University of Michigan Retirement Research Center

Working Paper
WP 2006-140

Social Security Privatization with Income-Mortality Correlation

Shinichi Nishiyama and Kent Smetters

MRRC

Project #: UM06-07

“Social Security Privatization with Income-Mortality Correlation”

Shinichi Nishiyama
Georgia State University

Kent Smetters
The Wharton School

November 2006

Michigan Retirement Research Center
University of Michigan
P.O. Box 1248
Ann Arbor, MI 48104
http://www.mrrc.isr.umich.edu/
(734) 615-0422

Acknowledgements

This work was supported by a grant from the Social Security Administration through the Michigan Retirement Research Center (Grant # 10-P-98358-5). The findings and conclusions expressed are solely those of the author and do not represent the views of the Social Security Administration, any agency of the Federal government, or the Michigan Retirement Research Center.

Social Security Privatization with Income-Mortality Correlation

Shinichi Nishiyama and Kent Smetters

Abstract

While privatizing Social Security can improve labor supply incentives, it can also reduce risk sharing. We simulate a 50-percent privatization using an overlapping-generations model where heterogeneous agents with elastic labor supply face idiosyncratic earnings shocks and longevity uncertainty. When wage shocks are insurable, privatization produces about $30,100 of extra resources for each future household after all transitional losses have been paid. When wages are not insurable, privatization *reduces* efficiency by about $8,100 per future household. We check the robustness of these results to different model specifications as well as policy reforms and arrive at several surprising conclusions. First, privatization performs better in a closed economy, where interest rates decline with capital accumulation, than in an open economy. Second, privatization also performs better when an actuarially-fair private annuity market does *not* exist. Third, government matching of private contributions on a progressive basis is not very effective at restoring efficiency and can actually harm.

Authors' Acknowledgements

Our research was supported by the U.S. Social Security Administration through a grant to the Michigan Retirement Research Consortium as part of the SSA Retirement Research Consortium. The first stage of this research was done when Nishiyama was at the Congressional Budget Office. The opinions and conclusions expressed are solely those of the authors and do not represent the opinions or policy of SSA, CBO, any agency of the Federal Government, or the MRRC. Helpful comments were received from participants at the 2004 NBER Summer Institute, 2005 ASSA Meeting, Congressional Budget Office, Georgia State University, two anonymous referees and the Editor.

I. Introduction

It has been known for some time that shutting down ("privatizing") a pay-as-you-go social security system would simply reallocate resources between generations when all economic variables are deterministic and labor supply is inelastic. In particular, no new resources would be created in present value once the "winners" have fully compensated the "losers."[1] Allowing for *elastic* labor supply as well as various *risks* that are difficult to insure in the private market, however, changes things considerably.

On one hand, privatization could produce efficiency gains by reducing the effective tax rate on labor supply. Payroll taxes distort labor supply decisions for many participants because a mature pay-as-you-go social security provides an internal rate of return on average payroll contributions that is below the return that participants could have received in the private market. As a result, for every dollar contributed to a mature social security, future benefits increase by less than a dollar in present value – the difference is an effective tax. This tax services the implicit debt inherited from past generations who received more from Social Security than they paid. In addition, the U.S. Social Security system, in particular, is designed to be progressive in nature by giving a household with a lower Average Index of Monthly Earnings (AIME) a larger Social Security benefit *relative* to their AIME, i.e., a larger "replacement rate". This intra-generational redistribution increases effective marginal tax rates on households with AIME's above the economy-wide average while reducing effective marginal tax rates on households with smaller AIME's.

On the other hand, the U.S. Social Security system also provides two sources of risk sharing that could be reduced by privatization. First, the progressive benefit formula shares wage shocks among participants that are difficult to insure in the private market. Privatization, therefore, could reduce this insurance unless it were complemented with some other form of redistribution. Second, Social Security pays benefits until the beneficiary and spouse die rather than over a fixed number of years. To the extent that longevity uncertainty is also difficult to insure privately, privatization could also reduce annuity protection.

I.A. Overview of Our Approach

Determining the overall change in efficiency from privatization requires simulation analysis. We use a heterogeneous overlapping-generations model in which agents with elastic labor supply face

[1] See, e.g., Breyer [1989], Feldstein [1995], Geanakoplos, Mitchell and Zeldes [1998], Murphy and Welch [1998], Mariger [1999], Shiller [1999], and Diamond and Orszag [2003].

idiosyncratic earnings shocks and longevity uncertainty. The economy's entire transition path after privatization is calculated.[2] To determine the Hicksian efficiency gain or loss from this reform, each household of every generation and income class receives a lump-sum rebate or tax to return their expected remaining lifetime utility to their pre-reform levels. If the *net* amount of new resources remaining after these rebates and taxes is positive, then privatization produces an efficiency gain; if negative, an efficiency loss. Following Auerbach and Kotlikoff (1987), new net resources (positive or negative) are distributed to future households in equal amounts (growth adjusted over time).

We consider a stylized partial privatization where traditional Social Security benefits are reduced *slowly* across time; the initial elderly are fully protected. Benefit levels eventually reach 50 percent of their original scheduled value. Payroll taxes, which cover Social Security benefits on a pay-as-you-go basis, are, therefore, also reduced over time. While younger workers alive at the time of the reform will eventually see their payroll taxes decline, their *effective* labor income tax rates actually increase throughout a large period of their lifetime because they help pay for a large part of the transitional benefits owed to retirees and older workers without receiving full benefits themselves. In this sense, the "transition costs" to personal accounts are effectively financed with a labor income tax. However, workers born in the long run enjoy smaller effective tax rates on their labor income.

I.B. Summary of Our Findings

We find that privatization can substantially improve labor supply incentives. When wage shocks are assumed to be insurable in the private market, our stylized partial privatization produces new net resources equal to $30,100 per future household in our benchmark model. However, when, more realistically, wages are not insurable, privatization *reduces* efficiency by about $8,100 per future household despite improving labor supply incentives. This loss occurs even though privatization substantially increases the welfare of those born in the *long run* by increasing the capital stock and reducing the effective tax rates on labor income.

The efficiency loss that we calculate when wages are not insurable, though, makes four key assumptions that might appear at first glance to be biased against privatization. Several surprising insights emerge as we investigate each of these assumptions more closely.

[2]This paper builds upon the work by İmrohoroğlu, İmrohoroğlu, and Joines [1995], Huang, Imrohoroğlu, and Sargent [1997], and Conesa and Krueger [1999]. İmrohoroğlu *et al.* focus on steady states and find that the value of risk sharing is outweighed by the reduction in capital. Huang *et al.* and Conesa and Krueger consider transition dynamics and present welfare calculations for different generations. Our analysis finds that while privatization typically raises long-run welfare it often fails to increase efficiency due to larger losses of transitional generations.

First, our benchmark economy is closed to international capital flows. As a result, capital accumulation after privatization reduces interest rates, discouraging more accumulation. If, instead, capital could flow across borders then more capital could be accumulated with no reduction in the rate of return to saving. However, interestingly, we find that the efficiency *losses* from partial privatization are even *larger* (equal to about $10,100 per future household) in a small open economy version of our model that allows for perfect international capital flows. As expected, privatization leads to substantially more capital accumulation with open capital markets. But, for the purpose of determining *efficiency* gains, the higher interest rate in the open economy case, relative to privatization in the closed economy, also means that it is more costly to borrow against the long-run gains from privatization in order to compensate households alive during the transition that would otherwise lose from privatization. This finding emphasizes the fact that gains to macroeconomic variables alone are not necessarily good metrics for inferring efficiency gains.

Second, our benchmark calculations assume that a private annuity market does not exist, and so the pre-reform Social Security system not only provides insurance against wage uncertainty but against longevity as well. Households, therefore, must guard against outliving their resources after privatization by relying more on precautionary saving, which is less efficient at smoothing consumption across states than insurance markets. However, rather surprisingly, we find that allowing for an actuarially-fair private annuity market also *increases* efficiency *losses* (to about $10,900 per future household) relative to our benchmark case. This result can be mostly traced to the relative smaller amount of precautionary saving after privatization when private annuity markets exist. The higher interest rate in this case increases the cost at which compensation can be made across generations. Less precautionary savings and lower labor supply also reduce the tax bases relative to the benchmark privatization, thereby increasing the income tax rates that are required to fund the rest of government.

Third, our benchmark model ignores the fact that poor households might not live as long as wealthier households. As a result, the initial Social Security system in our benchmark model might overestimate the amount of redistribution – and, hence, risk sharing – that is being provided. However, we find that reducing progressivity in the baseline Social Security system does not have a monotonic impact on the efficiency gains following privatization: efficiency losses can actually worsen relative to the baseline. The reason is that a lower assumed amount of progressivity also reduces the amount of distortion caused by the payroll tax in the initial baseline economy, leaving less opportunity to produce efficiency gains from privatization.

Fourth, the stochastic nature of wages in our benchmark economy is calibrated to the Panel Study of Income Dynamics, which is likely measured with a fair amount of error. The insurance value provided by Social Security, therefore, might be over-estimated in our baseline economy. But we find that reducing transitory shocks in our model also does not have a monotonic impact on efficiency gains. The reason is that a reduction in transitory shocks also effectively makes any shock that does occur more permanent. Only in the limit, when the shocks go to zero, are efficiency gains guaranteed from privatization.

Another potential line of criticism of our results is that the privatization plan that we consider is fairly stylized and does not explicitly include any mechanism that shares the idiosyncratic wage shocks that were previously partially insured under Social Security. We, therefore, consider two modifications to our privatization plan itself.

First, we also simulate privatization where the government matches contributions made by poorer households. The match is financed with general-revenue income taxes and is reduced linearly across income classes so that a household with median income receives no match. We find that matching contributions does not have a monotonic effect on efficiency gains either. Too much matching can actually worsen efficiency by introducing new marginal tax rates associated with the match phase-out as well as increased income taxes.

Second, we show that simply increasing the progressivity of the smaller Social Security program that remains after partial privatization is more effective. Compared to contribution matching, this alternative produces smaller *marginal* tax rates because Social Security benefits are computed based on lifetime earnings whereas the match is based on contemporaneous earnings. However, efficiency losses still emerge because, while more progressive, the new system is smaller in scale than the traditional system being replaced. But we show that additional progressivity allows privatization to produce efficiency *gains* if the transition were financed with a *consumption* tax.

I.C. Outline of Rest of the Paper

The rest of the paper is as follows: Section II describes the model; Section III explains the calibration of the model; Section IV presents simulation results from privatization with and without insurance markets for wage uncertainty; Section V investigates the robustness of the results in Section IV to alternative model designs; Section VI considers alternative policy reforms; and, Section VII concludes the paper. The Appendices explain the computational algorithm in more detail.

II. Model

Our model has three sectors: heterogeneous households with elastic labor supply; a competitive representative firm with constant-returns-to-scale production technology; and a government with a full commitment technology. Like most previous analyses of Social Security reform, our model's pre-reform neoclassical economy is stationary by construction, and so we don't capture the effects of projected demographic changes.[3] We, however, are only interested in comparing the efficiency gains from privatization against the baseline, not examining the implications of demographics.

We investigated the U.S. federal tax system in Nishiyama and Smetters [2005*a*]. The current paper uses a more extensive model with a detailed social security sector to investigate arguably the most important public program for income risk-sharing inside the United States, the U.S. Social Security system. The more extensive model contained in this paper requires the addition of another state variable, which significantly increases the complexity of the model and the required computation time from several hours to typically several days per simulation. We also consider an extended range of modeling assumptions as well as policy experiments.

II.A. The Household Sector

Households are heterogeneous with respect to the following variables: age i; working ability e (measured by hourly wages); beginning-of-period wealth holdings a; and, average historical earnings b that determine their Social Security benefits. Each year, a large number (normalized to unity) of new households of age 20 enter the economy. Population grows at a constant rate ν. A household of age i observes an idiosyncratic working ability shock e at the beginning of each year and chooses its optimal consumption c, working hours h, and end-of-period wealth holding a', taking as given the government's policy schedule and future factor prices.[4] At the end of each year, a fraction of households die according to standard mortality rates; no one lives beyond age 109. For simplicity, all households represent two-earner married couples of the same age.

Let $\mathbf{s}$ denote the individual state of a household,

$$\mathbf{s} = (i, e, a, b),$$

[3] We are aware of only a few papers, including De Nardi, İmrohoroğlu and Sargent (1999), Kotlikoff, Smetters and Walliser (2001), and Nishiyama (2004), that attempt to capture the effect of non-stationary demographics on baseline factor prices.

