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Home Production of Dual Earner Couples and Consumption During Retirement

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Abstract

To study the role of home production in life—cycle behavior, this paper creates a theoretical model in which both spouses in a couple allocate their time between market and home work. It then derives a pair of regression equations for estimating the parameters of the model, and it carries out the estimation using panel data on household net worth and lifetime earnings from the Health and Retirement Study and pseudo—panel data on household consumption expenditures from the Consumer Expenditure Survey. We estimate that the value of forgone home production is roughly 10-15 cents for every dollar that a married man earns, but 30-35 cents per dollar of married women's market earnings. Our findings imply male labor supply elasticities that are very near zero and female elasticities in the range of 0.50. Our model predicts a substantial decline in measured consumption expenditure at a household's retirement, and it shows that Euler—equation models of consumption behavior should include terms reflecting home production.

1 Introduction

Although home production of child care, cooking, residence maintenance, etc., is a potentially significant part of total economic activity, conventional surveys seldom try to measure its value — nor do the National Income and Product Accounts endeavor to do so. Some analyses propose to circumvent the lack of measurements by valuing a household's entire time budget at market wages rates. A person's current wage may, however, fail to provide a good indication of the average value of his or her time. The present paper attempts a new evaluation of home production, including comparison of men's and women's contributions, as follows: it develops a dynamic, theoretical life–cycle framework in which both spouses in a couple allocate their time between market and home production, with the costs and benefits of different allocations explicitly specified; on the basis of the theoretical framework, it derives regression equations for estimating the parameters of the model; and, it then carries out the estimation using microeconomic data on household net worth and lifetime earnings from the Health and Retirement Study (HRS) and data on household consumption expenditure from the Consumer Expenditure Survey (CEX).

Recent work by House, Laitner, and Stolyarov (HLS) [2006] employs HRS data to try to quantify the value of home production that married households lost in the course of recent increases in female labor force participation. HLS argue that a "traditional" household in which the male works in the market and the female works at home would have more "real income" than a "modern" household with the same market earnings but with both spouses participating in the labor market. The intuitive idea is that as a woman participates in the labor market, her household loses her corresponding hours of home production. HLS attempt to quantify the value of the loss indirectly, using HRS data on household net worth at retirement. In their formulation, all households have full-time home production after retirement, but a household that sacrificed home production prior to retirement should have a lower standard of living relative to its measured, market earnings and, hence, should have saved less for retirement in proportion to its measured lifetime wage and salary income.

While HLS assume that men work in the market full time prior to retirement, the present paper adopts a more symmetric setting in which both men and women divide their time between market and home production. With the new model, one could, in principle, estimate the value of forgone home production for both sexes from the data in HLS. We show this below. Such an approach, however, places very heavy demands on the net worth and earnings data — and, in practice, large standard errors emerge. The present paper develops a two–equation formulation for statistical analysis, which brings expenditure data from the CEX into the analysis.

This paper generalizes the theoretical framework of HLS [2006]. The new model determines one regression equation that, as before, relates a household's net worth to its lifetime market earnings. As in HLS, we utilize HRS lifetime data on individual households to estimate this equation. One of the unusual strengths of the HRS is its inclusion of Social Security–record measurements of annual earnings for both men and women. Although this data does not report work hours, our analysis (like that of HLS), surprisingly, does not require such information.

Our theoretical model also determines a second equation for statistical analysis. The

new condition relates a household's market expenditures in one year to those in the next. Consumption expenditure has been comparatively difficult for surveys to measure, and the BLS Consumer Expenditure Survey has provided virtually the only comprehensive disaggregative data. Nevertheless, by construction, the CEX does not provide observations on individual households. However, the CEX sample is so large that we can extract average consumption for married households of individual ages, for individual years. This paper uses the CEX data to construct a pseudo panel — i.e., to construct measures of changes in expenditure between an average household of age s at time t and an average household at age $s+1$ in year $t+1$, where the latter (average) household represents a one–year–older version of the first. We estimate our two equations simultaneously, taking into account cross–equation parameter restrictions from the theory.

Our results suggest that the value of forgone male home production is roughly 15 cents per dollar of male earnings, while the value of forgone home production per dollar of female earnings is 30-35 cents. Our findings add support to the original estimates of the value of forgone home production in HLS, though the new estimates are somewhat higher. They show that male losses per dollar of earnings are noticeably smaller. Moreover, the estimates imply that total household expenditures on replacements for lost home production are large in aggregate. In terms of the well–being of older citizens, our results suggest that simple comparisons of income flows before and after retirement can be misleading — because retirees have much more time for home production.

Our results also enable us to estimate male and female labor supply elasticities. The female elasticity is appreciably larger than the male. Existing papers suggest that home production may be an important element in explaining changes in employment over the business cycle (e.g., Rupert *et al.* [1995] McGrattan *et al.* [1997], and Benhabib *et al.* [1991]), for example, and our elasticity estimates may be able to contribute to the literature.

Since the majority of males work standard hours until retirement, our empirical analysis of men relies on comparing household expenditures from ages before retirement to ages after. A number of recent studies find significant drops in household expenditure at, and after, retirement (e.g., Banks *et al.* [1998], Bernheim *et al.* [2001], Hurd and Rohwedder [2003], Laitner and Silverman [2005], and Aguiar and Hurst [2005]). The literature considers a number of possible explanations: insufficient household preparation for retirement (e.g., Bernheim *et al.*), smoothing of service flows from consumption and leisure after expansion of the latter upon retirement (e.g., Laitner and Silverman), and greater availability of time for home production after retirement (e.g., Augiar and Hurst). The present paper has the most in common with the last hypothesis. Our model predicts a decline in expenditures upon retirement. Indeed, it assumes that any decline observable in the data stems from opportunities after retirement for males and females to expand their home production. The apparent plausibility of our parameter estimates conditional on this assumption may be deemed to support the assumption. What is more, we hope that the explicit grounding of our analysis in economic theory will suggest avenues in the future for testing the relative importance of competing explanations for household expenditure changes at retirement.

The second equation in our statistical specification has a familiar form — namely, it is a microeconomic version of the well–known Euler equation of Hall [1978] and many

others (e.g., Hansen and Singleton [1982]). Our specification differs from most in the existing literature. In particular, our model distinguishes between market expenditures on consumption goods (and services) and total consumption, with the latter additionally including home production. The distinction leads to a statistical model with male and female earnings terms, which are atypical for the literature, as well as market expenditure terms, which are standard.

The organization of this paper is as follows. Section 2 presents our basic model. Section 3 describes our data. Sections 4-5 present the empirical analysis and interpret the results. Section 6 concludes.

2 Model

We consider a life–cycle model of a dual–earner household. The household can produce a home good with male and/or female time and can also purchase market goods, and market goods can substitute for home good. The model's basic structure follows HLS [2006]; however, the present paper extends home–production options to males as well as females.

Each household seeks to maximize its discounted lifetime utility. For household i, let h_{is}^m be male market hours at (household) age s and w_{is}^m the net–of–tax market wage, let h_{is}^f be female market hours at (household) age s and w_{is}^f the female's net–of–tax wage, let x_{is} be market expenditure, let N_{is} be number of "equivalent adults" (see below), let S_i be household's the starting age, let T be its terminal age, let r be the net–of–tax real interest rate, let a_{is} be household net worth (i.e., "assets"), and let c_{is} be household "consumption" at age s. Let R_i be the household's retirement age, assume spouses retire together, and assume that R_i is exogenously given.[1] The household's objective is

$$\max_{c_{is} \geq 0, \, h_{is}^m \geq 0, \, h_{is}^f \geq 0} \int_{S_i}^{T} e^{-\rho \cdot s} \cdot N_{is} \cdot u\Big(\frac{c_{is}}{N_{is}}\Big) \, ds \,, \tag{1}$$

$$\text{subject to:} \quad c_{is} = x_{is} - A_{is}^f \cdot [h_{is}^f]^{\xi^f} - A_{is}^m \cdot [h_{is}^m]^{\xi^m} \,, \tag{2}$$

$$\dot{a}_{is} = r \cdot a_{is} + h_{is}^f \cdot w_{is}^f + h_{is}^m \cdot w_{is}^m - x_{is} \,, \tag{3}$$

$$a_{s,S_i} = 0 \quad \text{and} \quad a_{iT} = 0 \,, \tag{4}$$

$$h_{is}^f = h_{is}^m = 0 \quad \text{all} \quad s \geq R_i \,. \tag{5}$$

When the male or female spouse works h^m or h^f hours in the market, output of the home good is reduced by $A^m \cdot [h^m]^{\xi^m}$ and $A^f \cdot [h^f]^{\xi^f}$, respectively. This paper's formulation assumes that the home good is essential for household operation; consequently, a household must purchase market–good replacements for all reductions in home–good

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[1] HLS [2006] show that exogeneity for R_i is, in general, inessential. See Gustman and Steinmeier [2000] for evidence that couples tend to retire together in practice.

