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Social Security, Retirement and Wealth: Theory and Implications

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Abstract

The effect of Social Security rules on the age people choose to retire can be critical in evaluating proposed changes to those rules. This research derives a theory of retirement that views retirement as a special type of labor supply decision. This decision is driven by wealth and substitution effects on labor supply, interacting with a fixed cost of working that makes low hours of work unattractive.

The theory is tractable analytically, and therefore well-suited for analyzing proposals that affect Social Security. This research examines how retirement age varies with generosity of Social Security benefits. A ten-percent reduction in the value of benefits would lead individuals to postpone retirement by between one-tenth and one-half a year. Individuals who are relatively buffered from the change—because they are wealthier or because they are younger and therefore can more easily increase saving to offset the cut in benefits—will have smaller changes in their retirement ages.

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1 Introduction

Understanding the determinants of the age of retirement is crucial for the ongoing discussions about Social Security reform. At a minimum, the age of retirement should respond to (a) the resources available to the household for retirement, (b) the marginal net earnings from an extra year of work and (c) the health of the household members nearing retirement. To the extent that these factors affect the age of retirement, changes in public policy and in health care technology will cause important movements in the average age of retirement. Movements in the average age of retirement in turn have important deadweight loss and budgetary consequences. We refer to the size of the response of the age of retirement to these factors as the "retirement elasticity."

From an *a priori* theoretical point of view, there is good reason to think that the retirement elasticity might be substantial. Although some people gradually cut back their hours over many years as they move toward full retirement, most retirees make a fairly sudden transition from working 20, 30, 40 or more hours per week to essentially zero hours. This sudden transition strongly suggests the existence in many jobs of fixed costs of going to work that make it inefficient to continue working but work only small number of hours per week. If these costs are borne by the worker, the worker will insist on working a substantial number of hours or not at all. If these costs are borne by the firm, the firm will insist on the worker putting in a substantial number of hours or quitting entirely. The outcome will be similar either way. Hours of work will make a discrete jump at some critical level of the underlying determinants.

Kimball and Shapiro (2003) propose a model of consumption and labor supply that (1) allows for the kind of fixed costs that generate sudden transitions between zero and substantial positive hours of work, (2) maintains the traditional view that the income and substitution effects on labor supply of a permanent increase in the real wage should (approximately) cancel, and (3) allows for nonseparability between consumption and labor. In this model, there are two types of forces ultimately leading to retirement even when there is no drop in the effective wage W. One is the increasing disutility of work with increasing age and declines in health. The other is the extra wealth accumulated with each extra year of work, which creates a wealth effect in favor of retirement. The following section comes from Kimball and Shapiro (2003), with minor modifications.

2 Structural Model of Labor Supply

Consider a two-member household with no bequest motive that faces uncertainty about mortality but no other risks. Assume fair annuities and life insurance are available but that there are no other risky assets. The household faces the optimization problem

$$\max_{C,N_1,N_2} E_0 \int_0^\infty e^{-\rho t} U(C, N_1, N_2, \nu) dt,$$

s.t.,

$$A_0 = E_0 \int_0^\infty e^{-rt} [C - W_1 N_1 - W_2 N_2 - \Pi]$$

where C is total consumption expenditure by the household, N_1 and N_2 are labor hours with corresponding after-tax wages W_1 and W_2, Π is government lump-sum transfers and $\nu = (\nu_1, \nu_2)$ is an exogenous vector of indicator variables for who is alive. Consumption C, labor hours N_1 and N_2 and transfers Π can be functions of the life-state ν as well as of time, while after-tax wages W_1 and W_2 are functions only of time. Note that any effect of earnings on the expected present value of Social Security payments would need to show up in $W_i(t)$. Π is only the lump-sum aspect to transfers. The only significance of time zero will be that it is a moment when new information about life-time resources arrives. It can be at any point in the life cycle. If the flow of utility, consumption, labor hours and transfers are uniformly zero when both members of the household are dead, and $\Gamma = \{(0,1),(1,0),(1,1)\}$ is the set of life-states when at least one person is alive, then the optimization problem can be spelled out as

$$\max_{C(\nu,t),N_1(\nu,t),N_2(\nu,t)} \sum_{\nu\in\Gamma} \int_0^\infty p(\nu,t)e^{-\rho t}U(C(\nu,t),N_1(\nu,t),N_2(\nu,t),\nu)dt, \tag{1}$$

s.t.,

$$A_0 = \sum_{\nu\in\Gamma} \int_0^\infty e^{-rt}p(\nu,t)[C(\nu,t)-W_1(t)N_1(\nu,t)-W_2(t)N_2(\nu,t)-\Pi(\nu,t)], \tag{2}$$

where $p(\nu,t)$ is the probability of a life-state. The form of the budget constraint reflects the existence of fair annuity and life-insurance markets.