[4] Because there are no aggregate shocks in the present model, households can perfectly foresee factor prices and policy variables using the current distribution of households and the current policy variables. Yet, their own future working ability and mortality are uncertain.

where $i \in I = \{20, ..., 109\}$ is the household's age, $e \in E = [e_{\min}, e_{\max}]$ is its age-dependent working ability (the hourly wage), $a \in A = [a_{\min}, a_{\max}]$ is its beginning-of-period wealth, and $b \in B = [b_{\min}, b_{\max}]$ is its average historical earnings for Social Security purposes.[5]

Let $\mathbf{S}_t$ denote the state of the economy at the beginning of year t,

$$\mathbf{S}_t = (x_t(\mathbf{s}), W_{LS,t}, W_{G,t}),$$

where $x_t(\mathbf{s})$ is the joint distribution of households where $\mathbf{s} \in I \times E \times A \times B$. $W_{LS,t}$ is the beginning-of-period net wealth held by the Lump-Sum Redistribution Authority (LSRA), which as described below, is used to determine the efficiency gain or loss from privatization. $W_{G,t}$ is the net wealth of the rest of the government.

Let $\mathbf{\Psi}_t$ denote the government policy schedule known at the beginning of year t,

$$\mathbf{\Psi}_t = \{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, \tau_{I,s}(.), \tau_{P,s}(.), tr_{SS,s}(\mathbf{s}), tr_{LS,s}(\mathbf{s})\}_{s=t}^{\infty},$$

where $C_{G,s}$ is government consumption, $\tau_{I,s}(.)$ is an income tax function, $\tau_{P,s}(.)$ is a payroll tax function for Social Security (OASDI), $tr_{SS,s}(\mathbf{s})$ is a Social Security benefit function, and $tr_{LS,s}(\mathbf{s})$ is an LSRA wealth redistribution function. The specifications of these functions are described below.

The household's problem is

$$v(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t) = \max_{c,h} u_i(c,h) + \beta(1+\mu)^{\alpha(1-\gamma)}\phi_i E\left[v\left(\mathbf{s}', \mathbf{S}_{t+1}; \mathbf{\Psi}_{t+1}\right)|e\right] \tag{1}$$

subject to

$$\begin{aligned} a' = \frac{1}{1+\mu}\{&w_t eh + (1+r_t)(a + tr_{LS,t}(\mathbf{s})) - \tau_{I,t}(w_t eh, r_t(a + tr_{LS,t}(\mathbf{s})), tr_{SS,t}(\mathbf{s})) \\ &- \tau_{P,t}(w_t eh) + tr_{SS,t}(\mathbf{s}) - c\} \geq a'_{\min,t}(\mathbf{s}), \\ a = 0 &\text{ if } i = 20, \quad a \geq 0 \text{ if } i \geq 65, \end{aligned} \tag{2}$$

where the utility function, $u_i(.)$, takes the Cobb-Douglas form nested within a time-separable isoelastic

[5] The average historical earnings are used to calculate the Social Security benefits of each household. The variable b approximates the average indexed monthly earnings (AIME) multiplied by 12 as of age i.

specification,

$$u_i(c,h) = \frac{\{((1+n_i/2)^{-\zeta}c)^{\alpha}(h_{\max}-h)^{1-\alpha}\}^{1-\gamma}}{1-\gamma}; \tag{3}$$

γ is the coefficient of relative risk aversion; n_i is the number of dependent children at the parents' age i; ζ is the "adult equivalency scale" that converts the consumption by children into their adult equivalent amounts; and, $h_{\max}$ is the maximum working hours. Wages are stochastic and follow a Markov process that is described in more detail below.

The constant β is the rate of time preference; ϕ_i is the survival rate at the end of age i; w_t is the wage rate per efficiency unit of labor (accordingly, $w_t eh$ is total labor compensation at age i in time t); and r_t is the rate of return to capital. Individual variables of the model are normalized by the exogenous rate of labor augmenting technological change, μ. Our choice for $u_i(.)$ is consistent with the conditions that are necessary for the existence of a long-run steady state in the presence of constant population growth. Hence, μ is also equal to the per-capita growth rate of output and capital in steady state. The term $\beta(1+\mu)^{\alpha(1-\gamma)}$, therefore, is the *growth-adjusted* rate of time preference.

$a'_{\min,t}(\mathbf{s})$ is the state-contingent minimum level of end-of-period wealth that is sustainable, that is, even if the household receives the worst possible shocks in future working abilities.[6] At the beginning of the next period, the state of this household when private annuity markets do not exist becomes

$$\mathbf{s}' = (i+1, e', a'+q_t, b'), \tag{4}$$

where q_t denotes accidental bequests that a household receives at the end of the period. In the presence of perfect annuity markets, the household's state in the next period is instead

$$\mathbf{s}' = (i+1, e', a'/\phi_i, b'). \tag{5}$$

[6]In particular, $a'_{\min,t}(\mathbf{s})$ is allowed to be negative but cannot exceed in magnitude the present value of the worst possible future labor income stream at maximum working hours, sometimes called the "natural borrowing limit." Although not shown explicitly in equation (2) in order to save on notation, any borrowing (i.e., $a' < 0$) by an agent age i at time t must be done at rate $(1+r_t)/\phi_i - 1$ in order cover the chance that they will die before repaying their loan.

The average historical earnings for this household, b, follows the following process,

$$b' = \begin{cases} 0 & \text{if } i \leq 24 \\ \frac{1}{i-24}\{(i-25)b\frac{w_t}{w_{t-1}} + \min(w_t eh/2, weh_{\max,t})\} & \text{if } 25 \leq i \leq 59 \\ b/(1+\mu) & \text{if } i \geq 60 \end{cases}, \tag{6}$$

where $weh_{\max,t}$ is the Old-Age, Survivors, and Disability Insurance (OASDI) tax cap, which is $80,400 in 2001. U.S. Social Security benefits are computed on the basis of the highest 35 years of earnings. For simplicity, the model assumes that the highest 35 years of earnings correspond to ages between 25 and 59.

Let $x_t(\mathbf{s})$ denote the measure of households, and let $X_t(\mathbf{s})$ be the corresponding cumulative measure. The measure of households is adjusted by the steady-state population growth rate, ν. The population of age 20 households is normalized to unity in the baseline economy along the balanced growth path, that is,

$$\int_E \mathrm{d}X_t(20, e, 0, 0) = 1.$$

Let $\mathbf{1}_{[a=y]}$ be an indicator function that returns 1 if $a = y$ and 0 if $a \neq y$. Then, the law of motion of the measure of households is

$$x_{t+1}(\mathbf{s}') = \frac{\phi_i}{1+\nu}\int_{E\times A\times B} \mathbf{1}_{[a'=a'(\mathbf{s},\mathbf{S}_t;\mathbf{\Psi}_t)+q_t]}\mathbf{1}_{[b'=b'(w_t eh(\mathbf{s},\mathbf{S}_t;\mathbf{\Psi}_t),b)]}\pi_{i,i+1}(e'|e)\mathrm{d}X_t(\mathbf{s}),$$

where $\pi_{i,i+1}$ denotes the transition probability of working ability from age i to age $i+1$.

The *aggregate* value of accidental bequests each period is deterministic in our model because all risks are idiosyncratic and, therefore, uncorrelated across households. Accidental bequests could, therefore, be simply distributed equally and deterministically across all surviving households, as in previous work. That approach, however, suffers from two shortcomings. First, households would anticipate receiving a bequest with certainty, thereby artificially crowding out their pre-bequest savings. This savings reduction would be mitigated if bequests were random. Second, empirically, the inequality of bequests is important in generating a realistic measure of wealth inequality.

Our alternative strategy, therefore, distributes bequests randomly to surviving working-age households. Each household receives a bequest q_t with constant probability η:

$$q_t = \frac{\sum_{i=20}^{109}(1-\phi_i)\int_{E\times A\times B} a'(\mathbf{s},\mathbf{S}_t;\mathbf{\Psi}_t)\mathrm{d}X_t(\mathbf{s})}{\sum_{i=20}^{109}(1-\phi_i)\int_{E\times A\times B}\mathrm{d}X_t(\mathbf{s})},$$

$$\eta = \frac{\sum_{i=20}^{109}(1-\phi_i)\int_{E\times A\times B}\mathrm{d}X_t(\mathbf{s})}{\sum_{i=20}^{64}\phi_i\int_{E\times A\times B}\mathrm{d}X_t(\mathbf{s})}.$$

where q_t is the average wealth left by deceased households, and η is the ratio of deceased household to the surviving working-age households. In other words, a constant fraction η of households across all income groups will receive a bequest of size q_t while a constant fraction $(1-\eta)$ of households will not. But ex-ante, each household only knows it will receive a bequest with probability η.[7]

Despite this concentration of bequests, however, our model still does not capture a realistic concentration of wealth, a well-known problem for this class of models [Diaz-Jimenez *et al.*, 1997]. While our model produces a plausible wealth Gini Index equal to 67.9%, the top 1% of households in our model hold only about 12% of the economy's wealth in our baseline and up to 15% of wealth under some of our alternative model assumptions.

II.B. Government

Government tax revenue consists of federal income tax $T_{I,t}$, and payroll tax for Social Security (OASDI) $T_{P,t}$. These revenues are

$$T_{I,t} = \sum_{i=20}^{109}\int_{E\times A\times B}\tau_{I,t}(w_t e h(\mathbf{s},\mathbf{S}_t;\mathbf{\Psi}_t), r_t(a+tr_{LS,t}(\mathbf{s})), tr_{SS,t}(\mathbf{s}))\mathrm{d}X_t(\mathbf{s}), \tag{7}$$

$$T_{P,t} = \sum_{i=20}^{109}\int_{E\times A\times B}\tau_{P,t}(w_t e h(\mathbf{s},\mathbf{S}_t;\mathbf{\Psi}_t))\mathrm{d}X_t(\mathbf{s}). \tag{8}$$

Social Security (OASDI) benefit expenditure $Tr_{SS,t}$ is

$$Tr_{SS,t} = \sum_{i=20}^{109}\int_{E\times A\times B} tr_{SS,t}(\mathbf{s})\mathrm{d}X_t(\mathbf{s}). \tag{9}$$

[7] Future work could go even a step further and allow for a correlation between a household's own income and the size of the bequest that they receive. At this point, however, we are not aware of any careful empirical work that would allow us to include this correlation into our model.

The law of motion of the government wealth (normalized by productivity growth and population growth) is

$$W_{G,t+1} = \frac{1}{(1+\mu)(1+\nu)}\{(1+r_t)W_{G,t} + T_{I,t} + T_{P,t} - Tr_{SS,t} - C_{G,t}\}, \tag{10}$$

where $C_{G,t}$ is government consumption.

II.C. Measuring Efficiency Gains

We follow Auerbach and Kotlikoff (1987) by measuring efficiency gains from Social Security privatization using a Lump-Sum Redistribution Authority that compensates households who would otherwise lose from reform.[8] To be clear, the LSRA is not being proposed as an actual government institution. Instead, it is simply a hypothetical mechanism that allows us to measure the standard Hicksian efficiency gains in general equilibrium associated with privatization. A policy reform that increases Hicksian efficiency is *potentially* Pareto improving whereas a reform that reduces efficiency cannot be Pareto improving.[9]

To see how the LSRA works, suppose that a new policy is announced at the beginning of period 1. First, the LSRA makes a lump-sum compensating variation transfer or tax, $tr_{CV,1}(\mathbf{s})$, to each *living* household of age i in order to return its expected remaining lifetime utility at state $\mathbf{s}$ to its pre-reform level in the baseline (pre-reform) economy. Next, the LSRA makes a lump-sum transfer or tax, $tr_{CV,t}(\mathbf{s})$, to *each future* household (born in periods 2, 3, ...) to make it as well off as in the baseline economy, conditional on its initial state at age 20. Thus far, however, the net present value of these taxes and transfers across living and future households will generally not sum to zero. So, finally, the LSRA makes an additional lump-sum transfer (tax), Δtr, to each *future* household so that the net present value across all transfers is zero. For illustrative purposes, we assume, like Auerbach and Kotlikoff, that these additional transfers are uniform across future generations on a growth-adjusted basis.

[8] We, however, extend the Auerbach and Kotlikoff approach to a heterogeneous-agent environment.

[9] Of course, constructing a policy that is *actually* Pareto improving from a policy that improves efficiency is a tougher task. A " horse race" exist between the amount of household heterogeneity in the model versus the degrees of freedom that the modeler believes that the government has in its policy toolbox. With lots of heterogeneity but few degrees of freedom, it will be hard to construct a policy that is actually Pareto improving even if it is potentially Pareto improving. With little heterogeneity and more degrees of freedom, such a possibility materializes. The actual degrees of freedom, of course, depend on the perceived informational constraints, constitutional issues and political beliefs (e.g., are age-indexed policies acceptable?). Public economists, therefore, have historically taken the minimalist approach and simply focused on potentially Pareto improving policies. Our paper also falls within that modest tradition.

The lump-sum transfers made by the LSRA, therefore, are

$$(11)\qquad tr_{LS,t}(\mathbf{s}) = \begin{cases} tr_{CV,t}(\mathbf{s}) & \text{if } t = 1 \\ tr_{CV,t}(\mathbf{s}) + \Delta tr & \text{if } t > 1 \text{ and } i = 20 \\ 0 & \text{otherwise} \end{cases}.$$

If $\Delta tr > 0$ then privatization has produced net new resources and so we say that this reform “increases efficiency." Conversely, if $\Delta tr < 0$ then privatization “reduces efficiency."