output. Household "consumption," c_{is}, the argument in the household's utility function, equals expenditure, x_{is}, net of the cost of market goods that substitute for lost home–good output,

$$A^f \cdot [h^f]^{\xi^f} + A^m \cdot [h^m]^{\xi^m} .$$

Our treatment of men and women is symmetric; however, we allow arbitrary variation in home productivity — i.e., A_{is}^f and A_{is}^m can differ across households, across time, and between sexes. This paper assumes that the marginal cost of forgone home production is increasing; therefore, we assume

$$\xi^m > 1 \quad \text{and} \quad \xi^f > 1 . \tag{6}$$

Because females exhibit much longer hours of home production in practice, one might expect

$$\xi^m > \xi^f . \tag{7}$$

Household i lives from age S_i to T, and both men and women retire at exogenously specified household age $R_i < T$. A household knows these ages with certainty. As is familiar in the literature, this paper takes $u(.)$ to be isoelastic:

$$u(c) = \begin{cases} \frac{1}{\gamma} \cdot c^\gamma , & \text{for } \gamma < 1 \text{ and } \gamma \neq 0 , \\ \ln(c) , & \text{for } \gamma = 0. \end{cases}$$

We follow Tobin [1967] (and HLS) in constructing the measure of a household's "equivalent adult" membership as

$$N_{is} = 1 + \alpha^S \cdot \chi^S(is) + \alpha^K \cdot \chi^K(is) , \tag{8}$$

where $\chi^S(is)$ is 1 if the household has a spouse present at age s and 0 otherwise, $\chi^K(is)$ gives the number of children (i.e., "kids") present, and α^S and α^K are parameters.

The following two propositions summarize the implications of optimizing behavior:

Proposition 1. *Assume that N_{is}, A_{is}^m, A_{is}^f, w_{is}^m, w_{is}^f, and r are continuous everywhere except a finite set of ages $\{s_j\}_{j=1}^J$. Let h_{is}^m, h_{is}^f, and c_{is} be the optimal solution to (1)-(5). Then for $s \neq s_j$,*

$$A_{is}^m \cdot [h_{is}^m]^{\xi^m} = \frac{1}{\xi^m} \cdot h_{is}^m \cdot w_{is}^m , \tag{9}$$

$$A_{is}^f \cdot [h_{is}^f]^{\xi^f} = \frac{1}{\xi^f} \cdot h_{is}^f \cdot w_{is}^f , \tag{10}$$

$$c_{is} = c_i^* \cdot N_{is} \cdot e^{\frac{r-\rho}{1-\gamma} \cdot s} , \tag{11}$$

$$c_i^* \equiv \frac{\int_{S_i}^T e^{-r \cdot s} \cdot \left[h_{is}^m \cdot w_{is}^m + h_{is}^f \cdot w_{is}^f - A_{is}^m \cdot [h_{is}^m]^{\xi^m} - A_{is}^f \cdot [h_{is}^f]^{\xi^f} \right] ds}{\int_{S_i}^T e^{-r \cdot s} \cdot N_{is} \cdot e^{\frac{r-\rho}{1-\gamma} \cdot s} ds} . \tag{12}$$

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Proof: See Appendix.

Proposition 2.   *Let retirement age R_i be given. Then for any $s \geq R_i$, solution of (1)-(5) implies*

$$\frac{a_{is}}{(1 - \theta^m) \cdot Y_{is}^m + (1 - \theta^f) \cdot Y_{is}^f} = \frac{\int_s^T N_{it} \cdot \sigma^{t-s} \, dt}{\int_{S_i}^T N_{it} \cdot \sigma^{t-s} \, dt}, \tag{13}$$

where

$$\sigma \equiv e^{-r + \frac{r - \rho}{1 - \gamma}}, \quad \theta^m \equiv \frac{1}{\xi^m}, \quad \theta^f \equiv \frac{1}{\xi^f},$$

$$Y_{is}^m \equiv \int_{S_i}^{R_i} e^{-r \cdot (t-s)} \cdot w_{it}^m \cdot h_{it}^m \, dt \quad and \quad Y_{is}^f \equiv \int_{S_i}^{R_i} e^{-r \cdot (t-s)} \cdot w_{it}^f \cdot h_{it}^f \, dt. \tag{14}$$

Proof: See Appendix.

An interpretation of Proposition 1 is as follows. Both men and women face time–allocation decisions at all ages prior to retirement. As men, for example, equate the marginal value of their time in alternative uses, one has

$$\xi^m \cdot A_{is}^m \cdot [h_{is}^m]^{\xi^m - 1} = w_{is}^m \iff A_{is}^m \cdot [h_{is}^m]^{\xi^m} = \frac{1}{\xi^m} \cdot h_{is}^m \cdot w_{is}^m.$$

The right–hand expression is equation (9). Similarly for (10). Condition (11) is a well–known implication of the isoelastic utility function: consumption should grow at a constant rate, faster for a higher interest rate, slower for a higher rate of subjective impatience.

Turning to Proposition 2, the household budget constraint implies that after retirement, the household's assets must cover its expenditures; hence,

$$a_{is} = \int_s^T N_{is} \cdot c_{is} \cdot e^{-r \cdot (t-s)} \, dt \quad \text{any} \quad s \geq R_i. \tag{15}$$

The same accounting implies that over a household's entire life span, the present value of earnings must cover expenditures, which, through (2), include consumption and replacements for lost home production. Thus,

$$Y_{is}^m + Y_{is}^f = \int_{S_i}^T e^{-r \cdot (t-s)} \cdot A_{it}^m \cdot [h_{it}^m]^{\xi^m} \, dt +$$

$$\int_{S_i}^T e^{-r \cdot (t-s)} \cdot A_{it}^f \cdot [h_{it}^f]^{\xi^f} \, dt + \int_{S_i}^T e^{-r \cdot (t-s)} \cdot c_{it} \, dt. \tag{16}$$

Proposition 1 shows that the lifetime value of lost home production is proportional to lifetime earnings — i.e.,

$$\int_{S_i}^{T} e^{-r \cdot (t-s)} \cdot A_{it}^m \cdot [h_{it}^m]^{\xi^m} \, dt = \theta^m \cdot Y_{is}^m \, .$$

And, similarly for females. Equations (15)-(16) determine the numerators and denominators of (13) — after one substitutes from (11).

Propositions 1-2 determine our two estimation conditions. The first resembles the sole estimation condition from HLS. Define $\bar{\theta}$ from

$$1 - \bar{\theta} = \frac{1 - \theta^f}{1 - \theta_m} \, . \tag{17}$$

Then separating a_{is} by itself on the left–hand side of (13), taking logarithms, and appending an error term ϵ_{is}^1, one has

$$\ln(a_{is}) = \ln(1 - \theta^m) + \ln\big(Y_{is}^m + (1 - \bar{\theta}) \cdot Y_{is}^f\big) + \ln\Big(\int_s^T N_{it} \cdot \sigma^{t-s} \, dt\Big) -$$

$$\ln\Big(\int_{S_i}^T N_{it} \cdot \sigma^{t-s} \, dt\Big) + \epsilon_{is}^1 \, . \tag{18}$$

Focus, for a moment, on the error term. Assets and debts are difficult to capture precisely on a survey; so, ϵ_{is}^1 reflects measurement error in a_{is}. If the latter errors are lognormally distributed, ϵ_{is}^1 is normal; and, $E[\epsilon_{is}^1] = 0$ if the (log) survey data are unbiased.

Beyond this, we worry that asset prices rose precipitously in the late 1990s, and fell thereafter — all for reasons beyond the scope of this paper's model. Rather than resort to a very elaborate components–of–error structure for ϵ^1 with separate, aggregative time effects, we introduce time–dummy variables into (18). Our HRS data (see below) covers 1992, 1994, 1996, 1998, 2000, and 2002. Let

$$D^v(is) = \begin{cases} 1 & \text{if } s = v, \\ 0 & \text{otherwise.} \end{cases}$$

Let δ_v^1 be the coefficient on D^v. Assume that the new dummies make no difference to levels in total over all survey waves — i.e., assume

$$\sum_{v=1992,1994,1996,1998,2000,2002} \delta_v^1 = 0 \, .$$

Then we include

$$\sum_{v=1992,1994,1996,1998,2002} \delta_v^1 \cdot [D^v(is) - D^{2000}(is)]$$

on the right–hand side of (18).