The Lagrangian for (1) is

$$\begin{aligned}\mathcal{L} &= \lambda_0 A_0 + \sum_{\nu\in\Gamma} \int_0^\infty p(\nu,t)e^{-\rho t}\left\{U(C(\nu,t),N_1(\nu,t),N_2(\nu,t),\nu)\right.\\ &\qquad \left.+\lambda_0 e^{(\rho-r)t}[W_1(t)N_1(\nu,t)+W_2(t)N_2(\nu,t)+\Pi(\nu,t)-C(\nu,t)]\right\}dt,\end{aligned}$$

where the Lagrange multiplier λ_0 is a scalar that is constant across both time and life-states. Regardless of the shape of the utility function, maximizing the intertemporal Lagrangian requires maximizing the integrand

$$I = U(C,N_1,N_2,\nu) + \lambda_0 e^{(\rho-r)t}[W_1N_1+W_2N_2+\Pi-C] \tag{3}$$

at each time t and life-state ν. In particular, the integrand I needs to be maximized even if the utility function is nonconcave because of fixed costs of going to work.[1]

[1] If one defines $\lambda(t) = \lambda_0 e^{(\rho-r)t}$, the integrand differs from the current-value Hamiltonian in Kimball and Shapiro (2003) only by the term $\lambda(t)rA(t)$—a term needed only to get the Euler equation that integrates to $\lambda(t) = \lambda_0 e^{(\rho-r)t}$. The term $\lambda(t)rA(t)$ does not affect the optimization over consumption and labor conditional on $\lambda(t)$.

In order to build in the observed cancellation of income and substitution effects on labor supply of a permanent increase in the real wage, let

$$U(C, N_1, N_2, \nu) = -\frac{(1-\alpha)}{\alpha} C^{-\alpha/(1-\alpha)}[\psi(\nu) + \alpha g(N_1, N_2, t)]^{1/(1-\alpha)} \tag{4}$$

when $\alpha \in (0, 1)$ and

$$U(C, N_1, N_2) = \psi(\nu)\ln(C) - g(N_1, N_2, t). \tag{5}$$

when $\alpha = 0$. α measures the degree of substitutability between consumption and leisure—*or equivalently, the degree of complementarity between consumption and labor*. $\alpha = 0$ corresponds to additive separability, while $\alpha = 1$ corresponds to perfect substitutability between consumption and leisure. $\alpha \in (0, 1)$ corresponds to consumption being a partial substitute for leisure.

$g(N_1, N_2, t)$ is an increasing function of N_1 and N_2. We assume that the household acts as a unit, maximizing the joint utility of its members. For any given total amount of household expenditure, the utility from consumption depends on how many people that expenditure is spread over. $\psi(\nu)$ is a household equivalence scale:

$$\psi(1, 0) = \psi(0, 1) = 1,$$

$$1 < \psi(1, 1) \leq 2$$

and

$$g(0, 0) = 0.$$

It is possible to obtain some results that apply quite generally for any shape of the disutility of work function $g(N_1, N_2, t)$. To deal with possible additive nonseparability between consumption and labor, it is helpful to maximize first over consumption conditional on labor quantities.

The first-order condition for optimal consumption is

$$\frac{\partial I}{\partial C} = C^{-1/(1-\alpha)}[\psi(\nu) + \alpha g(N_1, N_2, t)]^{1/(1-\alpha)} = \lambda_0 e^{(\rho - r)t}.$$

Solving for consumption,

$$C = \lambda_0^{-(1-\alpha)} e^{-(1-\alpha)(\rho-r)t}[\psi(\nu) + g(N_1, N_2, t)]. \tag{6}$$

Define "baseline consumption" B by

$$B = \lambda_0^{-(1-\alpha)} e^{-(1-\alpha)(\rho-r)t}\psi(\nu) \tag{7}$$

and total "job-induced consumption" J by

$$J = \alpha\lambda_0^{1-\alpha} e^{-(1-\alpha)(\rho-r)t} g(N_1, N_2, t) = \frac{\alpha B g(N_1, N_2, t)}{\psi(\nu)}. \tag{8}$$

Then total consumption equals baseline consumption plus the job-induced consumption of each worker in the household:

$$C = B + J = B\left(1 + \frac{\alpha g(N_1, N_2, t)}{\psi(\nu)}\right). \tag{9}$$

Because it represents every interaction between work and consumption, "job-induced consumption" must be construed quite broadly. It includes both (1) *work-related consumption* (such as childcare, transportation to and from work, the extra expense of food at work, and the extra expense of clothes suitable for work), and (2) *extra time-saving consumption* (such as easy-to-prepare foods at home, house-cleaning and house-repair services, and household conveniences).