The aggregate net lump-sum transfers / taxes to living households at time t, $Tr_{LS,t}$, is

$$(12)\qquad Tr_{LS,t} = \sum_{i=20}^{109} \int_{E\times A\times B} tr_{LS,t}(\mathbf{s}) \mathrm{d}X_t(\mathbf{s}).$$

The law of motion of the LSRA wealth (normalized by productivity growth and population growth), therefore, is

$$(13)\qquad W_{LS,t+1} = \frac{1}{(1+\mu)(1+\nu)}(1+r_t)(W_{LS,t} - Tr_{LS,t}).$$

II.D. Aggregation and Production

National wealth W_t is the sum of total private wealth, government net wealth $W_{G,t}$, and LSRA net wealth $W_{LS,t}$; and total labor supply L_t is measured in efficiency units:

$$(14)\qquad W_t = \sum_{i=20}^{109} \int_{E\times A\times B} a \, \mathrm{d}X_t(\mathbf{s}) + W_{LS,t} + W_{G,t},$$

$$(15)\qquad L_t = \sum_{i=20}^{109} \int_{E\times A\times B} e \, h(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t) \, \mathrm{d}X_t(\mathbf{s}).$$

In a closed economy, capital stock is equal to national wealth, that is, $K_t = W_t$, and gross national product Y_t is determined by a constant-returns-to-scale production function,

$$Y_t = F(K_t, L_t).$$

The profit-maximizing condition for this competitive firm is

$$F_K(K_t, L_t) = r_t + \delta, \tag{16}$$

$$F_L(K_t, L_t) = w_t, \tag{17}$$

where δ is the depreciation rate of capital.

In a small open economy, factor prices, r_t^* and w_t^* are fixed at international levels, and domestic capital stock $K_{D,t}$ and labor supply L_t are determined so that the firm's profit maximizing condition satisfies,

$$F_K(K_{D,t}, L_t) = r_t^* + \delta,$$
$$F_L(K_{D,t}, L_t) = w_t^*.$$

Gross domestic product $Y_{D,t}$ is determined by the production function,

$$Y_{D,t} = F(K_{D,t}, L_t),$$

and gross national product Y_t is determined by

$$Y_t = (r_t^* + \delta)W_t + w_t^* L_t.$$

II.E. Recursive Competitive Equilibrium

Definition **Recursive Competitive Equilibrium.** Let $\mathbf{s} = (i, e, a, b)$ be the individual state of households, let $\mathbf{S}_t = (x_t(\mathbf{s}), W_{LS,t}, W_{G,t})$ be the state of the economy, and let $\mathbf{\Psi}_t$ be the government policy schedule known at the beginning of year t,

$$\mathbf{\Psi}_t = \{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, \tau_{I,s}(.), \tau_{P,s}(.), tr_{SS,s}(\mathbf{s}), tr_{LS,s}(\mathbf{s})\}_{s=t}^{\infty}.$$

A series of factor prices $\{r_s, w_s\}_{s=t}^{\infty}$, accidental bequests $\{q_s\}_{s=t}^{\infty}$, the policy variables $\{W_{LS,s+1}, W_{G,s+1}, C_{G,s}, tr_{LS,s}(\mathbf{s})\}_{s=t}^{\infty}$, the parameters of policy functions $\{\boldsymbol{\varphi}_s\}_{s=t}^{\infty}$, the value function of households $\{v(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s)\}_{s=t}^{\infty}$, the decision rule of households

$$\{\mathbf{d}(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s)\}_{s=t}^{\infty} = \{c(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s), h(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s), a'(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s)\}_{s=t}^{\infty},$$

and the measure of households $\{x_s(\mathbf{s})\}_{s=t}^{\infty}$, are in a recursive competitive equilibrium if, in every period $s = t, ..., \infty$, each household solves the utility maximization problem (1)–(6) taking $\mathbf{\Psi}_t$ as given; the firm solves its profit maximization problem, the capital and labor market conditions (14)–(17) clear, and the government policy schedule satisfies (7)–(13).

In steady-state,

$$\mathbf{S}_{t+1} = \mathbf{S}_t$$

for all t and $\mathbf{s} \in I \times E \times A \times B$.

III. Calibration

III.A. Households

The coefficient of relative risk aversion, γ, is assumed to be 2.0. The number of dependent children at the parents' age i, n_i, is calculated using the Panel Study of Income Dynamics (PSID) 2003 Family Data as shown in Table III. The "adult equivalency scale," ζ, is set at 0.6.[10] As discussed later, β is chosen to hit a target capital-output ratio that produces an interest rate of 6.25 percent in the initial steady state.[11] The maximum working hours of husband and wife, $h_{\max}$, is set at 8,760, equal to 12 hours per day per person $\times$ 365 days $\times$ two persons.[12] A smaller value for $h_{\max}$ would reduce the effective labor supply elasticity, and tend to reduce the gains from privatization. The parameter α is chosen so that the average working hours of households between ages 20 and 64 equals 3,576 hours in the initial steady-state economy, the average number of hours supplied by married households in the 2003 PSID. Many of these parameters are summarized in Tables I and II. The parameters shown in Table I are the same for all of our privatization simulations. The parameters shown in Table II depend on the specification of the model that we assume in the initial steady state.

[10]Hence, a married couple with two dependent children must consume about 52 percent (i.e., $2^{0.6} = 1.517$) more than a married couple with no children to attain the same level of utility, ceteris paribus.

[11]It is well known that the exact choice of interest rate in a model without aggregate uncertainty is ambiguous [e.g., Blanchard and Fisher 1989, 104]. On one hand, an interest rate of 6.25% is larger than the historic risk-free rate. On the other hand, it is below the historic marginal product of capital. Some papers calibrate to the risk-free rate while others target capital's marginal product.

[12]The 95th and 99th percentiles of the working hours per married couple of aged 20-64 in the 2003 PSID are 5,719 and 6,810, respectively.

The working ability in this calibration corresponds to the hourly wage (labor income per hour) of each household in the 2003 PSID.[13] The average hourly wage of a married couple ("head" and "wife" in PSID) used in the calibration is calculated by

$$\text{Hourly Wage} = \frac{\text{Labor Income (head + wife) + Payroll Taxes / 2}}{\max\{\text{Total Hours Worked (head + wife), 2080}\}}.$$

We adjusted the salaries in the numerator by adding imputed payroll taxes paid by their employers, which allows us to levy the entire payroll tax on employees in order to incorporate the payroll tax ceiling. The max operator in the denominator adjusts the hourly wage for a small fraction of households in the PSID with large reported salaries but few reported working hours such as the self-employed.

Table IV shows the eight discrete levels of working abilities of five-year age cohorts. We use a shape-preserving cubic spline interpolation between each five-year age cohort to obtain the working ability for each age cohort.[14] In the version of our model where we "turn off" the idiosyncratic wage shocks, the hourly wages of the representative household are assumed to be those of the 40–60th percentile households.

Table IV, however, only shows the different potential "wage buckets" by age as well as the proportion of households in each bucket. It does not itself capture the uncertainty over wages. Using PSID, therefore, we estimate Markov transition matrixes that specify the probabilities that a household's wage will move from one level to a different level the next year. Separate transition matrixes were constructed for four age ranges—20-29, 30-39, 40-49, and 50-59—in order to capture the possibility that the probabilities themselves might change over the lifecycle. For households aged 60 or older, we used the matrix for ages 50-59. The Appendix reports the matrixes in detail. We check the sensitivity of our simulation results to this specification later in the paper.

The population growth rate ν is set to one percent per year while the survival rate ϕ_i at the end of age $i = \{20, ..., 109\}$ are the weighted averages of the male and female survival rates, as calculated by the Social Security Administration (2001). The survival rates at the end of age 109 are replaced by zero, thereby capping the maximum length of life.

[13] According to Bureau of Labor Statistics data, the average hourly earnings of production workers have increased by 2.9 percent from 2001 to 2002. Since the 2003 PSID wages correspond to year 2002 while our tax function introduced below is calibrated to the year 2001, we divide the PSID wages shown in Table IV by 1.029 to convert the hourly wages in 2002 into growth-adjusted wages in 2001.

[14] An alternative approach of estimating eight different wage rates for each age would have relied on too few observations.

III.B. Production

Capital K is the sum of private fixed assets and government fixed assets. In 2000, private fixed assets were \$21,165 billion, government fixed assets were \$5,743 billion, and the public held about \$3,410 billion of government debt.[15] Government net wealth, therefore, is set equal to 9.5 percent of total private wealth in the initial steady-state economy. Moreover, the time preference parameter β is chosen in each variant of our model explored below so that the capital-GDP ratio in the initial steady state economy is 2.74, the empirical value in 2000.[16]

Production takes the Cobb-Douglas form,

$$F(K_t, L_t) = A_t K_t^{\theta} L_t^{1-\theta}.$$

where, recall, L_t is the sum of working hours in efficiency units. The capital share of output is given by

$$\theta = 1 - \frac{\text{Compensation of Employees } + (1-\theta) \times \text{Proprietors' Income}}{\text{National Income } + \text{ Consumption of Fixed Capital}}.$$

The value of θ in 2000 was 0.30.[17] The annual per-capita growth rate μ is assumed to be 1.8 percent, the average rate between 1869 to 1996 (Barro, 1997). Total factor productivity A is set at 0.949, which normalizes the wage (per efficient labor unit) to unity.

The depreciation rate of fixed capital δ is chosen by the following steady-state condition,

$$\delta = \frac{\text{Total Gross Investment}}{\text{Fixed Capital}} - \mu - \nu.$$

In 2000, private gross fixed investment accounted for 17.2 percent of GDP, and government (federal and state) gross investment accounted for 3.3 percent of GDP.[18] With a capital-output ratio of 2.74, the ratio of gross investment to fixed capital is 7.5 percent. Subtracting productivity and population growth rates, the annual depreciation rate is 4.7 percent.

III.C. The Government

Federal income tax and state and local taxes are assumed to be at the level in year 2001 before the passage of the "Economic Growth and Tax Relief Reconciliation Act of 2001" (EGTRRA). Since

[15] Source: Department of Commerce, Bureau of Economic Analysis.
[16] *Ibid.*
[17] *Ibid.*
[18] *Ibid.*

households in our model are assumed to be married, we use a standard deduction of $7,600. However, following Altig *et al.* (2001), we allow higher income households to itemize deductions when it is more valuable to do so, and we assume that the value of the itemized deduction increases linearly in the Adjusted Gross Income.[19] The additional exemption per dependent person is $2,900 where the number of dependent children is consistent with Table III. Table V shows the statutory marginal tax rates before EGTRRA.[20] As noted earlier, a household's labor income in this calibration includes the imputed payroll tax paid by its employer. Thus, taxable income is obtained by subtracting the employer portion of payroll tax from labor income.

The standard deduction, the personal exemption, and all tax brackets grow with productivity over time so that there is no real bracket creep; this indexing is also needed for the initial economy to be in steady state. Since the effective tax rate on capital income is reduced by investment tax incentives, accelerated depreciation and other factors (Auerbach, 1996), the tax function is further adjusted so that the cross-sectional average tax rate on capital income is about 25 percent lower than the average tax on labor income.[21] In 2000, the ratio of total individual federal income tax revenue (not including Social Security and Medicare taxes) to GDP was 0.102 and the ratio of corporate income tax to GDP was 0.021. Each statutory federal income tax rate shown in Table V, therefore, is multiplied by φ_I so that tax revenue (including corporate income tax) totals 12.3 percent of GDP in the initial steady state. The adjustment factor is between 0.82 and 0.87 for heterogeneous-agent economies with idiosyncratic wage shocks and 1.0 for the representative-agent economy without wage shocks. State and local income taxes are modeled parsimoniously with a 4.0 percent flat tax on income above the deduction and exemption levels used at the federal level.

The tax rate levied on employees for Old-Age, Survivors, and Disability Insurance (OASDI) is 12.4 percent, and the tax rate for Medicare (HI) is 2.9 percent. In 2001, employee compensation above $80,400 was not taxable for OASDI. (See Table VI.) Workers with wages above $80,400, therefore, don't face a marginal tax or distortions from the Social Security system.

Social Security benefits are based on each worker's Average Indexed Monthly Earnings (AIME), $b/12$, and the replacement rate schedule shown in Table VII. The replacement rates are 90 percent for

[19]In particular, the deduction taken by a household is the greater of the standard deduction and $0.0755\times$AGI, or $\max\{\$7600, 0.0755\times\text{AGI}\}$.

[20]The key qualitative results reported herein are unaffected if the tax function were instead modeled as net taxes, that is, after substracting transfers indicated in the Statistics of Income.

[21]This relative reduction to the tax rate on capital is commonly used by the Congressional Budget Office, and it balances the legal tax preferences given to capital versus the legal tax benefits given to labor, including tax-preferred fringe benefits.

the first $561, 32 percent for amounts between $561 and $3,381, and 15 percent for amounts above $3,381. Social Security, therefore, is progressive in that a worker's benefit relative to AIME (the "replacement rate") is decreasing in the AIME.