Finally, since our HRS data forms a panel with observations for 1-6 waves for each household i, we model the residual error in (18) with

$$\mu_i + \eta_{is} \, ,$$

where μ_i is a random effect peculiar to household i — possibly reflecting household idiosyncrasies not otherwise captured by our model — and η_{is} is purely random. The condition that this paper actually estimates is then

$$q_{is}^1(\vec{\beta}, \vec{\delta^1}) \equiv \ln(a_{is}) - \{\beta_1 + \ln\big(Y_{is}^m + (1 - \beta_2) \cdot Y_{is}^f\big) + \ln\big(\int_s^T N_{it} \cdot [\beta_3]^{t-s} \, dt\big) -$$

$$\ln\big(\int_{S_i}^T N_{it} \cdot [\beta_3]^{t-s} \, dt\big) + \sum_{v=1992,\ldots,1998,2002} \delta_v^1 \cdot [D^v(is) - D^{2000}(is)]\} =$$

$$\mu_i + \eta_{is} \, , \tag{19}$$

where

$$\vec{\beta} \equiv (\beta_1 \, , \, \beta_2 \, , \, \beta_3) \equiv (\ln(1 - \theta^m) \, , \, \bar{\theta} \, , \, \sigma) \, , \tag{20}$$

and

$$\vec{\delta^1} \equiv (\delta_{1992}^1 \, , \, \delta_{1994}^1 \, , \, \ldots, \, , \delta_{1998}^1 \, , \, \delta_{2002}^1) \, .$$

Our second estimation condition, which is new to this paper, comes from equation (11) of Proposition 1. Using the notation σ from (13) and the definition of c_{is} (see (2)), and setting

$$y_{is}^f \equiv h_{is}^f \cdot w_{is}^f \quad \text{and} \quad y_{is}^m = h_{is}^m \cdot w_{is}^m \, ,$$

(11) yields

$$x_{i,s+1} - \frac{1}{\xi^f} \cdot y_{i,s+1}^f - \frac{1}{\xi^m} \cdot y_{i,s+1}^m = \sigma \cdot e^r \cdot [x_{is} - \frac{1}{\xi^f} \cdot y_{is}^f - \frac{1}{\xi^m} \cdot y_{is}^m] \, .$$

Adding a subscript for the time, when household i is age s at, say, time t, the preceding becomes

$$x_{i,s+1,t+1} - \frac{1}{\xi^f} \cdot y_{i,s+1,t+1}^f - \frac{1}{\xi^m} \cdot y_{i,s+1,t+1}^m = \sigma \cdot e^r \cdot [x_{ist} - \frac{1}{\xi^f} \cdot y_{ist}^f - \frac{1}{\xi^m} \cdot y_{ist}^m] \, . \tag{21}$$

The CEX (described below) collects data on household expenditures. Survey observations on individual households are not available (even in principle); however, we can derive weighted averages for married couples by (st)–cell, where the cells represent U.S. population averages for married couples. In other words, the CEX provides data on $\bar{x}_{st}$ where, for weights ω_{ist},

$$\bar{x}_{st} = \sum_i \omega_{ist} \cdot x_{ist} \, .$$

Think of s as being the male's age. Rather than trusting CEX earnings data (the CEX is, after all, primarily an expenditure survey), we turn to the March Current Population Surveys for 1984-2002. From earnings for married men and women, classified by the man's age, we generate population averages

$$\bar{y}_{st}^m \equiv \sum_j \bar{\omega}_{jst} \cdot y_{jst}^m \quad \text{and} \quad \bar{y}_{st}^f \equiv \sum_k \bar{\omega}_{kst} \cdot y_{kst}^f \,.$$

The linearity of (21) allows us to take weighted averages of each of its terms, giving

$$\bar{x}_{s+1,t+1} - \frac{1}{\xi^f} \cdot \bar{y}_{s+1,t+1}^f - \frac{1}{\xi^m} \cdot \bar{y}_{s+1,t+1}^m = \sigma \cdot e^r \cdot [\bar{x}_{st} - \frac{1}{\xi^f} \cdot \bar{y}_{st}^f - \frac{1}{\xi^m} \cdot \bar{y}_{st}^m] \,.$$

The CEX and CPS provide data on $\bar{x}$, $\bar{y}^f$, and $\bar{y}^m$. Noticing that knowledge of $\vec{\beta} \equiv (\beta_1, \beta_2, \beta_3)$ determines θ_f via (17), define

$$g_{st}(\vec{\beta}) \equiv \bar{x}_{s+1,t+1} - \frac{1}{\xi^f} \cdot \bar{y}_{s+1,t+1}^f - \frac{1}{\xi^m} \cdot \bar{y}_{s+1,t+1}^m -$$

$$\sigma \cdot e^r \cdot [\bar{x}_{st} - \frac{1}{\xi^f} \cdot \bar{y}_{st}^f - \frac{1}{\xi^m} \cdot \bar{y}_{st}^m] \,. \tag{22}$$

Then our second estimation condition is

$$g_{st}(\vec{\beta}) = \epsilon_{st}^2 \,, \tag{23}$$

with ϵ_{st}^2 a random error with mean 0.

Turn to ϵ_{st}^2. Because expenditures on consumption goods and services are probably more difficult to measure than household net worth, we continue to be concerned with measurement error. Although the asset bubble of the late 1990s is perhaps less of an issue for (23) than for (19), Hall [1978] calls attention to the possibility that unforeseen events force households to adjust their consumption permanently from time to time. Hence, to simplify ϵ^2, we again consider a system of dummy variables. Since our expenditure data covers each year 1984-2002, (22) uses differences for $t = 1984, ..., 2001$; thus, our new life–hand side component for (23) is

$$\sum_{v=1984,1985,...,1999,2001} \delta_v^2 \cdot [D^v(st) - D^{2000}(st)] \,.$$

Because (23) has expenditure terms for $(s+1, t+1)$ and (st), our complete specification is

$$q_{st}^2(\vec{\beta}, \vec{\delta^2}) \equiv g_{st}(\vec{\beta}) - \sum_{v=1984,1985,...,1999,2001} \delta_v^2 \cdot [D^v(st) - D^{2000}(st)] =$$

$$v_{s+1,t+1} - \sigma \cdot e^r \cdot v_{st} \,, \tag{24}$$

where each v is white noise with mean 0.

3 Data

This paper uses data from three sources. First, we use data on earnings and net worth at retirement for individual households from the Health and Retirement Study. Our other two data sources are the Current Population Survey and the Consumer Expenditure Survey. These surveys provide cross–sectional data on individuals' earnings and on households' consumption expenditures.

We estimate (19) from the HRS. HLS [2006] describe this data in detail. It has linked Social Security–record earnings histories — see Section 1. We correct for missing employer benefits by multiplying each year's earnings by the year's ratio of NIPA "total labor compensation" to NIPA "wages and salaries." We subtract personal income taxes at a proportional rate equaling the year's average rate. We derive the latter from NIPA "personal current taxes" divided by NIPA "personal income" plus "contributions for government social insurance" less one–half of transfers received.[2] We assume that one–half of Social Security benefits are subject to the income tax. We also subtract OASDI taxes at the statutory rate for each year, assessing the tax up to the year's cap. In all cases, we deflate with the NIPA PCE deflator. We assign household Social Security benefits according to the statutory formulas, by year. Our net worth variable corresponds to the same in HLS.

We estimate (24) from the other two data sets.

The CEX provides data on market expenditures x, as well as corresponding demographic information on households. Let s be a the male respondent's age and t the current date. Then the CEX provides observations on $\bar{x}_{st}$ described in the preceding section. With SAS software provided by BLS, we are able to derive $\bar{x}_{st}$ for all couples of age s at time t, and for married couples alone.

The CEX covers consumption categories closely corresponding to National Income and Product Accounts (NIPA) personal consumption expenditures (PCE). Following Laitner and Silverman [2005], we assume that totals in the NIPA are correct and adjust the CEX data to match. The adjustments are as follows. First, we exclude pension contributions and life-insurance contributions from the CEX. Then we subdivide both NIPA PCE and the CEX data into eleven expenditure categories: (1) food, (2) apparel, (3) personal care, (4) shelter, (5) household operations, (6) transportation, (7) medical, (8) entertainment, (9) education, (10) personal business, and (11) miscellaneous. For each year, we scale the CEX composite data for all households to match category by category the NIPA consumption data. We then apply the category adjustments one–by–one to the individual age cells of the CEX data for married couples.

Second, we adjust the CEX "shelter" category. For "own dwelling," the CEX measure of expenditure is mortgage payments, maintenance, etc. The CEX separately measures the market value of the house, however. We replace the CEX "own dwelling" component of "shelter" for couples, age–by–age, with the CEX value of own house multiplied by the factor that in aggregate makes the CEX service flow, for all households (of all ages), equal to the NIPA flow from own houses for the same year.

Third, we adjust the CEX "personal business" category. NIPA "personal business" consumption includes bank and brokerage fees, many of which are hidden in the form of

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[2] This paper's treatment of income taxes differs from HLS [2006].

low interest rates on checking accounts, etc., and hence do not appear in CEX survey responses from households. Assuming that all bank and brokerage fees make their way into our life–cycle model through lower interest rates on savings, we drop them from the NIPA data, and we drop more limited measures of the same from the CEX data. We then adjust the remainder of "personal business" spending in the CEX for all households to match the corresponding remainder in the NIPA, carrying the adjustment factor to the CEX data for couples.