Returning to the underlying expression for optimized consumption, and substituting into the integrand I, the maximized integrand $\bar{I}$ is

$$\bar{I} \;=\; \lambda_0 e^{(\rho-r)t}\left\{\Pi + W_1N_1 + W_2N_2 - \lambda_0^{-(1-\alpha)} e^{-(1-\alpha)(\rho-r)t}\left[\frac{\psi(\nu)}{\alpha} + g(N_1, N_2, t)\right]\right\}$$

Maximizing I over C, N_1 and N_2 is the same as maximizing $\bar{I}$ over N_1 and N_2. This in turn requires solving the optimization subproblem

$$\max_{N_1,N_2} \; W_1N_1 + W_2N_2 - \lambda_0^{1-\alpha} e^{-(1-\alpha)(\rho-r)t} g(N_1, N_2, t). \tag{10}$$

The structure of (10), the optimization subproblem for N_1 and N_2, is the same regardless of the shape of $g(N_1, N_2, t)$. A model of endogenous retirement calls for some sort of nonconcavity. Section 3 considers $g(N_1, N_2, t)$ additively separable in N_1 and N_2 with employers allowing complete flexibility in work hours with proportional pay, but a fixed utility cost of going to work. Conversely, Section 4 examines an employer-imposed restriction to work $\bar{N}_1$ and $\bar{N}_2$ hours or not at all. In either case, the objective is to relate endogenous retirement ages to the resources available to a household and to a pair of multiplicative work aversion parameters M_1 and M_2. Section 5 considers the case of a single-earner facing employer-imposed hours restrictions and develops a set of numerical examples.

3 Endogenous Retirement and Labor Supply with Fixed Costs of Going to Work

Before going on, one observation is in order. In the next section, an interior solution for N_1 and N_2 is often relevant. Given the optimization subproblem, (10), the first order condition for an interior solution for N_i is

$$W_i = \lambda_0^{1-\alpha} e^{-(1-\alpha)(\rho-r)t} \frac{\partial g(N_1, N_2, t)}{\partial N_i}. \tag{11}$$

This first order condition, combined with equation (8), yields the useful fact

$$\frac{\partial J}{\partial N_i} = \alpha \lambda_0^{1-\alpha} e^{-(1-\alpha)(\rho-r)t} \frac{\partial g(N_1, N_2, t)}{\partial N_i} = \alpha W_i. \tag{12}$$

In particular, if g is additively separable in N_1 and N_2,

$$g(N_1, N_2, t) = \sum_{i=1,2} g_i(N_i, t),$$

then

$$J = J_1 + J_2$$

where

$$J_i = \alpha \lambda_0^{1-\alpha} e^{-(1-\alpha)(\rho-r)t} g_i(N_i) = \frac{\alpha B g_i(N_i, t)}{\psi(\nu)}. \tag{13}$$

and at an interior optimum,

$$\frac{\partial J_i}{\partial N_i} = \alpha \lambda_0^{1-\alpha} e^{-(1-\alpha)(\rho-r)t} \frac{\partial g(N_i, t)}{\partial N_i} = \alpha W_i. \tag{14}$$

Now, specify g_i by

$$g_i(N_i, t) = \chi(N_i) M_i(t)[F_i + v_i(N_i)], \tag{15}$$

where χ is the indicator function for working:

$$\chi(N_i) = \begin{cases} 0 & \text{if } N_i = 0 \\ 1 & \text{if } N_i > 0 \end{cases}.$$

Here $M_i(t)$ is the "work aversion" factor, F is a positive number that models the fixed utility cost of going to work and v_i is a function satisfying $v_i(0) = 0$, $v_i'(N) > 0$, $v_i''(N) > 0$ and $v_i'(168) = \infty$. (N is measured in weekly hours. 24 hours a day, seven days a week is 168 hours a week.)