The U.S. OASDI also pays spousal, survivors' and disability benefits in addition to the standard retirement benefit described above. Indeed, retiree benefits accounted for only 69.1 percent of total OASDI benefits in December 2000.[22] OASDI benefits, therefore, are adjusted upward by the proportional adjustment factor φ_{SS} so that total benefit payments equal total payroll tax revenue. The adjustment factor φ_{SS} equals about 1.46 in our model with wage shocks and 1.32 in our model without wage shocks (Table II). This adjustment proportionally distributes non-retiree OASDI payments across retirees.

IV. Baseline Policy Experiments

We simulate a stylized phased-in partial privatization of Social Security that begins in year 1. Statutory (or, sometimes called " scheduled") Social Security benefits are reduced linearly over time. Households age 65 and older in year 1 receive the current-law (baseline) benefits throughout the rest of their lifetime; households of age 65 in year 2 receive benefits that are 1.25 percent lower than the current-law level throughout the rest of lifetime; households of age 65 in year 3 receive benefits 2.5 percent lower than the current law-level, and so on. Households age 25 or younger in year 1, therefore, receive one half of their traditional Social Security benefits when they turn 65. Pay-as-you-go payroll taxes, therefore, are also reduced over time. But the *effective* tax rates on younger workers alive at the time of the reform actually *increase* during the transition because these workers help pay for the traditional benefits owed to retirees and older workers but do not receive full benefits themselves. However, workers born in the long run enjoy smaller effective tax rates on their labor income.

As is implicit in most previous work on privatization, assets in the new private accounts are assumed to be perfect substitutes with other private assets, including earning the same market rate of return and being subject to the same income tax schedule, as outlined above. As a result, the new private accounts do not have to be explicitly modeled; households will increase their savings in response to a decline in retirement benefits.

We first consider the representative-agent economy without wage shocks (equivalently, with *insurable* wage shocks) where all households have the wage profile of the 40–60th percentile in Table IV,

[22] See Table 5.A1 in Social Security Administration (2001).

i.e., lifetime income group e^3. We then turn to a heterogeneous-agent economy with *uninsurable* wage shocks. We initially assume that both economies are closed to international capital flows, and that a private annuity market does not exist.

IV.A. Representative-Agent Economy without Wage Shocks

As shown in Run 1 in Table VIII, 50% privatization of Social Security in the representative-agent economy increases national wealth by 26.7 percent in the long run compared to the baseline economy. GNP increases by 12.3 percent in the long run, while labor supply increases by 6.7 percent. These large gains are driven by pre-funding a portion of Social Security's liabilities that were previously financed on a pay-as-you-go (unfunded) basis. Because the *effective* tax rate on labor income actually increases in the short run, labor supply initially contracts by 0.1 percent, which requires a small increase in the *statutory* payroll tax rate. Over the long run, however, the statutory payroll tax rate declines by 51.9%. (The payroll tax rate declines by more than 50% in the long run because the wage rate increases with more capital; labor supply is also larger.) An expanded labor and capital base also allows for federal income tax rates to be reduced, by 25.5% in the long run.

Despite these positive gains to economic variables, not everyone wins from privatization. As shown for Run 1 in lefthand column in Table IX ("Without LSRA"), all households alive at the time of the reform (that is, aged 20 or older) are worse off. For example, the age-20 household at the time of policy change *loses* \$41,700, as measured by the equivalent variation of an one-time wealth transfer made at the time of the reform. The age-40 household loses \$78,300. Both of these households help pay for the traditional Social Security benefits received by retirees and older workers, but these younger households do not receive as large of Social Security benefits themselves. The age-40 household is especially "stuck in the middle" by paying for traditional benefits but being too old to gain much from the eventual reduction in payroll taxes. In a closed economy, this household is also hurt by the fall in the interest rate shown in Table VIII given its large accumulation of wealth at the time of the reform.[23]

Future households, who pay very little of the transition costs, however, gain substantially from privatization. For example, newborns in Year 1 gain about \$18,500 per couple while generations born 20 years later gain about \$57,200. Generations born in the long run (i.e., are age $-\infty$ in Year 1) gain \$66,100. These gains arise mainly from higher wages, smaller payroll taxes, and reduced federal income taxes.

[23] For example, the welfare loss of a 40-year old is reduced to \$50,700 in a small open economy (not shown), where the interest rate does not change.

Overall, privatization produces large *efficiency gains*, that is, extra resources after the "winners" compensate the "losers" in present value. This fact can be seen in the righthand column in Table IX where we simulate the same economy and policy experiment but with an operative Lump-Sum Redistribution Authority (LSRA). Recall that the LSRA transfers exactly enough wealth to would-be losing households alive at the time of the reform so that their remaining expected lifetime utilities return to their pre-reform levels. These transfers must be financed with borrowing that is financed with the gains to future generations. All net new resources (positive or negative) are then distributed equally to future households (growth adjusted over time). Because the LSRA is part of the general-equilibrium solution, the macroeconomic outcomes shown in Table VIII will, of course, also change in the presence of lump-sum transfers. Indeed, including the general-equilibrium effects associated with these lump-sum transfers are important for calculating efficiency gains, as first noted by Auerbach and Kotlikoff (1987). The new macroeconomic outcomes with an operative LSRA, though, are not reported in order to economize on space; only the resulting efficiency gains are shown.[24] As demonstrated in Table IX, privatization produces about $30,100 (in 2001 growth-adjusted dollars) in additional net resources per each future household that enters the economy in year 2 and later, that is, after all the would-be losing households have been fully compensated.

IV.B. Heterogeneous-Agent Economy with Wage Shocks

Run 2 in Table VIII also shows the effect of the same stylized privatization experiment in a more realistic economy with *uninsurable* wage shocks. Unlike in the deterministic economy, Social Security's progressive benefit formula now shares some wage risks, thereby providing some insurance that is unattainable in the private market. National wealth now increases by 18.8 percent in the long run, but by less than in the representative-agent economy (Run 1). A portion of private saving in the pre-reform economy is now for precautionary motives, which is less sensitive to changes in Social Security policy. Indeed, unlike Run 1, a non-trival portion of the build-up in national weath in Run 2 is due to increased precautionary savings as privatization reduces the risk sharing of wage shocks. Labor supply increases by 3.3 percent in the long run and GNP is 7.7 percent higher.

Similar to the representative-agent economy, most households alive at the time of the reform are worse off because they have to pay higher taxes to finance the transition. Run 2 in Table IX shows that a 40-year old in the top one percent of the wage distribution (e^8) at the time of privatization loses

[24] These tables are available from the authors.

$134,100. As with Run 1, future households, however, gain considerably from reduced payroll taxes, smaller income tax rates, and higher wages. Even households in the lowest 20 percent of the wage distribution (e^1) born in the long run gain $72,600 (in 2001 growth-adjusted dollars). Overall, privatization, though, no longer improves efficiency. After the LSRA returns the welfare of all households to their pre-reform levels, it distributes a *negative* $8,100 to each future household. This loss contrasts sharply with the gain of $30,100 in the deterministic economy discussed above.

V. Alternative Assumptions in the Heterogeneous-Agent Economy

The efficiency loss in our benchmark economy shown in Run 2, however, is based on four key model assumptions that might appear at first glance to be biased against privatization. We now investigate each of these assumptions, which leads to several surprising insights. In each case, the model is recalibrated in order to hit specific observable targets on economic relationships in the initial steady state that were discussed earlier. The resulting parameters are summarized for each Run in Table II.

V.A. In a Small Open Economy

Our benchmark economy shown in Run 2 is closed to international capital flows. When privatization increases capital accumulation, interest rates drop, thereby discouraging even more accumulation. If, instead, capital could flow across borders then more capital could be accumulated with no reduction in the rate of return to saving.

Run 3, reported in Table X, therefore, shows the effect of privatization in the setting of a small open economy where any changes that the capital-labor ratio would have on factor prices are nullified immediately by international capital flows. National wealth does indeed increase by substantially more in the open-economy case, by 35.7% or almost double the amount reported earlier for the closed economy setting (Run 2, Table VIII). The gain in labor supply, though, is considerably smaller – 0.8% versus 3.3% – since the wage rate does not rise. The net effect is to increase GNP by 11.3 percent in the long run, compared to only 7.7 percent in Run 2.

Table XI shows that the welfare losses of households alive at the time of the reform tend to be smaller in the case of a small-open economy (Run 3) relative to our closed benchmark economy (Run 2, Table IX). A fixed interest rate protects the value of wealth that was accumulated prior to reform. The *gains* to future households born in the long run also tend to be a little smaller in the small open economy, mostly due to a fixed wage rate per efficiency unit. One notable exception is that wealthier households born in the long run gain more in the open economy because interest rates do not fall. Overall, therefore, it might appear at first that privatization in an open economy setting would produce a smaller efficiency loss than in the closed economy. This hunch, however, is incorrect. When the LSRA is operative, Table XI shows that the efficiency losses are actually slightly *larger* in the small open economy, equal to $10,100 per each future household, compared to a loss of $8,100 with a closed economy.

One reason for this suprising result is that the LSRA's cost of borrowing is higher in the small open economy setting since the interest rate does not decrease over time after privatization, as in the closed-economy setting. Recall that the LSRA must, in effect, borrow from future generations, who would otherwise win from reform, in order to return current workers to their pre-reform level of utility, who would otherwise lose from reform. A larger interest rate increases the cost of these inter-temporal transfers.

Another reason that the open-economy case performs worse is that income taxes can decrease by relatively more in the closed-economy case in order to balance the government's budget because wages increase. Notice from Table X that the income tax drops by 10.8% in the long run with an open economy, compared to 11.2% in the closed economy.

To partially separate these two different effects, we re-ran both the closed- and open-economy cases holding the income tax rate fixed at its steady state level. In order to satisfy the government's dynamic budget constraint in general equilibrium, we first experimented with reducing government consumption (the results are not shown in the tables herein in order to save on space[25]). In that case, the open economy setting again produce a larger efficiency loss than the closed-economy case. We then experimented with using lump-sum taxes / rebates to satisfy the government's dynamic budget constraint; lump-sum transfers were given equally to each household. Again, the open economy setting produced relatively larger efficiency losses (again, not shown). These results demonstrate the importance of interest-rate channel in producing relatively larger efficiency losses in the open economy setting.

V.B. Perfect Annuity Markets

Thus far, we have assumed private annuity markets do not exist and so, in addition to sharing wage uncertainty, the Social Security system shares *longevity* uncertainty in a way the private market cannot. It would appear at first glance, therefore, that privatization has a better chance of producing efficiency

[25]Tables are available from the authors.

gains if we instead assumed that a private annuity market is available. Surprisingly, this intuition also turns out to be incorrect.

Run 4 in Table XI shows that the efficiency losses are actually *larger* with perfect private annuity markets than without (Run 2). In particular, each household loses $10,900, compared to $8,100 shown earlier without an annuity market (Run 2, Table IX). Privatization with perfect private annuities leads to only a 14.5 percent increase in national wealth in the long run (Table X), compared to a 18.8 percent increase without a private annuity market (Run 2, Table VIII). The reason is that, without private annuities available, households increase their precautionary savings in Run 2 after privatization as the annuity insurance provided by Social Security is reduced; in contrast, households can rely more on the private annuity insurance market rather than precautionary savings in Run 4. The smaller amount of precautionary savings in Run 4 produces larger efficiency losses for three reasons: (i) the LSRA must borrow at a relatively higher interest rate; (ii) income taxes are higher since there is less capital and labor income; and (iii) the interest elasticity of saving is higher, increasing the role that falling interest rates have on discouraging additional saving.

V.C. Less Initial Redistribution

Social Security's progressive benefit formula, as shown in Table VII, is intended to redistribute resources toward the lifetime poor. A recent literature, though, has suggested that Social Security might be less redistributive than traditionally thought because, in part, poorer people don't live as long as wealthier people, thereby collecting fewer; poor people are also less likely to qualify for a spousal benefit.[26] Our benchmark model ignores these complications and, therefore, might over-estimate the amount of risk sharing that is being provided by Social Security under the baseline. To test the importance of this redistribution, we changed the marginal replacement rates shown in Table VII from the levels 90 / 32 / 15 to 60 / 31 / 22.5, which has the effect of reducing by about one-half the partial-equilibrium intra-generational redistribution from households with above-average AIME to households with below-average AIME.

As shown in Table XI, however, a static reduction in redistribution has a non-monotonic impact on efficiency gains. In particular, the efficiency loss from 50% privatization is now $9,400 per future household, which is slightly larger than our baseline measure of $8,100. Intuitively, a system with less

[26]See, for example, Gustman and Steinmeier (2001), Brown (2002), Liebman (2002), and Fullerton and Mast (2005). Incorporating an income-mortality correlation directly into our model produces technical challenges when $\gamma > 1$. We instead reduce the progressivity of our benefit formula in a manner consistent with the conclusions of these papers, which also captures the inequality in spousal benefits.

initial redistribution also has contains fewer distortions to labor supply in the baseline and, hence, less opportunity for efficiency gains from reform.