Our final adjustment applies to "medical" spending. The CEX data, because it is based on a survey of households, omits both employer contributions to health insurance and governmental Medicare and Medicaid spending. As a result, the CEX captures a particularly small share of NIPA medical spending. So, we drop the CEX medical spending and use an alternative measure. The Center for Medicare and Medicaid Services (CMS) provides information on personal health care expenditures by age.[3] Specifically, for the years 1987, 1996, and 1999, the CMS provides total health spending, and per capita health care spending, for 7 age groups. (The totals in the CMS data reflect the NIPA medical spending component of PCE quite well.) We linearly interpolate and extrapolate the totals and per capital figures to all years 1984-2002. We then justify the annual totals to amounts in the NIPA. Next, after carrying the latter adjustment proportionately to per capita amounts, we year–by–year interpolate and extrapolate the per capita figures to the individuals in our CEX couples data. Assuming wives are two years younger than their husbands, we then impute couples' medical spending to each CEX age–year cell.

The adjusted CEX consumption data serves as our data on market expenditures. We deflate the final consumption data with the NIPA PCE deflator (normalized to 1 in 2000).

As stated, we use earnings data come from the Current Population Survey (CPS). We restrict our attention to two-member households that are married couples living together.[4] The CPS provides annual earnings data for husbands and their wives. As above, we define a household's age to be the husband's age. We correct for missing benefits and taxes as with the HRS data. We deflate with the NIPA PCE deflator (normalized to 1 in 2000). Using CPS weights, this generates our variables $\bar{y}_{st}^m$ and $\bar{y}_{st}^f$.

Changes in family composition for young households present complications for our analysis (due to varying N_{is}). To minimize these, we limit our attention to the subsample of CEX and CPS households between the age of 55 and 69. For this group, children are less important; moreover, older people who are married are likely to remain so. We have CEX and CPS cross sections for 1984-2002.

Finally, we assume a real interest rate of 5 percent per year.[5] To derive our variable r, we subtract incomes taxes at a constant average rate of 0.1169 — which is the long–run average of our personal current tax rate above.

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[3] Public access to this data is available through the CMS web site

www.cms.hhs.gov/NationalHeathExpendData.

[4] This contrasts slightly with the CEX data described above, which includes all couples.

[5] See the discussion in Laitner and Silverman [2005].

4 Estimation

This section describes our estimation steps and outcomes. We use the notation q_{is}^1 and q_{st}^2 from Section 3; let q^1 be the vector of the former for all is, and, similarly, let q^2 be the vector with elements q_{st}^2 all st. Let $\vec{p}$ be the vector of parameters to be estimated, including, at most, $\vec{\beta}$, $\vec{\delta^1}$, and $\vec{\delta^2}$.

HLS [2006] estimate (19) alone, using NLLS with White standard errors. Since this paper's estimation procedure is different, and since we have computed taxes on a different (i.e., a yearly) basis, for the sake of comparison, we begin with new, single–equation results.

Table 1 presents outcomes based on (19) by itself. We use a two–stage estimation procedure. The first generates consistent point estimates of β_1, β_2, β_3, and δ_v^1. In the case of (19), stage-1 uses NLLS. From the residuals, we estimate σ_μ^2 and σ_η^2 (e.g., Greene [1990, p. 490-491]). Throughout this paper, stage-1 estimates of $\sigma_\mu^2/(\sigma_\mu^2 + \sigma_\eta^2)$ are always about 0.25. We form a matrix Ω_1 with $n_i \times n_i$ submatrices along its principal diagonal (and zeros elsewhere), where n_i is the number of HRS observations for household i, and where each submatrix has 1 along its principal diagonal and $\sigma_\mu^2/(\sigma_\mu^2 + \sigma_\eta^2)$ elsewhere.[6] In this case, $\vec{p} = (\vec{\beta}, \vec{\delta^1})$. We form a matrix of instruments Z_1, with one row for each observation, and each row having the form

$$\left(\begin{array}{ccccccc} \frac{\partial q_1(\vec{p}^1)}{\partial \beta_1} & \frac{\partial q_1(\vec{p}^1)}{\partial \beta_2} & \frac{\partial q_1(\vec{p}^1)}{\partial \beta_3} & \frac{\partial q_1(\vec{p}^1)}{\partial \delta_{1992}} & \cdots & \frac{\partial q_1(\vec{p}^1)}{\partial \delta_{2002}} \end{array} \right),$$

with each partial derivative evaluated at the stage–1 consistent parameters estimates, say, $\vec{p}^1$.

Stage 2 chooses a new parameter vector $\vec{p}$ solving

$$\min_{\vec{p}} \left(q_1^T(\vec{p}) \cdot [\Omega_1]^{-1} \cdot Z_1 \right) \cdot \left(Z_1^T \cdot [\Omega_1]^{-1} \cdot Z_1 \right)^{-1} \cdot \left(Z_1^T \cdot [\Omega_1]^{-1} \cdot q_1(\vec{p}) \right). \qquad (25)$$

Letting Q_1 be the matrix whose rows are the partial derivatives of q_1 with respect to each element of the parameter vector in turn, Gallant [1987] shows the covariance matrix for our second–stage parameter–vector estimate, say, $\widehat{p}$, is

$$C = \sigma_{11}(\vec{p}^1) \cdot \left[\left(Q_1^T(\widehat{p}) \cdot [\Omega_1]^{-1} \cdot Z_1 \right) \cdot \left(Z_1^T \cdot [\Omega_1]^{-1} \cdot Z_1 \right)^{-1} \cdot \left(Z_1^T \cdot [\Omega_1]^{-1} \cdot Q_1(\widehat{p}) \right) \right]^{-1}, \quad (26)$$

where if n is the number of observations and p the number of elements in $\vec{p}^1$,

$$\sigma_{11}(\vec{p}^1) \equiv q_1^T(\vec{p}^1) \cdot [\Omega_1]^{-1} \cdot q_1(\vec{p}^1)/(n - p).$$

Table 1 presents stage-2 parameter estimates. In HLS [2006], $\theta^m = 0$ by assumption, and $\theta^f = \overline{\theta}$. In this case, Table 1 shows that households require about 25 cents worth of market goods to substitute for the lost home production implicit in each dollar's worth of female market earnings.

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[6] Notice that the stage–1 estimates are consistent. We do not re–estimate Ω_1 in stage 2.

Para–meters[a]	Specification:		
	$\alpha^S = 0.50$; $\alpha^K = 0.00$	$\alpha^S = 0.50$; $\alpha^K = 0.15$	$\alpha^S = 0.50$; $\alpha^K = 0.50$
θ^M	0.1371 (0.2573/0.3244)	0.1042 (0.2640/0.4223)	0.0459 (0.2647/0.5466)
$\overline{\theta}$	0.2576** (0.1220/2.0400)	0.2567** (0.1221/2.0385)	0.2567** (0.1223/2.0412)
σ	0.9812*** (0.0104/93.4028)	0.9798*** (0.0101/95.9510)	0.9777*** (0.0093/104.5581)
	Sample Size:		
Observations	954	954	954
Households	441	441	441

Table 1. Estimated Coefficients Single–Equation Model: Estimated Parameter (Std. Error/T Stat.)

Significant at: 10% level (*), 5% level (**), 1% level (***).
a. Year dummies not reported.

As in Section 2, we can interpret row 1 of Table 1 as estimating θ^m (as shown) and deduce an estimate of θ^f from (17). The magnitude of the estimates of θ^m in the three columns may be plausible, yet the estimates, unfortunately, are imprecise — having large standard errors. Our hope is that our 2–equation specification can do better. Indeed, we believe that it does.

To estimate stage–1 for the 2–equation statistical formulation, run the first stage for (19) as above — treating β_1 as an unrestricted constant. NLLS can, as is well–known, be viewed as a method of moments procedure, solving

$$\min_{\vec{\beta},\,\vec{\delta 1}} \left(q_1^T(\vec{\beta},\,\vec{\delta 1}) \cdot Z_1(\vec{\beta},\,\vec{\delta 1}) \right) \cdot \left(Z_1^T(\vec{\beta},\,\vec{\delta 1}) \cdot q_1(\vec{\beta},\,\vec{\delta 1}) \right).$$

Form matrix Z_2 of instruments for (24) with rows

$$\left(1,\, D^{1984}(st) - D^{2000}(st),\, ...,\, D^{1999}(st) - D^{2000}(st),\, D^{2001}(st) - D^{2000}(st)\right).$$

Fixing equation (19) stage–1 estimates of σ and $\overline{\theta}$, obtain stage–1 estimates of θ^m and $\vec{\delta 2}$ from

$$\min_{\theta^m,\,\vec{\delta 2}} \left(q_2^T(\theta^m,\,\vec{\delta 2}) \cdot Z_2 \right) \cdot \left(Z_2^T \cdot q_2(\theta^m,\,\vec{\delta 2}) \right),$$

where we constrain θ^f with

$$\theta^f = 1 - (1 - \overline{\theta}) \cdot (1 - \theta^m)$$

(recall (17)). Following the preceding discussion and Section 3, form the matrix Ω_2 with $n_j \times n_j$ submatrices along the principal diagonal (and zeros elsewhere), where n_j is the number of years length for CEX pseudo panel j, and where each submatrix has 1 along its principal diagonal and

$$-\frac{\sigma \cdot e^r \cdot \sigma_v^2}{\sigma_v^2 + [\sigma]^2 \cdot e^{2 \cdot r} \cdot \sigma_v^2} = -\frac{\sigma \cdot e^r}{1 + [\sigma]^2 \cdot e^{2 \cdot r}} \qquad (27)$$

for the diagonals immediately above and below the principal one. We use the stage–1 estimate of σ in the construction of Ω_2, and fix Ω_2 thereafter.[7]

