONSTANT	9.4959 (0.06844)	9.5520 (0.07560)
EXP	0.0371 (0.008198)	0.03800 (0.008801)
EXP**2/100	-0.05846 (0.009326)	-0.05472 (0.009729)
DUMMY 1951-60	0.01122 (0.01586)	-0.006491 (0.01796)
DUMMY 1961-65	0.06665 (0.01229)	0.06867 (0.01370)
DUMMY 1966-70	0.07328 (0.01243)	0.07550 (0.01382)
DUMMY 1971-75	-0.003701 (0.01100)	-0.000185 (0.01191)
DUMMY 1976-80	-0.05524 (0.009030)	-0.05733 (0.009515)
DUMMY 1981-85	0.01592 (0.008025)	0.01786 (0.008531)
DUMMY 1986-90	0.004728 (0.008093)	0.000344 (0.008546)
DUMMY 1991-95	0.02648 (0.009430)	0.01825 (0.009733)
DUMMY 1996+	0.01445 (0.01195)	0.01156 (0.01204)
PRECISION h_ϵ	2.6634 (0.02895)	2.7875 (0.03202)
PRECISION h_μ	1.9494 (0.07996)	1.9495 (0.08442)
Summary Statistics		
DFE	7692	6582
-ln(Likelihood)	3,461	2,851

a. See text and Table 1.

Table A3. Sample Characteristics[a]				
Statistic	$E[\mu]$	$E[e^{\mu}]$	$E[Y^m(S)]$	$Y^f(S)$
High School Male				
Minimum	-1.0258	0.36	519,000	0
Lower Quartile	-0.2492	0.77	1,116,000	125,000
Median	-0.03420	0.97	1,392,000	261,000
Upper Quartile	0.2179	1.25	1,791,000	429,000
Maximum	1.4194	4.16	5,570,000	1,413,000
Mean	-0.02069	1.04	1,473,000	302,000
College Male				
Minimum	-1.3174	0.27	546,000	0
Lower Quartile	-0.3896	0.68	1,315,000	147,000
Median	-0.03911	0.97	1,821,000	290,000
Upper Quartile	0.1969	1.22	2,349,000	473,000
Maximum	1.8011	6.12	11,658,000	1,414,000
Mean	-0.04365	1.11	2,157,000	353,000

a. See text. Basic samples only.

Table A3. Sample Characteristics (cont.)[a]				
Statistic	$Y^{post}(S)$	$E[\ln(Y^m(S) + Y^{post}(S) + Y^f(S))]$	$E[\ln(y_S^m)]$	$E[y_S^m]$
High School Male				
Minimum	0	13.51	9.06	9000
Lower Quartile	0	14.14	9.87	19,000
Median	0	14.36	10.11	25,000
Upper Quartile	7000	14.55	10.34	31,000
Maximum	160,000	15.59	11.49	99,000
Mean	7000	14.35	10.11	26,000
College Male				
Minimum	0	13.40	9.49	13,000
Lower Quartile	0	14.34	10.39	33,000
Median	0	14.63	10.70	45,000
Upper Quartile	30,000	14.85	10.96	58,000
Maximum	1,739,000	16.27	12.54	283,000
Mean	32,000	14.63	10.72	52,000

a. See text. Basic samples only.