V.D. Smaller Transitory Shocks

Another key assumption in our model is the size of the transitory working ability shocks and their persistence. Recall that we constructed the age-working ability transition matrices from the 1989-92 Panel Study of Income Dynamics (PSID). Floden and Lindé (2001), however, persuasively argue that measurement error in the PSID might be as large as the size of the real fluctuation.

Run 6 shown in Tables X and XI show the economic and welfare effects, respectively, of privatization when the transitory shocks are reduced to only half of their previous values we used in the main calibration. National wealth increases by 20.9 percent in the long run compared to 18.8 percent for our baseline, Run 2. However, notice that the efficiency losses actually *increase* to $14,000 per future household (relative to a $8,100 loss in Run 2) under the LSRA. This counter-intuitive result can be explained by the fact that a reduction in transitory shocks also increases the *persistence* of any shock. As a result, the effect of any negative shock becomes more permanent, potentially increasing the value of the risk sharing in the former Social Security system.[27] In the limit, however, the model collapses to one with no wage uncertainty if the transitory shocks are eliminated, thereby resulting in efficiency gains.

VI. Alternative Experiments in the Heterogeneous-Agent Economy

Another potential criticism of our simulations is that the experiment itself is too simple in that it makes no attempt to incorporate any type of redistribution within the new privatizated system. We, therefore, consider two alternatives for reintroducing redistribution into the reform: matching contributions made to private accounts as well as increasing the progressivity in the smaller traditional system that remains. For each case, we use our baseline model (Run 2) and the corresponding parameter assumptions shown in Table II. Only the policy reform itself changes.

VI.A.Contribution Matching

Run 7 considers privatization in which working households with low levels of labor income receive a fairly generous match equal to 10 percent of their *earnings*. This matching rate declines linearly to

[27]We benefited from a helpful conversation with Dirk Krueger on this point.

zero as labor income approaches $60,000, which is slightly above the median household income in the model economy.[28] While ensuing Social Security deficits continue to be financed a pay-as-you-go payroll tax, the contribution match is financed each year from general revenue using income taxes.

As shown in Run 7 in Table XII, privatization with contribution matching decreases labor supply, GNP and national wealth over the first decade; in contrast privatization under our baseline (Run 2) produced gains to these variables throughout the entire transition path. Eventually, privatization with contribution matching leads to a 5.9% gain in GNP, compared to a 7.7% increase under our baseline shown in Run 2. Contribution matching also produces only a 2.0% increase in labor supply in the long run, compared to a 3.3% increase under the baseline. The gains to macroeconomic variables under contribution matching are generally smaller relative to the baseline simulation for two reasons. First, contribution matching produces positive marginal tax rates associated with the phase-out.[29] Second, the matching itself must be financed with a distorting income tax.

The welfare gains for Run 7 reported in Table XIII show that contribution matching tends to improve the welfare of poorer households relative to Run 2 without the match. Whereas the poorest households (e^1) born 20 years after reform gains $62,800 without the match, they gain $65,900 with the match. Not surprising, the richest households, however, are worse off since they don't receive any of the match but must help finance it; they gained $67,000 without the match in the long run but only $58,800 with the match.

With the LSRA operative, Table XIII shows that privatization now leads to an efficiency loss of $13,200, which is actually larger than efficiency loss under the baseline. Recall that privatization already increases effective marginal tax rates on labor for many workers caught in the transition. Contribution matching enhances those distortions with a phase-out range as well as with general-revenue financing. While contribution matching reintroduces some risk sharing that was lost during privatization, the additional distortions to labor supply – which increase by roughly the square of the tax rate – is enough to reverse the potential benefits.

In order to understand this surprising result some more, we also simulated privatization, both with

[28]We also simulated contribution matching without a phase-out. The efficiency losses were actually larger than those reported in this section. Whereas eliminating the phase-out eliminates some implicit marginal tax rates, it enhances the income tax distortions since more revenue is needed to finance the contribution matches. We also considered financing the phased-out match with a *negative* match on those with above-average incomes. Although potentially more efficient at redistribution than an income tax since the poor are not financing their own match, it also performed poorly. Labor supply tends to be fairly elastic in our model whereas the savings elasticity is relatively low with precautionary saving.

[29]This matching schedule is equivalent with the marginal labor income tax of -10 percent at $0 of labor income, 0 percent at $30,000, 10 percent at $60,000, and 0 percent for labor income above $60,000.

and without contribution matching, in which the transition is financed using a *consumption* tax that is more efficient at raising revenue than payroll taxes.[30] A 10% match rate now leads to *smaller* efficiency losses relative to the baseline privatization. However, a 20% match produces larger efficiency losses relative to the baseline. These non-monotonic reductions in efficiency losses can be traced to the trade-off between risk sharing and labor supply distortions: *some* match is beneficial but is dominated by distortions at higher tax rates. In fact, there is no match rate that allows privatization to produce efficiency gains.

VI.B. Progressive Benefit Schedule

Run 8 takes a different approach to maintaining some progressivity after privatization. It immediately increases the progressivity of the Social Security benefit that remains after privatization by raising the replacement rate of the lowest wage income bracket from 90 percent to 120 percent while reducing the replacement rate of the highest wage income bracket from 15% to 7.5%. Run 9 is even more aggressive in the redistribution by raising the replacement rate of the lowest wage income bracket to 150 percent while reducing the top replacement rate to 0%.

Table XIII shows that increasing the progressivity of the smaller Social Security system that remains after privatization is better than contribution matching at protecting the welfare of the poor at the time of reform as well as reducing efficiency losses. Now, privatization reduces efficiency by only $7,100 per future household under the LSRA in Run 8, and by $6,700 in Run 9.

Both of these losses are smaller than our baseline loss of $8,100. Intuitively, increasing the progressivity of the remaining system performs better than contribution matching because more redistribution can be accomplished with less distortion to labor supply. Whereas contribution matching is based on the labor income in any given year, Social Security's progressive benefit is based on a household's lifetime earnings, which is harder to change.

To give partial privatization the best chance of succeeding, we re-consider Run 9 but were the transition is now financed with a consumption tax.[31] The payroll tax is immediately reduced in half and the difference in cash flow between payable benefits and collected payroll taxes is financed with a flat consumption tax. Run 10 in Table XIII shows that progressive privatization in this case can now

[30] Simulation results with a consumption tax using a previous version of our model are reported in our working paper, Nishiyama and Smetters [2005*b*].

[31] Kotlikoff, Smetters and Walliser (1998, 2001) also considered different tax bases for financing the transition path. However, their analyses used a deterministic OLG model and only examined changes to macroeconomic variables. They did not examine efficiency gains.

lead to an efficiency *gain* equal to $2,800 per each future household. This improvement in efficiency can be traced to replacing part of the distorting payroll tax with a consumption tax that incorporates an efficient lump-sum levy on existing assets. Of course, this same improvement in efficiency could be achieved inside of the traditional system. Some proponents of privatization, however, have suggested that the transition costs associated with private accounts presents a unique political opportunity for implementing a retail sales tax.

VII. Concluding Remarks

This paper investigated whether a stylized Social Security privatization generates efficiency gains or losses in the presence of an overlapping-generations economy with elastic labor supply, idiosyncratic wage shocks and longevity uncertainty. We found that the privatization of Social Security produces efficiency gains in a representative-agent economy without wage shocks (or, equivalently, if these shocks are insurable). In a heterogeneous-agent economy with idiosyncratic and uninsurable wage shocks, however, the overall efficiency of the economy is reduced by our stylized privatization since the existing Social Security system provides a valuable source of risk sharing through its progressive benefit formula. This result was fairly robust to a wide range of model considerations as well as policy reforms. Privatization leads to efficiency gains only when combined with an efficient tax to finance the transition as well as a properly-constructed restoration of progressivity.

Appendix 1

The Markov transition matrixes of working ability are constructed for four age groups—20-29, 30-39, 40-49, and 50-59—from the hourly wages in the PSID individual data 1990, 91, 92, and 93. The transition matrix of each age group is the average of three transition matrixes, from 1989 to 90, from 90 to 91, and from 91 to 92. For households aged 60 or older, we used the matrix for ages 50-59.

$$\Gamma_{i\in\{20,\ldots,29\}} = \begin{pmatrix} 0.5964 & 0.2499 & 0.0875 & 0.0464 & 0.0118 & 0.0048 & 0.0029 & 0.0003 \\ 0.2093 & 0.4594 & 0.2322 & 0.0756 & 0.0104 & 0.0088 & 0.0042 & 0.0001 \\ 0.1044 & 0.1902 & 0.4084 & 0.2385 & 0.0342 & 0.0153 & 0.0048 & 0.0042 \\ 0.0642 & 0.0831 & 0.2016 & 0.4576 & 0.1314 & 0.0380 & 0.0241 & 0.0000 \\ 0.0313 & 0.0202 & 0.0784 & 0.2947 & 0.4285 & 0.0882 & 0.0408 & 0.0179 \\ 0.0246 & 0.0005 & 0.0898 & 0.1084 & 0.2462 & 0.3216 & 0.1862 & 0.0227 \\ 0.0108 & 0.0248 & 0.0432 & 0.0373 & 0.1163 & 0.2858 & 0.3923 & 0.0895 \\ 0.0376 & 0.0440 & 0.0000 & 0.0012 & 0.2615 & 0.0291 & 0.3714 & 0.2552 \end{pmatrix},$$

$$\Gamma_{i\in\{30,\ldots,39\}} = \begin{pmatrix} 0.6936 & 0.2078 & 0.0546 & 0.0330 & 0.0031 & 0.0018 & 0.0061 & 0.0000 \\ 0.1972 & 0.5587 & 0.2001 & 0.0341 & 0.0077 & 0.0006 & 0.0000 & 0.0016 \\ 0.0620 & 0.1796 & 0.5233 & 0.2018 & 0.0154 & 0.0110 & 0.0069 & 0.0000 \\ 0.0214 & 0.0413 & 0.2024 & 0.5411 & 0.1526 & 0.0281 & 0.0116 & 0.0015 \\ 0.0272 & 0.0068 & 0.0348 & 0.3065 & 0.4581 & 0.1182 & 0.0484 & 0.0000 \\ 0.0163 & 0.0309 & 0.0084 & 0.0907 & 0.2946 & 0.3798 & 0.1512 & 0.0281 \\ 0.0404 & 0.0000 & 0.0007 & 0.0621 & 0.0830 & 0.2624 & 0.4869 & 0.0645 \\ 0.0000 & 0.0302 & 0.0000 & 0.0334 & 0.0379 & 0.0384 & 0.3209 & 0.5392 \end{pmatrix},$$

$$\Gamma_{i\in\{40,\ldots,49\}} = \begin{pmatrix} 0.7111 & 0.2340 & 0.0352 & 0.0110 & 0.0070 & 0.0017 & 0.0000 & 0.0000 \\ 0.1847 & 0.5571 & 0.2078 & 0.0261 & 0.0142 & 0.0052 & 0.0020 & 0.0029 \\ 0.0579 & 0.1520 & 0.5429 & 0.1996 & 0.0339 & 0.0117 & 0.0020 & 0.0000 \\ 0.0214 & 0.0430 & 0.1833 & 0.5587 & 0.1576 & 0.0311 & 0.0027 & 0.0022 \\ 0.0191 & 0.0145 & 0.0217 & 0.3155 & 0.4644 & 0.1055 & 0.0593 & 0.0000 \\ 0.0416 & 0.0089 & 0.0512 & 0.1385 & 0.1427 & 0.3653 & 0.2094 & 0.0424 \\ 0.0247 & 0.0086 & 0.0354 & 0.0493 & 0.0777 & 0.2486 & 0.4942 & 0.0615 \\ 0.0000 & 0.0543 & 0.0000 & 0.0475 & 0.0786 & 0.1300 & 0.2502 & 0.4394 \end{pmatrix},$$

$$\Gamma_{i\in\{50,\ldots,78\}} = \begin{pmatrix} 0.7000 & 0.2164 & 0.0514 & 0.0121 & 0.0110 & 0.0015 & 0.0076 & 0.0000 \\ 0.2215 & 0.5452 & 0.2117 & 0.0189 & 0.0027 & 0.0000 & 0.0000 & 0.0000 \\ 0.0439 & 0.1743 & 0.5353 & 0.2043 & 0.0197 & 0.0123 & 0.0102 & 0.0000 \\ 0.0170 & 0.0525 & 0.1651 & 0.6075 & 0.1220 & 0.0190 & 0.0169 & 0.0000 \\ 0.0276 & 0.0085 & 0.0352 & 0.2608 & 0.4774 & 0.1690 & 0.0215 & 0.0000 \\ 0.0002 & 0.0127 & 0.0429 & 0.0898 & 0.2605 & 0.3345 & 0.2444 & 0.0150 \\ 0.0189 & 0.0210 & 0.0409 & 0.0213 & 0.2008 & 0.2026 & 0.4079 & 0.0866 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0127 & 0.1701 & 0.2386 & 0.5786 \end{pmatrix},$$

where $\Gamma_i(j,k) = \pi(e_{i+1} = e^k_{i+1} | e_i = e^j_i)$. For Run 6 (the 1/2 wage shock case), the transition matrixes are modified to $\Gamma_{1/2,i} = (\Gamma_i + I_8)/2$, where I_8 is the 8×8 identity matrix.