For stage 2, this paper considers 3 specifications. The first allows an arbitrary constant in (19); so,

$$\vec{p} = (\beta_1, \overline{\theta}, \theta^m, \sigma, \vec{\delta^1}, \vec{\delta^2}).$$

The second sets $\beta_1 = \ln(1 - \theta^m)$; hence, the preceding $\vec{p}$ loses its first element. The third specification adds two additional instruments for (24), y_{st}^m and y_{st}^f. The latter then constitute two additional columns in Z_2. The stage–2 estimates $\widehat{p}$ come from

$$\min_{\vec{p}} \{ \left(q_1^T(\vec{p}) \cdot [\Omega_1]^{-1} \cdot Z_1 \right) \cdot \left(Z_1^T \cdot [\Omega_1]^{-1} \cdot Z_1 \right)^{-1} \cdot \left(Z_1^T \cdot [\Omega_1]^{-1} \cdot q_1(\vec{p}) \right) / \sigma_{11}(\vec{p}^1) +$$

$$\left(q_2^T(\vec{p}) \cdot [\Omega_2]^{-1} \cdot Z_2 \right) \cdot \left(Z_2^T \cdot [\Omega_2]^{-1} \cdot Z_2 \right)^{-1} \cdot \left(Z_2^T \cdot [\Omega_2]^{-1} \cdot q_2(\vec{p}) \right) / \sigma_{22}(\vec{p}^1) \}. \quad (28)$$

Setting $Q^2(\vec{p})$ analogously to the way we set $Q_1(\vec{p})$ above, the covariance matrix for the stage–2 parameter estimates is

$$C = \left[\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} \right]^{-1}, \qquad (29)$$

where

$$C_{21} = 0 = C_{12},$$

$$C_{11} = \frac{1}{\sigma_{11}(\vec{p}^1)} \cdot \left(Q_1^T(\widehat{p}) \cdot [\Omega_1]^{-1} \cdot Z_1 \right) \cdot \left(Z_1^T \cdot [\Omega_1]^{-1} \cdot Z_1 \right)^{-1} \cdot \left(Z_1^T \cdot [\Omega_1]^{-1} \cdot Q_1(\widehat{p}) \right),$$

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[7] In fact, the estimate of (27) is almost exactly -0.50 in all cases, and the ratio of consistent estimates of $E[q_{s+1,t+1}^2 \cdot q_{st}^2]$ to $E[q_{st}^2 \cdot q_{st}^2]$ for the calculations in Tables 2-4 below is always between -0.54 and -0.55.

$$C_{22} = \frac{1}{\sigma_{22}(\vec{p}^1)} \cdot \left(Q_2^T(\widehat{p}) \cdot [\Omega_2]^{-1} \cdot Z_2\right) \cdot \left(Z_2^T \cdot [\Omega_2]^{-1} \cdot Z_2\right)^{-1} \cdot \left(Z_2^T \cdot [\Omega_2]^{-1} \cdot Q_2(\widehat{p})\right).$$

Tables 2-4 present the second–stage estimates from the two–equation formulation. Our family–composition weights (recall (8)) follow the Social Security system in setting $\alpha^S = 0.50$. This also agrees with the estimates in Laitner and Silverman [2005]. The Social Security system's treatment of orphans implicitly sets $\alpha^K = 0.50$ and, as this paper does, allows the weight for a maximum of 2 children at a time. Many papers simply impose $\alpha^K = 0.00$. Laitner and Silverman estimate $\alpha^K = 0.15$. Although tables 2-4 try all three possibilities for α^K, results seem insensitive to one's choice.

In each case, we began with a complete set of 5 time dummies for (19) and 17 time dummies for (24). Coefficient estimates for the former dummies always have a methodical pattern (in particular, they increase through 2000) and always are jointly statistically significantly different from zero at the 10 percent significance level — in fact, at the 5 percent level except for each table's first column. The time dummies for (24), on the other hand, show little discernible pattern and are only jointly statistically significantly different from zero (under a Wald test) at the 10 percent level in one of the nine columns. Out of concern for adequate sample size for the remaining estimates, in the end we drop the dummies from (24). In terms of Hall's [1978] model, our pseudo–panel CEX data present average consumption expenditures; thus, our analysis cannot address the possible role of idiosyncratic shocks to individual households. In terms of aggregative shocks, 1984-2002 was not a period with a sharp or prolonged recession. It is also possible that the measurement errors in our expenditure data simply overwhelm Hall's error. Future work will consider formulations with random time effects.

Since columns 2-3 in Tables 2-4 have more instruments than parameters, we present χ^2 tests of the over–identifying restrictions. These tests never reject our specification at even the 20 percent significance level.

Tables 2-4 show estimates of θ^f of 0.30-0.35, which is somewhat larger than the estimate from Table 1 under the assumption, for the latter, that $\theta^f = \overline{\theta}$. Our estimates of θ^m are about half as large, ranging in magnitude, in columns 2-3, of 0.13 to 0.17. In contrast to Table 1, standard errors for θ^m are now as small — at least in columns 2-3 — as those for θ^f. Table 2-4 estimates of θ^f are always different from 0.0 at the 5 percent significance level, and the estimates of θ^m are different from 0.0 at the 10 percent significance level in column 3. The tables show that a Wald test of $\theta^m = 0 = \theta^f$ invariably strongly rejects.

A number of economists have argued that policymakers should judge household well–being on the basis of consumption rather than dollar income (e.g., Hurd and Rohwedder [2006]). The present analysis suggests that household expenditures on market goods and services are, in turn, potentially misleading indicators of overall consumption, because total consumption includes home production as well. The distinction between total consumption and market expenditures on goods and services is particularly important for comparisons of households before and after retirement. Tables 2-4 show that a married household's gains in home production at retirement may amount to 15 percent of a husband's pre–retirement earnings and 30 percent of the same for his wife.

Para–meters[a]	Specification:		
	Separate Constant EQ1	EQ1-Constant Constrained to Equal $\ln(1 - \theta^m)$	EQ1-Constant= $\ln(1 - \theta^m)$; Added Instruments $\bar{y}_{st}^m$ and $\bar{y}_{st}^f$
EQ1 Constant	-0.1474 (0.2981/-0.4945)	NA	NA
$\bar{\theta}$	0.2576** (0.1220/2.1117)	0.2577** (0.1220/2.1129)	0.1940 (0.1213/1.5996)
θ^m	0.1396 (0.1831/0.7623)	0.1444 (0.0944/1.5292)	0.1682* (0.0920/1.8291)
σ	0.9812*** (0.0104/94.1357)	0.9814*** (0.0044/224.7665)	0.9822*** (0.0044/222.6810)
Sample Size: EQ1/EQ2			
Observations	954/252	954/252	954/252
Households[b]	441/31	441/31	441/31
Calculated Parameters:[c]			
θ^f	0.3613** (0.1588/2.2753)	0.3649*** (0.1051/3.4720)	0.3296*** (0.1062/3.1033)
Wald Test of $\theta^m = 0 = \theta^f$: Statistic (P-value)			
	$\chi^2(2) = 7.3322$** (0.9744)	$\chi^2(2) = 12.1820$*** (0.9977)	$\chi^2(2) = 9.9703$*** (0.9932)
Test of Overidentifying Restrictions: Statistic (P-value)			
	NA	$\chi^2(1) = 0.0009$ (0.0246)	$\chi^2(3) = 4.4779$ (0.7857)

Table 2. Estimated Coefficients Two–Equation Model: $\alpha^S = 0.5$ and $\alpha^K = 0.00$ Estimated Parameter (Std. Error/T Stat.)

Significant at: 10% level (*), 5% level (**), 1% level (***).

  a. Year dummies not reported.

  b. EQ2 "households" from pseudo–panel.

  c. Asymptotic standard error from delta method.

Table 3. Estimated Coefficients Two–Equation Model: $\alpha^S = 0.5$ and $\alpha^K = 0.15$
Estimated Parameter (Std. Error/T Stat.)

Para–meters[a]	Specification:		
	Separate Constant EQ1	EQ1-Constant Constrained to Equal $\ln(1-\theta^m)$	EQ1-Constant= $\ln(1-\theta^m)$; Added Instruments $\bar{y}_{st}^m$ and $\bar{y}_{st}^f$
EQ1 Constant	-0.1100 (0.2947/-0.3732)	NA	NA
$\bar{\theta}$	0.2567** (0.1221/2.1034)	0.2569** (0.1222/2.1012)	0.1927 (0.1214/1.5878)
θ^m	0.1167 (0.1831/0.6372)	0.1396 (0.0938/1.4888)	0.1630* (0.0912/1.7875)
σ	0.9798*** (0.0101/96.6633)	0.9812*** (0.0043/228.3513)	0.9819*** (0.0043/226.7019)

Sample Size: EQ1/EQ2

Observations	954/252	954/252	954/252
Households[b]	441/31	441/31	441/31

Calculated Parameters:[c]

θ^f	0.3434** (0.1589/2.1612)	0.3606*** (0.1050/3.4348)	0.3243*** (0.1059/3.0615)

Wald Test of $\theta^m = 0 = \theta^f$: Statistic (P-value)

	$\chi^2(2) =6.7638$** (0.9660)	$\chi^2(2) =11.9372$*** (0.9974)	$\chi^2(2) =9.7307$*** (0.9923)

Test of Overidentifying Restrictions: Statistic (P-value)

	NA	$\chi^2(1) = 0.0217$ (0.1172)	$\chi^2(3) = 4.4941$ (0.7872)

Significant at: 10% level (*), 5% level (**), 1% level (***).
  a. Year dummies not reported.
  b. EQ2 "households" from pseudo–panel.
  c. Asymptotic standard error from delta method.