The survival rates, calculated from the period life table in Social Security Administration (2001), can be found in Nishiyama and Smetters [2005*b*]. The remainder of this Appendix focuses on how the solution is calculated.

Appendix 2

2.1. The Discretization of the State Space

The algorithm to solve the model for a steady-state equilibrium and an equilibrium transition path is similar to that in Nishiyama and Smetters (2005) but is extended significantly to include Social Security. The state of a household is $\mathbf{s} = (i, e, a, b) \in I \times E \times A \times B$, where $I = \{20, ..., 109\}$, $E = [e^{\min}, e^{\max}]$, $A = [a^{\min}, a^{\max}]$, and $B = [b^{\min}, b^{\max}]$. To compute an equilibrium, the state space of a household is discretized as $\hat{\mathbf{s}} \in I \times \hat{E} \times \hat{A} \times \hat{B}$, where $\hat{E}_i = \{e_i^1, e_i^2, ..., e_i^{N_e}\}$, $\hat{A} = \{a^1, a^2, ..., a^{N_a}\}$, and $\hat{B} = \{b^1, b^2, ..., b^{N_b}\}$. For all these discrete points, the model computes the optimal decision of households, $\mathbf{d}(\hat{\mathbf{s}}, \mathbf{S}_t; \mathbf{\Psi}_t) = (c(.), h(.), a'(.)) \in (0, c^{\max}] \times [0, h_{\max}] \times A$, the marginal values, $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}, \mathbf{S}_t; \mathbf{\Psi}_t)$ and $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}, \mathbf{S}_t; \mathbf{\Psi}_t)$, and the values $v(\hat{\mathbf{s}}, \mathbf{S}_t; \mathbf{\Psi}_t)$, given the expected factor prices and policy variables.[32]

To find the optimal end-of-period wealth, the model uses the Euler equation and bilinear interpolation (with respect to a and b) of marginal values at the beginning of the next period.[33] In a heterogeneous-agent economy, N_e, N_a, and N_b are 8, 50, and 8, respectively. In a representative-agent economy, the numbers of grid points are 1, 71, and 6, respectively.[34]

2.2. A Steady-State Equilibrium

The algorithm to compute a steady-state equilibrium is as follows. Let $\mathbf{\Psi}$ denote the time-invariant government policy rule $\mathbf{\Psi} = (W_{LS}, W_G, C_G, \tau_I(.), \tau_P(.), tr_{SS}(\hat{\mathbf{s}}), tr_{LS}(\hat{\mathbf{s}}))$.

1. Set the initial values of factor prices (r^0, w^0), accidental bequests q^0, the policy variables (W_{LS}^0, C_G^0), lump-sum redistribution $tr_{LS}^0(\hat{\mathbf{s}})$, and the parameters ($\varphi_I^0$, φ_P^0, φ_{SS}^0) of policy functions ($\tau_I(.)$, $\tau_P(.)$, $tr_{SS}(.)$) if these are determined endogenously.[35]

2. Given $\mathbf{\Omega}^0 = (r^0, w^0, q^0, W_{LS}^0, C_G^0, \varphi_I^0, \varphi_P^0, \varphi_{SS}^0)$, find the decision rule of a household $\mathbf{d}(\hat{\mathbf{s}}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ for all $\hat{\mathbf{s}} \in I \times \hat{E} \times \hat{A} \times \hat{B}$.[36]

   (a) For age $i = 109$, find the decision rule $\mathbf{d}(\hat{\mathbf{s}}_{[i=109]}; \mathbf{\Psi}, \mathbf{\Omega}^0)$. Since the survival rate $\phi_{109} = 0$, the end-of-period wealth $a'(\hat{\mathbf{s}}_{[i=109]}; .) = 0$ for all $\hat{\mathbf{s}}_{[i=109]}$. Compute consumption and working hours $(c(\hat{\mathbf{s}}_{[i=109]}; .), h(\hat{\mathbf{s}}_{[i=109]}; .))$ and, then, marginal values $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_{[i=109]}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ and values $v(\hat{\mathbf{s}}_{[i=109]}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ for all $\hat{\mathbf{s}}_{[i=109]}$.[37]

---

[32] Because the marginal value with respect to historical earnings, $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}, \mathbf{S}_t; \mathbf{\Psi}_t)$, is difficult to obtain analytically, it is approximated by $(v(., b^{j+1}, \mathbf{S}_t; \mathbf{\Psi}_t) - v(., b^j, \mathbf{S}_t; \mathbf{\Psi}_t))/(b^{j+1} - b^j)$ where $j = 1, 2, ..., N_b$.

[33] The marginal values with respect to wealth, $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}, \mathbf{S}_t; \mathbf{\Psi}_t)$, are used in the Euler equation to obtain optimal savings, the marginal values with respect to historical earnings, $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}, \mathbf{S}_t; \mathbf{\Psi}_t)$, are used in the marginal rate of substitution condition of consumption for leisure to obtain optimal working hours, and the values, $v(\hat{\mathbf{s}}, \mathbf{S}_t; \mathbf{\Psi}_t)$, are used to calculate welfare changes measured by compensating and equivalent variations in wealth.

[34] The grid points on A and B are not equally spaced. In a heterogeneous-agent economy, $\hat{A}$ ranges from -$266,200 to $14,817,600 (in 2001 growth-adjusted dollars) and $\hat{B}$ ranges from $10,000 to $80,400. In a representative-agent economy, $\hat{A}$ and $\hat{B}$ range from -$333,000 to $2,626,900 and from $16,900 to $57,500, respectively.

[35] If we find the capital-labor ratio, both r and w are calculated from the given production function and depreciation rate. In this paper, the endogenous policy variables are C_G and φ_{SS} in baseline economies, and φ_I, φ_P, $tr_{LS}(\hat{\mathbf{s}})$, and W_{LS} in policy experiments.

[36] In the steady-state economy, the decision rule of a household $\mathbf{d}(\hat{\mathbf{s}}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ is not a function of the aggregate state of economy $\hat{\mathbf{S}} = (x(\hat{\mathbf{s}}), W_{LS}, W_G)$. The measure of household $x(\hat{\mathbf{s}})$ is determined uniquely by the steady-state condition, and the government's wealth W_G is determined by the policy rule $\mathbf{\Psi}$.

[37] The marginal value with respect to historical earnings, $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}; \mathbf{\Psi}, \mathbf{\Omega}^0)$, is zero when $i > 60$ in this paper.

(b) For age $i = 108, ..., 20$, find the decision rule $\mathbf{d}(\hat{\mathbf{s}}_{[i]}; \mathbf{\Psi}, \mathbf{\Omega}^0)$, marginal values $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_{[i]}; \mathbf{\Psi}, \mathbf{\Omega}^0)$, and values $v(\hat{\mathbf{s}}_{[i]}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ for all $\hat{\mathbf{s}}_{[i]}$, using $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_{[i+1]}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ and $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_{[i+1]}; \mathbf{\Psi}, \mathbf{\Omega}^0)$ recursively.

   i. Set the initial guess of $a'^0(\hat{\mathbf{s}}_{[i]}; .)$.

   ii. Given $a'^0(\hat{\mathbf{s}}_{[i]}; .)$, compute $(c(\hat{\mathbf{s}}_{[i]}; .),\ h(\hat{\mathbf{s}}_{[i]}; .))$, using $\frac{\partial}{\partial b} v(\hat{\mathbf{s}}_{[i+1]}; \mathbf{\Psi}, \mathbf{\Omega}^0)$. Plug these into the Euler equation with $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_{[i+1]}; \mathbf{\Psi}, \mathbf{\Omega}^0)$.

   iii. If the Euler error is sufficiently small, then stop. Otherwise, update $a'^0(\hat{\mathbf{s}}_{[i]}; .)$ and return to Step ii.

3. Find the steady-state measure of households $x(\hat{\mathbf{s}}_{[i]}; \mathbf{\Omega}^0)$ using the decision rule obtained in Step 2. This computation is done forward from age 20 to age 109. Repeat this step to iterate q for q^1.

4. Compute new factor prices (r^1, w^1), accidental bequests q^1, the policy variables (W_{LS}^1, C_G^1), lump-sum redistribution $tr_{LS}^1(\hat{\mathbf{s}})$, and the parameters $\left(\varphi_I^1, \varphi_P^1, \varphi_{SS}^1\right)$ of policy functions.

5. Compare $\mathbf{\Omega}^1 = (r^1, w^1, q^1, W_{LS}^1, C_G^1, \tau_C^1, \varphi_I^1, \varphi_{SS}^1)$ with $\mathbf{\Omega}^0$. If the difference is sufficiently small, then stop. Otherwise, update $\mathbf{\Omega}^0$ and return to Step 2.

2.3. An Equilibrium Transition Path

Assume that the economy is in the initial steady state in period 0, and that the new policy schedule $\mathbf{\Psi}_1$, which was not expected in period 0, is announced at the beginning of period 1, where $\mathbf{\Psi}_1 = \{W_{LS,t+1}, W_{G,t+1}, C_{G,t}, \tau_{I,t}(.), \tau_{P,t}(.), tr_{SS,t}(\hat{\mathbf{s}}), tr_{LS,t}(\hat{\mathbf{s}})\}_{t=1}^{\infty}$. Let $\hat{\mathbf{S}}_1 = (x_1(\hat{\mathbf{s}}), W_{LS,1}, W_{G,1})$ be the state of the economy at the beginning of period 1. The state of the economy $\hat{\mathbf{S}}_1$ is usually equal to that of the initial steady state. The algorithm to compute a transition path to a new steady-state equilibrium (thereafter, final steady-state equilibrium) is as follows.

1. Choose a sufficiently large number, T, such that the economy is said to reach the new steady state within T periods. Set the initial guess, $\mathbf{\Omega}_1^0 = \{r_s^0, w_s^0, q_s^0, W_{LS,s}^0, C_{G,s}^0, \varphi_{I,s}^0,\ \varphi_{P,s}^0\}_{s=1}^{T}$, on factor prices, accidental bequests, and the policy variables. Because there are no aggregate productivity shocks in this model, a time series $\mathbf{\Omega}_t = \{r_s^0, w_s^0, q_s^0,\ W_{LS,s}^0, C_{G,t}^0, \varphi_{I,s}^0, \varphi_{P,s}^0\}_{s=t}^{T}$ is deterministic, and each household perfectly foresees $\mathbf{\Omega}_t$ based on the information $\hat{\mathbf{S}}_t$ in an equilibrium. Since $\hat{\mathbf{S}}_t$ is in a household decision rule only to make the household expect $\mathbf{\Omega}_t$ rationally, in the computation, we can avoid the "curse of dimensionality" by replacing $\mathbf{d}(\hat{\mathbf{s}}, \hat{\mathbf{S}}_t; \mathbf{\Psi}_t)$ with $\mathbf{d}(\hat{\mathbf{s}}; \mathbf{\Psi}_t, \mathbf{\Omega}_t)$.

2. Given $W_{LS,T}$, find the final steady-state decision rule $\mathbf{d}(\hat{\mathbf{s}};\ \mathbf{\Psi}_T,\ \mathbf{\Omega}_T^0)$, marginal values, $\frac{\partial}{\partial a} v(\hat{\mathbf{s}};\ \mathbf{\Psi}_T,\ \mathbf{\Omega}_T^0)$, and values $v(\hat{\mathbf{s}};\ \mathbf{\Psi}_T,\ \mathbf{\Omega}_T^0)$ for all $\hat{\mathbf{s}} \in I \times \hat{E} \times \hat{A} \times \hat{B}$. (See the algorithm for a steady-state equilibrium.)

3. For period $t = T-1, T-2, ..., 1$, based on the guess, $\mathbf{\Omega}_t^0$, find backward the decision rule $\mathbf{d}(\hat{\mathbf{s}}_{[i]};\ \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$, marginal values $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_{[i]}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$, and values $v(\hat{\mathbf{s}}_{[i]};\ \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$ for all $\hat{\mathbf{s}} \in I \times \hat{E} \times \hat{A} \times \hat{B}$, using the next period marginal values $\frac{\partial}{\partial a} v(\hat{\mathbf{s}}_{[i+1]};\ \mathbf{\Psi}_{t+1}, \mathbf{\Omega}_{t+1}^0)$ and values $v(\hat{\mathbf{s}}_{[i+1]};\ \mathbf{\Psi}_{t+1}, \mathbf{\Omega}_{t+1}^0)$ recursively.