Table 4. Estimated Coefficients Two–Equation Model: $\alpha^S = 0.5$ **and** $\alpha^K = 0.50$
Estimated Parameter (Std. Error/T Stat.)

Para–meters[a]	Specification:		
	Separate Constant EQ1	EQ1-Constant Constrained to Equal $\ln(1 - \theta^m)$	EQ1-Constant= $\ln(1 - \theta^m)$; Added Instruments $\bar{y}^m_{st}$ and $\bar{y}^f_{st}$
EQ1 Constant	-0.0469 (0.2774/-0.1692)	NA	NA
$\bar{\theta}$	0.2567** (0.1223/2.0999)	0.2565** (0.1223/2.0984)	0.1917 (0.1215/1.5782)
θ^m	0.0775 (0.1760/0.4403)	0.1304 (0.0921/1.4152)	0.1530* (0.0894/1.7114)
σ	0.9777*** (0.0093/105.2148)	0.9807*** (0.0042/236.1722)	0.9814*** (0.0042/235.3521)
Sample Size: EQ1/EQ2			
Observations	954/252	954/252	954/252
Households[b]	441/31	441/31	441/31
Calculated Parameters:[c]			
θ^f	0.3143** (0.1555/2.0214)	0.3535*** (0.1049/3.3713)	0.3154*** (0.1055/2.9886)
Wald Test of $\theta^m = 0 = \theta^f$: Statistic (P-value)			
	$\chi^2(2)$ =5.9787* (0.9497)	$\chi^2(2)$ =11.5304*** (0.9969)	$\chi^2(2)$ =9.3246*** (0.9906)
Test of Overidentifying Restrictions: Statistic (P-value)			
	NA	$\chi^2(1) = 0.1307$ (0.2823)	$\chi^2(3) = 4.6060$ (0.7970)

Significant at: 10% level (*), 5% level (**), 1% level (***).
  a. Year dummies not reported.
  b. EQ2 "households" from pseudo–panel.
  c. Asymptotic standard error from delta method.

5 Implications

This section discusses implications of Section 4's outcomes for the elasticity of labor supply, consumption behavior at retirement (including the so–called "consumption puzzle"), and the literature on "Euler equation" models.

Labor Supply. The magnitude of labor supply elasticities has long been an important topic in labor economics, public finance, and macroeconomics. Labor economists usually argue that elasticities of labor supply for adult males are small, perhaps not much different from zero (e.g., Blundell and MaCurdy [1999]), while female elasticities may be much larger, perhaps 0.5 or more (e.g., Pencavel [1998]).

Results in Tables 2-4 generate a new set of labor–supply elasticity estimates. Taking logarithms of (9)-(10) and then differentiating with respect to the log of the wage, one has

$$\frac{d \ln(h_{is}^m)}{d \ln(w_{is}^m)} = \frac{1}{\xi^m - 1} = \frac{\theta^m}{1 - \theta^m} \quad \text{and} \quad \frac{d \ln(h_{is}^f)}{d \ln(w_{is}^f)} = \frac{1}{\xi^f - 1} = \frac{\theta^f}{1 - \theta^f} \,.$$

In Section 2's model, these constitute both long–run labor supply elasticities for men and women (i.e., the percentage response of labor supply with respect to a permanent change in the wage) and short–run elasticities (i.e., the percentage response of labor supply to temporary wage changes). Formally, this establishes

Proposition 3. *For $j = m, f$, let ϵ_{LR}^j give the elasticity of the labor supply with respect to a permanent change in the wage rate, and let ϵ_{SR}^j give the elasticity with respect to a momentary change in the wage rate. Then for the model of Section 2,*

$$\epsilon_{LR}^j = \epsilon_{SR}^j = \frac{d \ln(h_{is}^j)}{d \ln(w_{is}^j)} = \frac{\theta^j}{1 - \theta^j} \quad for \quad j = m, f \,. \tag{30}$$

Table 5 uses parameter estimates from Table 3 to derive numerical values of the elasticities. The so–called delta method yields standard errors and confidence intervals.

Despite large standard errors, Table 5's elasticity estimates are interesting in a number of respects. (i) Although our framework of analysis is very different from traditional labor–economics approaches, our point estimates are quite similar to those cited. This might increase one's confidence is both methodologies. (ii) Macroeconomists have been intrigued with the possibility that agents' responses to short–run wage changes might be larger than their responses to long–run or permanent changes. The idea would be that following a wage increase, the substitution effect makes an agent want to supply more labor but the income effect increases the agent's demand for leisure. In the case of a temporary wage change, the income effect would tend to be greatly diminished or absent. In our model, agents divide their time between home and market work. They do not withdraw hours for more leisure. As Proposition 1 shows, lifetime resources do not affect a man or woman's time allocation choices. In our context, therefore, a shock to the market wage leads to a substitution effect — tending to increase labor supply to the market — but no income effect on overall work hours. The small magnitude of our elasticity estimates is then somewhat surprising. There is no difference in size between our elasticities for short

Table 5. Estimated Labor Elasticities of Supply: $\alpha^S = 0.5$ and $\alpha^K = 0.15$ Point Estimate (Std. Error) [95% Confidence Interval]			
Sex	Estimation Specification:[a]		
	Separate Constant EQ1	EQ1-Constant Constrained to Equal $\ln(1-\theta^m)$	EQ1-Constant= $\ln(1-\theta^m)$; Added Instruments $\bar{y}^m_{st}$ and $\bar{y}^f_{st}$
Men	0.1321 (0.2347) $[-0.3283\,,\,0.5923]$	0.1623 (0.1267) $[-0.0863\,,\,0.4108]$	0.1948 (0.1302) $[-0.0606\,,\,0.4502]$
Women	0.5231 (0.3686) $[-0.2001\,,\,1.2463]$	0.5640 (0.2568) $[0.0601\,,\,1.0679]$	0.4800 (0.2320) $[0.0247\,,\,0.9353]$

a. Parameter values from Table 3.

and long–run wage changes. (iii) Because our estimation is closely tied to a fully specified life–cycle model, elaborations are reasonably straightforward, and they could increase the utility of our results. HLS's treatment of experiential human capital illustrates this point, and human capital is a direction in which we want to proceed with this work in the future.

<u>Consumption at Retirement</u>. The "retirement consumption puzzle" refers to the systematic reduction in expenditures on market goods and services for households that enter retirement. A number of recent papers document such a decline for households at, and after, their retirement age (e.g., Banks *et al.* [1998], Bernheim *et al.* [2001], Hurd and Rohwedder [2003], Laitner and Silverman [2005], and Aguiar and Hurst [2005]). Possible explanations include reduced work–related expenses after retirement, poor planning, nonseparability of consumption and leisure, and increased shopping time after retirement. The model in this paper offers another explanation, perhaps most similar to the shopping–time theory above. Our model implies that after retirement, lost home production due to market work ceases, yet lifetime utility maximization requires total consumption to vary smoothly with age; thus, with fewer home–production losses to offset, households reduce their expenditure on market goods and services after they retire.

To think about consumption near retirement age in more detail, recall Proposition 1. Near R_i, family composition may be quite stable, so that N_{is} is constant. If so, (11) implies

$$c_{i,s+\Delta} = e^{\frac{r-\rho}{1-\gamma}\cdot\Delta} \cdot c_{is}\,. \tag{31}$$

Let $s = R_i = R$ and $\Delta \downarrow 0$. Then

$$c_{i,R+0} \to c_{i,R-0}\,. \tag{32}$$

16

Suppose, for simplicity, that labor supply ceases entirely at retirement for household i, so that

$$h_{i,R+0}^{m} = 0 = h_{i,R+0}^{f} \,.$$

Then (2) and (32) imply

$$x_{i,R+0} = x_{i,R-0} - A_{i,R-0}^{f} \cdot [h_{i,R-0}^{f}]^{\xi^{f}} - A_{i,R-0}^{m} \cdot [h_{i,R-0}^{m}]^{\xi^{m}} \,.$$

Using Proposition 1 again, this yields

$$x_{i,R+0} - x_{i,R-0} = -\Big(\frac{1}{\xi^{f}} \cdot y_{i,R-0}^{f} + \frac{1}{\xi^{m}} \cdot y_{i,R-0}^{m}\Big) \equiv \big(\theta^{f} \cdot y_{i,R-0}^{f} + \theta^{m} \cdot y_{i,R-0}^{m}\big) \,.$$

Formally, we have

Proposition 4. *Assume that N_{is} is continuous at $s = R_i = R$. Then upon the retirement of household i, its expenditure on market goods and services, x_{is} for $s = R$, declines according to*

$$x_{i,R+0} - x_{i,R-0} = -\Big(\frac{1}{\xi^{f}} \cdot y_{i,R-0}^{f} + \frac{1}{\xi^{m}} \cdot y_{i,R-0}^{m}\Big) \,. \tag{33}$$

Thus, our model predicts a decline in a household's expenditures after retirement. In particular, in view of (33) and Tables 2-4, for couples the magnitude of the decline should be about 15 percent the size of pre–retirement male earnings plus about 30 percent of pre–retirement female earnings. Evidently, the model predicts a smaller decline per dollar of pre–retirement earnings for households relying on male income alone, for households in which only one spouse retires at $s = R$, and for households engaging in part–time market work after age R. For households that abruptly cease all market employment at age R, Tables 2-4 show the decline in market expenditure can be large. For example, Laitner and Silverman [2006] find average pre–retirement earnings for HRS couples to be about \$35,000 (in 1984 dollars) and average expenditures to be about \$40,000 (including, as we do here, service flows from owner–occupied houses and medical spending). With male earnings alone, the drop in average household expenditure at retirement would slightly exceed \$5,000, about 12.5 percent of average expenditures. With male and female earnings in a ratio 2:1, the fall would be \$7,000, about 17.5 percent of pre–retirement consumption. These figures match the "consumption puzzle" declines in the cited literature.

It is fair to note that our estimation strategy uses — and even relies upon — the magnitude of the empirical fall in household expenditures at retirement in deriving its estimates for Tables 2-4. We cannot, therefore, employ the same evidence to test our model against alternative explanations for the decline. On the other hand, one can say that our theoretical model is consistent with the empirical decline, and that parameter estimates based on the assumption that home production explains the entire decline are plausible in the sense that they obey our anticipations in (7) and yield labor–supply elasticities similar to those in the existing literature. We hope that our explicit theoretical modeling will aid

future work in developing tests of alternative hypotheses about consumption changes at retirement.

Euler Equations. A very large literature studies Euler–equation models that relate a household's consumption at different ages (e.g., Hall [1978, 1988], Hansen and Singleton [1982, 1983], Campbell and Mankiw [1989, 1991], Shapiro [1984], Altonji and Siow [1987], and many others]. Although we derive a certainly equivalent Euler equation from Proposition 1, many papers allow the greater generality of a stochastic environment.[8] However, most of the existing literature omits home production. When $\Delta = 1$ in (31), omitting home production would leave

$$x_{i,s+1} = c_{i,s+1} = e^{\frac{r-\rho}{1-\gamma}} \cdot c_{is} = e^{\frac{r-\rho}{1-\gamma}} \cdot x_{is} \, . \tag{34}$$

This paper argues instead for (21), which has

$$c_{i,s+1} = e^{\frac{r-\rho}{1-\gamma}} \cdot c_{is}$$

but

$$c_{is} = x_{is} - A_{is}^f \cdot [h_{is}^f]^{\xi^f} - A_{is}^m \cdot [h_{is}^m]^{\xi^m}$$

and similarly for $c_{i,s+1}$. Tables 2-4 present Wald tests of the hypothesis

$$H_0 : \ \theta^m = 0 = \theta^f \, .$$

Under H_0, (34) and (21) would be identical. But, the tests reject — at the 1 percent significance level in columns 2-3. Given positive values for θ^f and/or θ^m, the novel terms in (21) can easily become quite large.

6 Conclusion

Using data from the Health and Retirement Study, the Consumer Expenditure Survey, and the Current Population Survey, we estimate key parameters of a life–cycle model with home production. The estimates indicate that the value of forgone home production for working men is fairly modest — roughly 10-15 cents for every dollar that a married man earns in the formal labor market. The value of forgone home production for married, working women seems substantially larger, perhaps 30-35 cents per dollar of market earnings.

Our findings have implications for the study of several aspects of household economic activity and retirement behavior. First, our parameter estimates suggest that male labor supply elasticities should be quite low compared to those for married females. Specifically, the point estimates suggest that the male elasticity should be roughly 0.10-0.20 while the female elasticity should be 0.50-0.55. Other papers emphasize tradeoffs between home and market work as a potential source of business–cycle variation in measured production. Although our results confirm the existence of such a margin, they suggest that it may

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[8] Recall our discussion of time dummies in Section 4.

be of limited importance in practice for understanding sizeable labor supply responses, especially for men.

Second, several researchers have suggested that households may reduce their market expenditures at retirement because of enhanced opportunities thereafter for home production. We provide an explicit model of this potentiality and show that corresponding parameter estimates are plausible in several respects.

Finally, our findings have implications for the academic literature on intertemporal allocation. Many papers use the Euler–equation approach of Hall [1978] to test the optimality of consumption decisions. Our results suggest that these tests may be misspecified if their estimation equation omits home production.

Appendix

Proof of Proposition 1: For convenience, drop the household index i. Let $I_j = [s_{j-1}, s_j)$, $j = 1, ..., J-1$, and $[s_{J-1}, s_J]$, with $s_0 = S$, $s_{J-1} = R$ and $s_J = T$. We separately solve the optimization problem (1) on each interval I_j and then prove that the co-state variable is continuous at points s_j. Consider a subproblem of (1) on an interval I_j where N_s, A_s, y_s^m, w_s^f are all continuous, which makes it a standard optimal control problem. We can eliminate x_s by substituting (2) into (3) which gives

$$\dot{a}_s = r \cdot a_s + h_s^f \cdot w_s^f - A_s^f \cdot \left[h_s^f\right]^{\xi^f} + h_s^m \cdot w_s^m - A_s^m \cdot [h_s^m]^{\xi^m} - c_s. \tag{30}$$

Since the fixed cost has no effect upon marginal conditions for optimization, we write a Hamiltonian for this problem as

$$\mathcal{H} = N_s \cdot u\left(\frac{c_s}{N_s}\right) + \lambda_s \left(h_s^f \cdot w_s^f - A_s^f \cdot \left[h_s^f\right]^{\xi^f} + h_s^m \cdot w_s^m - A_s^m \cdot [h_s^m]^{\xi^m} - c_s\right)$$

Assume that I_j is such that $h_s^f > 0$ is optimal. Then the necessary conditions for optimality are

$$\frac{\partial \mathcal{H}}{\partial c_s} = 0 \iff \left(\frac{c_s}{N_s}\right)^{\gamma-1} = \lambda_s, \tag{31}$$

$$\frac{\partial \mathcal{H}}{\partial h_s^f} = 0 \iff \lambda_s \left(w_s^f - \xi^f A_s^f \cdot \left[h_s^f\right]^{\xi^f-1}\right) = 0, \tag{32}$$

$$\frac{\partial \mathcal{H}}{\partial h_s^m} = 0 \iff \lambda_s \left(w_s^m - \xi^m A_s^m \cdot [h_s^m]^{\xi^m-1}\right) = 0, \tag{33}$$

$$\dot{\lambda}_s = \rho \cdot \lambda_s - \frac{\partial \mathcal{H}}{\partial a_s} \iff \frac{\dot{\lambda}_s}{\lambda_s} = \rho - r, \tag{34}$$

$$\lambda_s \geq 0 \quad \text{and} \quad \lambda_s \cdot a_s = 0. \tag{35}$$

If I_j is such that $h_s^f = 0$, (32) is replaced by $h_s^f = 0$ (and similarly for h_s^m).

Now we establish continuity of λ_s by induction. Take the last interval of life I_J. (31) shows that $\lambda_s > 0$. Then (35) implies that $a_{s_J} = 0$. Therefore, we can solve the boundary value problem for the system of equations (30)-(34) for any initial value of assets $a_{s_{J-1}}$. Let $\varphi\left(a_{s_{J-1}}\right)$ denote the maximized criterion over the interval I_J. Observe that $\varphi(\cdot)$ is a differentiable function.

Then maximize over the interval I_{J-1} with a criterion

$$\int_{s_{J-2}}^{s_{J-1}} e^{-\rho s} \left[N_s \cdot u\left(\frac{c_s}{N_s}\right)\right] ds + \varphi\left(a_{s_{J-1}}\right)$$

This is a standard problem (e.g. Kamien and Schwartz, p. 153) that has the first order conditions (31)-(34) and a terminal condition

$$\lambda_{s_{J-1}-0} = \varphi'\left(a_{s_{J-1}}\right).$$

From the envelope theorem,

$$\varphi'\left(a_{s_{J-1}}\right) = \frac{\partial \mathcal{H}}{\partial a} = \lambda_{s_{J-1}+0}.$$

Thus λ_s is continuous at $s = s_{J-1}$. Induction over j establishes continuity of λ_s on $[S, T]$.