4. For period $t = 1, 2, ..., T-1$, compute forward $(r_t^0, w_t^0, q_t^0, W_{LS,t}^0, C_{G,t}^0, \varphi_{I,t}^0, \varphi_{P,t}^0)$ and the measure of households $x_{t+1}(\hat{\mathbf{s}})$, using the decision rule $\mathbf{d}(\hat{\mathbf{s}}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$ obtained in Step 3 and using the state of economy $\hat{\mathbf{S}}_t = (x_t(\hat{\mathbf{s}}), W_{LS,t}, W_{G,t})$ recursively.

5. Compare $\mathbf{\Omega}_t^1$ with $\mathbf{\Omega}_t^0$. If the difference is sufficiently small, then stop. Otherwise, update $\mathbf{\Omega}_t^0$ and return to Step 2.

2.4. The Lump-Sum Redistribution Authority

When the Lump-Sum Redistribution Authority (LSRA) is assumed, the following computation is added to the iteration process.

1. For period $t = T, T-1, ..., 2$, compute the lump-sum transfers to newborn households $tr_{CV}(\hat{\mathbf{s}}_{[i=20]};$ $\mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$ to make those households as well off as under the pre-reform economy.

   (a) Set the initial value of lump-sum transfers $tr_{CV}(\hat{\mathbf{s}}_{[i=20]}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$ to newborn households.

   (b) Given $tr_{CV}(\hat{\mathbf{s}}_{[i=20]}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$, find the decision rule of newborn households $\mathbf{d}(\hat{\mathbf{s}}_{[i=20]}; \mathbf{\Psi}_t,$ $\mathbf{\Omega}_t^0)$ and values $v(\hat{\mathbf{s}}_{[i=20]}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$.

   (c) Find the compensating variation in wealth $\Delta tr_{CV}(\hat{\mathbf{s}}_{[i=20]}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$ to make those households indifferent from the baseline economy. (The initial wealth of newborn households is assumed to be zero since they do not receive any bequests.) If the absolute value of $\Delta tr_{CV}(\hat{\mathbf{s}}_{[i=20]}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$ is sufficiently small, then go to Step (d). Otherwise, update $tr_{CV}(\hat{\mathbf{s}}_{[i=20]}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$ by adding $\Delta tr_{CV}(\hat{\mathbf{s}}_{[i=20]}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$ and return to Step (b).

   (d) Set the lump-sum transfers $tr_{LS,t}(\hat{\mathbf{s}}_{[i=20]}) = tr_{CV}(\hat{\mathbf{s}}_{[i=20]}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0) + \Delta tr$ where an additional lump-sum transfer Δtr is precalculated, and find the decision rule of newborn households $\mathbf{d}(\hat{\mathbf{s}}_{[i=20]}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$.

2. For period $t = 1$, compute the lump-sum transfers to all current households $tr_{CV}(\hat{\mathbf{s}}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$ to make those households as much better off as the pre-reform economy. The procedure is similar to Step 1. Set the lump-sum transfers $tr_{LS,1}(\hat{\mathbf{s}}) = tr_{CV}(\hat{\mathbf{s}}; \mathbf{\Psi}_t, \mathbf{\Omega}_t^0)$.

3. Compute an additional lump-sum transfer Δtr to newborn households so that the net present value of all transfers becomes zero. Compute the LSRA wealth, $\{W_{LS,t}^1\}_{t=1}^T$, which will be used to calculate national wealth. Recompute Δtr and $\{W_{LS,t}^1\}_{t=1}^T$ using new interest rates $\{r_t\}_{t=1}^T$.

GEORGIA STATE UNIVERSITY

THE WHARTON SCHOOL, UNIVERSITY OF PENNSYLVANIA

References

Altig, David, Alan Auerbach, Laurence Kotlikoff, Kent Smetters, and Jan Walliser, “Simulating Fundamental Tax Reform in the United States," *American Economic Review*, XC1 (2001), 574-595.

Auerbach, Alan, “Tax Reform, Capital Allocation, Efficiency and Growth," in H. Aaron and W. Gale, eds, *Economic Effects of Fundamental Tax Reform* (Washington, D.C.: Brookings Institution Press, 1996).

Auerbach, Alan and Laurence Kotlikoff, *Dynamic Fiscal Policy* (Cambridge, England: Cambride University Press, 1987).

Barro, Robert J, *Macroeconomics (5th edition)* (Cambridge, MA and London England: The MIT Press, 1997).

Blanchard, Olivier and Stanley Fischer, *Lectures on Macroeconomics* (Cambridge, MA and London England: The MIT Press, 1989).

Breyer, Friedrich, “On the Intergenerational Pareto Efficiency on Pay-as-you-go Financed Pension Systems," *Journal of Institutional and Theoretical Economics*, CXLV (1989), 643-58.

Brown, Jeffrey R. “Differential Mortality and the Value of Individual Account Retirement Annuities," in Martin Feldstein and Jeffrey B. Liebman, eds, *The distributional aspects of Social Security and Social Security reform* (Chicago and London: University of Chicago Press, 2002), 401 - 440.

Conesa, Juan and Dirk Krueger, “Social Security Reform with Heterogeneous Agents." *Review of Economic Dynamics*, II (1999), 757-795.

De Nardi, Mariacristina, Selahattin İmrohoroğlu, and Thomas J. Sargent, “Projected U.S. Demographics and Social Security," *Review of Economic Dynamics*, II (1999): 575-615.

Diamond, Peter A. and Peter R. Orszag. *Saving Social Security: A Balanced Approach*, (Washington, D.C.: Brookings Institution Press, 2003).

Díaz-Gimenez, Javier, Vincenzo Quadrini, Jose-Victor Ríos-Rull, “Dimensions of Inequality: Facts on the U.S. Distributions of Earnings, Income, and Wealth," *Federal Reserve Bank of Minneapolis Quarterly Review*, XXI (1997), 3-21

Feldstein, Martin, “Would Privatizing Social Security Raise Economic Welfare?," *NBER Working Paper* #5281 (1995).

Floden, Martin and Jesper Lindé, “Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?," *Review of Economic Dynamics*, IV (2001), 406-37.

Fullerton, Don and Brent Mast, *Income Redistribution from Social Security* (Washington, D.C.: AEI Press, 2005).

Geanakoplos, John, Olivia S. Mitchell and Stephen P. Zeldes, “Would a Privatized Social Security System Really Pay a Higher Rate of Return?" in D. Arnold, M. Graetz and A. Munnell, eds, *Framing the Social Security Debate: Values, Politics and Economics*, (Washington, D.C.: Brookings Institution Press, 1998, 137-157)

Gustman, Alan L. and Thomas L. Steinmeier, “How Effective Is Redistribution under the Social Security Benefit Formula?," *Journal of Public Economics*, LXXXII (2001), 1-28.

Huang, H., Imrohoroglu, S. and T. Sargent, “Two Computational Experiments to Fund Social Security," *Macroeconomic Dynamics* I (1997), 7-44.

İmrohoroğlu, Ayşe, Selahattin İmrohoroğlu, and Douglas Joines, “A Life Cycle Analysis of Social Security," *Economic Theory*, VI (1995), 83-114.

Kotlikoff, Laurence J., Kent Smetters, Jan Walliser, “Finding a Way Out of America’s Demographic Dilemma," *NBER Working Paper* #8258 (2001).

Kotlikoff, Laurence, Kent Smetters and Jan Walliser, “Social Security: Privatization and Progressivity," *American Economic Review (Papers and Proceedings)*, LXXXVIII (1998), 137-141.

Liebman, Jeffrey B., “Redistributional in the Current U.S. Social Security System," in Martin Feldstein and Jeffrey B. Liebman, eds, *The distributional aspects of Social Security and Social Security reform* (Chicago and London: University of Chicago Press, 2002), 11-41.

Mariger, Randall, “Social Security Privatization: What Are the Issues?," *National Tax Journal*, LII (1999), 783-802.

Murphy, Kevin and Finis Welch, “Perspectives on the Social Security Crisis and Proposed Solutions," *American Economic Review (Papers and Proceedings)*, LXXXVIII (1998): 142-50.

Nishiyama, Shinichi, “Analyzing an Aging Population—A Dynamic General Equilibrium Approach," Congressional Budget Office, Working Paper 2004-3 (2004).

Nishiyama, Shinichi and Kent Smetters, “Consumption Taxes and Economic Efficiency with Idiosyncratic Wage Shocks," *Journal of Political Economy*, CXIII (2005a), 1088-1115

Nishiyama, Shinichi and Kent Smetters, “Does Social Security Privatization Produce Efficiency Gains?" NBER Working Paper #11622 (2005b), 2005.

Shiller, Robert, “Social Security and Institutions for Intergenerational, Intragenerational and International Risk Sharing," *Carnegie-Rochester Conference Series on Public Policy*, L (1999), 165-204.

Social Security Administration, *Social Security Bulletin: Annual Statistical Supplement*, available at http://www.ssa.gov/statistics/supplement/2001/index.html (2001).

TABLE I

PARAMETERS INDEPENDENT OF MODEL ASSUMPTIONS

Coefficient of relative risk aversion	γ	2.0
Capital share of output	θ	0.30
Depreciation rate of capital stock	δ	0.047
Long-term real growth rate	μ	0.018
Population growth rate	ν	0.010
Probability of receiving bequests	η	0.0161
Total factor productivity[a]	A	0.949

a. Total factor productivity is chosen so that w equals 1.0.

TABLE II

PARAMETERS THAT VARY BY MODEL ASSUMPTIONS

		Representative-agent model without wage shocks (Run 1)	Heterogeneous-agent economy with wage shocks: Without annuity market (Runs 2, 3)	With annuity market (Run 4)	Less redist-ribution (Run 5)	Smaller transitory shocks (Run 6)
Time preference[a]	β	1.008	0.986	0.993	0.986	0.988
Consumption share[b]	α	0.455	0.504	0.494	0.500	0.489
Income tax adjustment[c]	φ_I	1.000	0.817	0.823	0.818	0.844
OASDI benefit adjustment[d]	φ_{SS}	1.315	1.463	1.459	1.695	1.468

a. The capital-GDP ratio is targeted to be 2.74 (r = 6.25 percent) without annuity markets.
b. The average annual working hours are 3,576 per married couple when $h_{\max}$ = 8,760.
c. In a heterogeneous-agent economy, the ratio of income tax revenue to GDP is 0.123.
d. OASDI benefits are pay-as-you-go.

TABLE III

NUMBER OF PEOPLE UNDER AGE 18 LIVING IN A MARRIED HOUSEHOLD

Age cohorts	Number of children	Age cohorts	Number of children
20-24	0.824	50-54	0.576
25-29	0.957	55-59	0.196
30-34	1.512	60-64	0.109
35-39	1.759	65-69	0.084
40-44	1.700	70-74	0.025
45-49	1.152	75-79	0.028

Source: Authors' calculations from the 2003 Panel Study of Income Dynamics (PSID).

TABLE IV

WORKING ABILITIES OF A HOUSEHOLD (IN U.S. DOLLARS PER HOUR)

	Percentile	Age cohorts					
		20-24	25-29	30-34	35-39	40-44	45-49
e^1	0-20th	6.59	6.79	7.23	7.58	6.63	7.06
e^2	20-40th	9.13	11.90	12.99	14.13	13.65	14.01
e^3	40-60th	11.13	15.15	17.63	19.43	18.76	19.84
e^4	60-80th	13.89	18.79	23.72	25.98	26.56	26.51
e^5	80-90th	17.89	23.07	31.94	36.66	37.30	34.38
e^6	90-95th	22.17	28.75	44.87	50.36	51.30	43.69
e^7	95-99th	28.92	37.02	70.45	90.33	74.86	76.14
e^8	99-100th	50.99	68.56	111.40	180.53	211.09	239.59
	Percentile	Age cohorts					
		50-54	55-59	60-64	65-69	70-74	75-79
e^1	0-20th	6.45	2.76	0.02	0.00	0.00	0.00
e^2	20-40th	14.02	11.90	4.54	0.01	0.00	0.00
e^3	40-60th	20.46	17.75	12.55	3.56	0.00	0.00
e^4	60-80th	27.89	25.24	20.40	12.35	1.64	0.35
e^5	80-90th	37.71	32.90	32.30	22.41	7.45	10.15
e^6	90-95th	47.60	43.79	42.47	34.78	12.52	20.57
e^7	95-99th	81.61	68.69	57.48	47.01	19.22	36.73
e^8	99-100th	247.47	443.14	89.02	101.28	100.08	51.30

Source: Authors' calculations from the 2003 PSID.

TABLE V

MARGINAL INDIVIDUAL INCOME TAX RATES IN 2001 (MARRIED HOUSEHOLD, FILED JOINTLY)

Taxable income			Marginal income tax rate (%)
\$0	–	\$45,200	$15.0 \times \varphi_I$
\$45,200	–	\$109,250	$28.0 \times \varphi_I$
\$109,250	–	\$166,500	$31.0 \times \varphi_I$
\$166,500	–	\$297,350	$36.0 \times \varphi_I$
\$297,350	–		$39.6 \times \varphi_I$

TABLE VI

MARGINAL PAYROLL TAX RATES IN 2001

Taxable labor income per worker			Marginal tax rate (%)	
			OASDI	HI
\$0	–	\$80,400	$12.4 \times \varphi_P$	2.9
\$80,400	–		$0.0 \times \varphi_P$	2.9

Note: The payroll tax adjustment factor φ_P equals 1.0 in the baseline economy.

TABLE VII

OASDI REPLACEMENT RATES IN 2001

AIME (b/12)			Marginal replacement rate (%)
\$0	–	\$561	$90.0 \times \varphi_{SS}$
\$561	–	\$3,381	$32.0 \times \varphi_{SS}$
\$3,381	–		$15.0 \times \varphi_{SS}$

Note: The OASDI benefit adjustment factor φ_{SS} is set so that the OASDI is pay-as-you-go in the baseline economies.

TABLE VIII

PERCENT CHANGE IN SELECTED MACRO VARIABLES RELATIVE TO BASELINE

Run #	Year t	GNP	National wealth	Labor supply	Interest rate	Wage rate	Income tax rate[b]	Payroll tax rate
1	1	-0.1	0.0	-0.1	-0.1	0.0	0.1	0.1
Representative	10	1.0	2.4	0.4	-2.4	0.6	-2.7	-4.8
agent without	20	3.5	6.4	2.3	-4.7	1.2	-9.3	-16.4
wage shocks[a]	40	8.9	16.6	5.8	-11.5	2.9	-20.7	-42.5
	Long run	12.3	26.7	6.7	-19.8	5.3	-25.5	-51.9
2	1	0.1	0.0	0.2	0.3	-0.1	-0.4	0.1
Heterogenous	10	0.8	1.7	0.5	-1.5	0.4	-1.5	-4.3
agents with	20	2.1	4.2	1.3	-3.5	0.9	-3.8	-14.9
wage shocks[a]	40	5.6	11.8	3.0	-9.8	2.5	-8.9	-40.9
	Long run	7.7	18.8	3.3	-16.4	4.3	-11.2	-52.1

a. Closed economy, no private annuity markets, and LSRA is off.
b. The proportional change in marginal tax rates across all households.

TABLE IX

CHANGE IN WELFARE PER HOUSEHOLD (1,000 DOLLARS IN 2001)

Run #	Age in year 1	Without LSRA[a] select productivities e^1	e^3	e^5	e^8	With LSRA[b] for all productivities
1	79	-	-0.6	-	-	0.0
Representative	60	-	-30.4	-	-	0.0
agent without	40	-	-78.3	-	-	0.0
wage shocks	20	-	-41.7	-	-	0.0
	0	-	18.5	-	-	30.1
	-20	-	57.2	-	-	30.1
	-∞	-	66.1	-	-	30.1
2	79	-0.2	-0.2	-0.3	-0.6	0.0
Heterogenous	60	-22.1	-29.8	-37.8	-56.8	0.0
agents with	40	-32.9	-51.9	-82.4	-134.1	0.0
wage shocks	20	-9.8	-12.8	-17.5	-30.0	0.0
	0	31.5	32.5	32.2	22.3	-8.1
	-20	62.8	67.8	72.3	67.0	-8.1
	-∞	72.6	79.1	84.8	81.3	-8.1

a. Standard equivalent variations measures.
b. Value of Δtr.

TABLE X

ALTERNATIVE ASSUMPTIONS IN THE HETEROGENOUS AGENT ECONOMY WITH WAGE SHOCKS
PERCENT CHANGE IN SELECTED MACRO VARIABLES RELATIVE TO BASELINE

Run #	Year t	GNP	National wealth	Labor supply	Interest rate	Wage rate	Income tax rate[b]	Payroll tax rate
3	1	0.4	0.0	0.6	0.0	0.0	-1.0	-0.3
Small open	10	1.4	3.1	0.7	0.0	0.0	-2.1	-4.2
economy[a]	20	3.2	8.0	1.1	0.0	0.0	-4.2	-14.2
	40	7.6	20.6	2.0	0.0	0.0	-8.9	-39.5
	Long run	11.3	35.7	0.8	0.0	0.0	-10.8	-50.8
4	1	0.0	0.0	0.1	0.1	0.0	-0.2	0.2
Perfect annuity	10	0.6	1.3	0.3	-1.3	0.2	-1.1	-4.1
markets[a]	20	1.7	3.4	0.9	-3.0	0.7	-3.0	-14.6
	40	4.5	9.3	2.6	-7.6	1.9	-7.6	-40.6
	Long run	6.3	14.5	2.9	-12.6	3.3	-9.9	-51.9
5	1	0.1	0.0	0.1	0.2	0.0	-0.3	0.2
Less	10	0.8	1.7	0.4	-1.6	0.4	-1.4	-4.1
initial	20	2.0	4.1	1.1	-3.6	0.9	-3.5	-14.7
redistribution[a]	40	5.3	11.6	2.8	-9.8	2.5	-8.5	-40.6
	Long run	7.4	18.4	3.0	-16.2	4.3	-10.8	-51.4
6	1	0.2	0.0	0.3	0.4	-0.1	-0.6	0.0
$\frac{1}{2}$ transitory	10	1.0	2.0	0.6	-1.6	0.4	-1.9	-4.5
shocks[a]	20	2.5	4.9	1.5	-4.0	1.0	-4.5	-15.2
	40	6.3	13.2	3.5	-10.7	2.7	-10.1	-41.2
	Long run	8.7	20.9	3.9	-17.7	4.7	-12.8	-52.3

a. Each Run represents one change in assumption relative to Run 2, i.e., the changes are not cumulative.
b. The proportional change in marginal tax rates across all households.

TABLE XI

CHANGE IN WELFARE PER HOUSEHOLD (1,000 DOLLARS IN 2001)

Run #	Age in year 1	Without LSRA[a] select productivities e^1	e^3	e^5	e^8	With LSRA[b] for all productivities
3	79	0.0	0.0	0.0	0.1	0.0
Small open	60	-20.3	-26.7	-31.1	-23.4	0.0
economy[c]	40	-29.0	-45.6	-67.6	-72.9	0.0
	20	-7.1	-9.0	-11.7	-16.7	0.0
	0	27.1	29.5	33.0	38.3	-10.1
	-20	52.3	59.2	68.4	84.4	-10.1
	$-\infty$	60.3	68.5	79.9	99.5	-10.1
4	79	-0.4	-0.4	-0.5	-0.8	0.0
Perfect annuity	60	-18.6	-23.6	-28.6	-42.9	0.0
markets[c]	40	-31.2	-47.5	-68.8	-99.7	0.0
	20	-12.1	-15.3	-20.1	-31.0	0.0
	0	22.3	21.9	20.0	9.4	-10.9
	-20	47.3	49.9	51.5	43.9	-10.9
	$-\infty$	53.9	57.2	59.8	54.0	-10.9
5	79	-0.25	-0.3	-0.4	-0.7	0.0
Less	60	-22.4	-30.1	-38.9	-60.7	0.0
initial	40	-32.6	-52.3	-84.2	-137.8	0.0
redistribution[c]	20	-10.1	-13.1	-17.9	-30.5	0.0
	0	31.1	32.0	31.5	21.5	-9.4
	-20	61.9	66.8	71.0	65.6	-9.4
	$-\infty$	70.9	77.2	82.6	78.9	-9.4
6	79	-0.2	-0.2	-0.3	-0.9	0.0
$\frac{1}{2}$ transitory	60	-19.3	-29.7	-39.3	-68.1	0.0
shocks[c]	40	-29.8	-56.2	-92.7	-161.8	0.0
	20	-11.7	-17.5	-25.8	-45.8	0.0
	0	27.0	27.2	25.4	8.1	-14.0
	-20	56.1	62.2	66.8	57.0	-14.0
	$-\infty$	65.2	73.2	79.6	72.7	-14.0

a. Standard equivalent variations measures.

b. Value of Δtr.

c. Each Run represents one change in assumption relative to Run 2, i.e., the changes are not cumulative.

TABLE XII

ALTERNATIVE EXPERIMENTS IN THE HETEROGENOUS AGENT ECONOMY WITH WAGE SHOCKS
PERCENT CHANGE IN SELECTED MACRO VARIABLES RELATIVE TO BASELINE

Run #	Year t	GNP	National wealth	Labor supply	Interest rate	Wage rate	Income tax rate[b]	Payroll tax rate
7	1	-1.3	0.0	-1.9	-2.3	0.6	5.9	2.0
Contribution	10	-1.0	-0.5	-1.2	-0.8	0.2	4.9	-2.3
matching	20	0.1	1.2	-0.3	-1.8	0.5	2.6	-13.3
starting at 10%[a]	40	3.6	8.2	1.7	-7.5	1.9	-3.3	-40.3
	Long run	5.9	15.3	2.0	-14.4	3.7	-6.1	-51.8
8	1	-0.1	0.0	-0.1	-0.1	0.0	0.0	0.3
More progressive	10	0.4	1.2	0.1	-1.4	0.3	-0.7	-2.8
S.S. bend points	20	1.3	3.1	0.6	-2.9	0.7	-2.4	-11.2
120 / 33 / 7.5%[a]	40	4.2	9.1	2.2	-7.9	2.0	-6.9	-34.3
	Long run	6.2	15.3	2.5	-13.8	3.6	-9.3	-45.6
9	1	-0.2	0.0	-0.4	-0.4	0.1	0.5	0.6
More progressive	10	0.0	0.6	-0.3	-1.1	0.3	0.1	-1.4
S.S. bend points	20	0.6	2.1	0.0	-2.5	0.6	-1.1	-7.5
150 / 34 / 0%[a]	40	3.0	6.8	1.4	-6.2	1.6	-5.0	-27.6
	Long run	4.8	12.1	1.8	-11.4	2.9	-7.3	-38.7
10	1	0.7	0.0	0.9	1.2	-0.3	-4.4	-50.0
More progressive	10	1.3	2.6	0.7	-2.2	0.5	-4.8	-50.0
S.S. bend points	20	2.1	4.9	0.9	-4.7	1.2	-5.6	-50.0
150 / 34 / 0%	40	4.0	9.6	1.8	-8.9	2.2	-7.5	-50.0
and consumption	Long run	5.1	12.9	1.9	-12.1	3.1	-8.3	-50.0
tax financing[a]								

a. Each Run represents one change in assumption relative to Run 2, i.e., the changes are not cumulative.
b. The proportional change in marginal tax rates across all households.

TABLE XIII

CHANGE IN WELFARE PER HOUSEHOLD (1,000 DOLLARS IN 2001)

Run #	Age in year 1	Without LSRA[a] select productivities e^1	e^3	e^5	e^8	With LSRA[b] for all productivities
7	79	-0.9	-1.0	-1.2	-1.6	0.0
Contribution	60	-22.7	-29.3	-43.4	-103.4	0.0
matching	40	-29.3	-55.5	-94.3	-179.1	0.0
starting at 10%[c]	20	-5.4	-10.0	-19.4	-38.2	0.0
	0	34.3	33.5	28.4	12.6	-13.2
	-20	65.9	69.3	69.1	58.8	-13.2
	-∞	76.0	80.5	81.7	73.5	-13.2
8	79	-0.3	-0.3	-0.4	-0.8	0.0
More progressive	60	-16.9	-22.5	-30.3	-51.6	0.0
S.S. bend points	40	-24.9	-41.7	-70.2	-115.5	0.0
120 / 33 / 7.5%[c]	20	-9.2	-11.9	-16.2	-27.0	0.0
	0	25.9	26.3	25.5	16.7	-7.1
	-20	54.1	58.4	62.0	57.5	-7.1
	-∞	63.3	68.7	73.8	71.3	-7.1
9	79	-0.4	-0.4	-0.5	-0.9	0.0
More progressive	60	-11.7	-15.3	-22.9	-46.4	0.0
S.S. bend points	40	-18.2	-31.9	-58.2	-97.7	0.0
150 / 34 / 0%[c]	20	-8.9	-11.5	-15.4	-24.7	0.0
	0	18.2	17.9	17.0	10.0	-6.7
	-20	45.2	48.7	51.5	47.3	-6.7
	-∞	53.7	58.2	62.3	60.5	-6.7
10	79	-9.7	-10.4	-12.9	-22.3	0.0
More progressive	60	-25.9	-34.5	-51.3	-186.0	0.0
S.S. bend points	40	-16.3	-28.9	-63.3	-188.3	0.0
150 / 34 / 0%	20	4.2	2.5	0.6	-8.8	0.0
and consumption	0	29.3	31.1	32.4	25.3	2.8
tax financing[c]	-20	50.1	54.5	58.4	54.6	2.8
	-∞	56.7	61.7	66.2	63.7	2.8

a. Standard equivalent variations measures.

b. Value of Δtr.

c. Each Run represents one change in assumption relative to Run 2, i.e., the changes are not cumulative.