Since $\lambda_s > 0$, $\frac{\partial \mathcal{H}}{\partial h_s^f} = 0$ immediately implies (9)-(10), and when $h_s^f = 0$, (9)-(10) also trivially hold.

Therefore, the first order conditions (31)-(34) apply on the whole interval $[S, T]$. Solving (34) on $[S, T]$ and substituting the result into (31) gives

$$c_s = N_s \left(\lambda_0\right)^{\frac{1}{\gamma-1}} \cdot e^{\frac{r-\rho}{1-\gamma}s}. \tag{36}$$

Since both the objective function and the law of motion are concave in the state and control variables, the first order conditions are sufficient for a maximum with respect to c_s for any given time path of h_s^m and h_s^f.

The value of $c^* \equiv \left(\lambda_0\right)^{\frac{1}{\gamma-1}}$ obtains from the budget constraint

$$\int_S^T e^{-r \cdot s}\left(h_s^f \cdot w_s^f - A_s^f \cdot \left[h_s^f\right]^{\xi^f} + h_s^m \cdot w_s^m - A_s^m \cdot \left[h_s^m\right]^{\xi^m} - c_s\right) = c^* \cdot \int_S^T e^{-rs} N_s \cdot e^{\frac{r-\rho}{1-\gamma}s} ds,$$

which yields (12).

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Proof of Proposition 2: Drop the household index i and fix $s \geq R$, Using the notation $\sigma = \exp\left(-r + \frac{r-\rho}{1-\gamma}\right)$, Euler equation (36) implies that for any $t \geq 0$

$$c_t = \frac{c_0}{N_0} \cdot \sigma^t e^{rt}.$$

Moreover, Proposition 1 showed that $a_T = 0$. Then integrating (3) from s to T and noting that ($h_s^m = h_s^f = 0$ for all $s \geq R$) gives

$$-a_s = -\int_s^T e^{-r \cdot (t-s)} \cdot c_t \, dt \Longleftrightarrow$$

$$a_s \cdot e^{-rs} = \frac{c_0}{N_0} e^{rs} \int_s^T \sigma^t N_t dt \tag{37}$$

On the other hand, intergrating (3) from S to T, and using (9), (10) and the definition of Y_s^M and Y_s^F we have

$$0 = a_S - a_T = \int_S^T e^{-rt}\left(h_t^f \cdot w_t^f - A_t^f \cdot \left[h_t^f\right]^{\xi^f} + h_t^m \cdot w_t^m - A_t^m \cdot [h_t^m]^{\xi^m} - c_t\right) dt \Longleftrightarrow$$

$$0 = \int_S^T e^{-rt}\left(\left(1 - \theta^f\right) h_t^f \cdot w_t^f + \left(1 - \theta^m\right) h_t^m \cdot w_t^m - c_t\right) dt$$

$$\Longleftrightarrow (1-\theta^m) \int_S^T e^{-rt} h_t^m \cdot w_t^m + \left(1-\theta^f\right) \int_S^T e^{-rt} h_t^f \cdot w_t^f = \int_S^T e^{-rt} \cdot c_t \, dt.$$

$$\Longleftrightarrow (1-\theta^m) Y_s^M + \left(1-\theta^f\right) Y_s^F = e^{rs} \int_S^T e^{-rt} \cdot c_t \, dt.$$

$$\Longleftrightarrow (1-\theta^m) Y_s^M + \left(1-\theta^f\right) Y_s^F = \frac{c_0}{N_0} e^{rs} \int_S^T \sigma^t N_t dt \tag{38}$$

Taking the ratio of (37) to (38), and multiplying the numerator and denominator of the ratio in the right hand side by σ^{-s} leads to (13). ∎

References

[1] Aguiar, Mark; and Erik Hurst "Consumption vs. Expenditure," *Journal of Political Economy*, October 2005, 113(5), pp. 919-948.

[2] Altonji, Joseph G.; Siow, Aloysius, "Testing the Response of Consumption to Income Changes with (Noisy) Panel Data", *Quarterly Journal of Economics*, May 1987; 102(2): 293-328

[3] Attanasio, Orazio P., 1999. "Consumption." in *Handbook of Macroeconomics*, Vol 1, J.B. Taylor and M. Woodford eds. Elsevier.

[4] Banks, James; Blundell, Richard; and Tanner, Sarah, "Is There a Retirement–Savings Puzzle?" *American Economic Review* 88, no. 4 (September 1998): 769-78.

[5] Benhabib, Jess; Rogerson, Richard; and Wright, Randall. "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy* 99, no. 6 (December 1991): 1166–1187.

[6] Bernheim, B. Douglas; Skinner, Jonathan; and Weinberg, Steven, "What Accounts for the Variation in Retirement Wealth Among U.S. Households," *American Economic Review* 91, no. 4 (September 2001): 832-857.

[7] Blundell, Richard and Thomas MaCurdy, "Labor Supply: A Review of Alternative Approaches," in A. Ashenfelter and D. Card, eds., Handbook of Labor Economics, Volume 3 (Amsterdam:Elsevier, 1999): 1569-1695.

[8] Campbell, John Y.; Mankiw, N. Gregory, "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence" in Blanchard, Olivier Jean; Fischer, Stanley, eds. *NBER macroeconomics annual: 1989*. Cambridge, Mass. and London: MIT Press 1989; 185-216.

[9] Campbell, John Y.; Mankiw, N. Gregory, "The response of consumption to income: a cross-country investigation", *European Economic Review* 35:715-721.

[10] Flavin, M., 1981. The Adjustment of Consumption to Changing Expectations about Future Income." *Journal of Political Economy* 89 (5): 974-1009.

[11] Gallant, A. Ronald. *Nonlinear Statistical Models.* New York: John Wiley & Sons, 1987.

[12] Michael Grossman, "On the Concept of Health Capital and the Demand for Health," *JPE*, 1972

[13] Greene, William H., *Econometric Analysis,* New York: Macmillan, 1990.

[14] Gustman Alan L., Steinmeier Thomas L., "Retirement in dual career families: A Structural Model", *Journal of Labor Economics* 18(3) (July 2000): 503–545.

[15] Hall, Robert E. "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence", *Journal of Political Economy.* Dec. 1978; 86(6): 971-87.

[16] Hansen, Lars-Peter; Singleton, Kenneth J., "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models", *Econometrica*, September 1982; 50(5): 1269-86.

[17] Hansen, Lars-Peter; Singleton, Kenneth J., "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns", *Journal of Political Economy*, April 1983; 91(2): 249-65.

[18] House, Christopher, John P. Laitner and Dmitriy Stolyarov, "Valuing Lost Home Production in Dual-Earner Couples", Micigan Retirement Research Center working paper WP 2005-097, March 2005 (UM04-14).

[19] House, Christopher, John P. Laitner and Dmitriy Stolyarov, "Valuing Lost Home Production in Dual-Earner Couples", mimeo, University of Michigan, Oct 2006.

[20] Hurd, Michael; Rohwedder, Susann "The Retirement-Consumption Puzzle: Anticipated and Actual Declines in Spending at Retirement" NBER working paper 9586 (2003).

[21] Hurd, Michael; Rohwedder, Susann, "Consumption and Economic Well-Being at Older Ages: Income- and Consumption-based Poverty Measures in the HRS", Micigan Retirement Research Center working paper 2006-110, January 2006.

[22] Kamien, Morton I.; Schwartz, Nancy L. *Dynamic optimization: The calculus of variations and optimal control in economics and management* Second edition. Advanced Textbooks in Economics, vol. 31. Amsterdam; London and Tokyo: North-Holland; distributed in the U.S. and Canada by Elsevier Science, New York, 1991.

[23] Laitner, John and Silverman, Daniel. "Estimating Life–Cycle Parameters from Consumption Behavior at Retirement," National Bureau of Economic Research working paper No. 11163, 2005.

[24] Laitner, John and Silverman, Daniel. "Consumption, Reitremtn and Social Security: Evaluating the Efficiency of Reform with a Life-Cycle Model", mimeo, University of Michigan, Oct 2006.

[25] McGrattan, Ellen R; Rogerson, Richard; Wright, Randall "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy", *International Economic Review*, May 1997; 38(2): 267-90.

[26] Pencavel, John, "The Market Work Behavior and Wages of Women: 1975-94" *Journal of Human Resources*. 33(4) (Fall 1998), 771-804.

[27] Rupert, Peter; Rogerson, Richard; Wright, Randall "Estimating Substitution Elasticities in Household Production Models" *Economic Theory*, June 1995; 6(1): 179-93.

[28] Shapiro, Matthew D., 1984. "The Permanent Income Hypothesis and the Real Interest Rate - Some Evidence from Panel Data." Economics Letters, 14 (1) : pp. 93-100.

[29] Shea John, 1994. "Should We Test The Life-Cycle Permanent Income Hypothesis With Food-Consumption Data." *Economics Letters* 45 (1): pp. 63-68 May.

[30] Tobin, James, "Life Cycle Saving and Balanced Growth", in W. Fellner (ed.), *Ten economic Studies in the Tradition of Irving Fisher*. New York: Wiley, 1967.