The additive separability of $g(N_1, N_2, t)$ in N_1 and N_2 also means that the maximization subproblem (10) can be broken into the two maximization subproblems

$$\max_{N_i} \quad W_i N_i - \lambda_0^{-(1-\alpha)} e^{-(1-\alpha)(\rho-r)t} \chi(N_i) M_i(t)[F_i + v_i(N_i)]. \tag{16}$$

If $N_i > 0$, the first order necessary condition for optimal N_i in (16) is

$$W_i = \lambda_0^{-(1-\alpha)} e^{-(1-\alpha)(\rho-r)t} M_i(t) v_i'(N_i).$$

Call the solution to this first order condition N_i^*. Then

$$N_i^* = v'^{-1}\left(\frac{\lambda_0^{(1-\alpha)} e^{(1-\alpha)(\rho-r)t} W_i}{M_i}\right). \tag{17}$$

Because this is a necessary condition for an optimum when $N_i > 0$, the optimal N_i must be either N_i^* or 0. To determine whether the optimal N_i is N_i^* or 0, we need to compare the value of the criterion function

$$W_i N_i - \chi(N_i) \lambda_0^{-(1-\alpha)} e^{-(1-\alpha)(\rho-r)t} M_i(t)[F_i + v_i(N_i)] \tag{18}$$

at $N_i = 0$ with its value at $N_i = N_i^*$ to see which one is greater.

When $N_i = 0$, the criterion function (18) is equal to zero. When $N_i = N_i^*$, the criterion function (18) is equal to

$$\lambda_0^{-(1-\alpha)} e^{-(1-\alpha)(\rho-r)t} M_i(t) \left\{N^* v_i'(N^*) - v_i(N^*) - F_i\right\} \tag{19}$$

The right-hand side of (19) is greater than or equal to zero if and only if

$$N^* v_i'(N^*) - v_i(N^*) \geq F.$$

Define the cutoff value for labor, $N_i^{\#}$, by

$$N_i^{\#} v_i'(N_i^{\#}) - v_i(N_i^{\#}) = F. \tag{20}$$

Since $\frac{d}{dN}[N v_i'(N) - v_i(N)] = N v_i''(N) > 0$ whenever $N > 0$, $N v_i'(N) - v_i(N)$ is an increasing function of N. This, plus the assumption that $v_i'(168) = \infty$ guarantees that there is a unique solution to (20) between 0 and 168. In terms of the cutoff value $N_i^{\#}$ that solves equation (20), the rule for optimal labor supply can be written as

$$N_i = \begin{cases} N_i^* & \text{if } N_i^* > N_i^{\#} \\ 0 & \text{if } N_i^* < N_i^{\#} \\ \text{either } N_i^* \text{ or } 0 & \text{if } N_i^* = N_i^{\#} \end{cases}, \tag{21}$$

In practice, we use empirical evidence to calibrate the cutoff value $N_i^{\#}$, and then determine the appropriate value of F. From this point of view, one can see equation (20) the other way around as a mapping from the cutoff value $N_i^{\#}$ to the fixed cost F. In terms of $N_i^{\#}$, the function $g_i(N_i)$ is given by

$$g_i(N_i) = \chi(N_i) M_i(t)[v_i(N_i) - v_i(N_i^{\#}) + N_i^{\#} v_i'(N_i^{\#})]. \tag{22}$$

3.1 Behavior Near the Moment of Retirement

At the moment of planned retirement R_i, a worker must be exactly indifferent between working and not working, which also implies that flexibly chosen work hours will be $N_i^{\#}$ at that moment. Setting the criterion function (18) equal to zero at $N_i^{\#}$ and making all dependence on time explicit,

$$W_i(R_i) N_i^{\#} = \lambda_0^{-(1-\alpha)} e^{-(1-\alpha)(\rho - r)R_i} M_i(R_i)[F_i + v_i(N_i^{\#})]. \tag{23}$$

As a result, by (13) and (22) , at the moment right before planned retirement, the job-induced consumption is

$$\begin{aligned} J_i(R_i) &= \alpha \lambda_0^{-(1-\alpha)} e^{-(1-\alpha)(\rho - r)R_i} g_i(N_i^{\#}, R_i) \\ &= \alpha \lambda_0^{-(1-\alpha)} e^{-(1-\alpha)(\rho - r)R_i} M_i(R_i)[F_i + v_i(N_i^{\#})] \\ &= \alpha W_i(R_i) N_i^{\#}. \end{aligned} \tag{24}$$

How is the moment of retirement determined? A worker is indifferent between working and not working at age t if

$$N_i^{\#} = N_i^*(t) = v'^{-1}\left(\frac{\lambda_0^{(1-\alpha)} e^{(1-\alpha)(\rho-r)t} W_i(t)}{M_i(t)}\right),$$

or equivalently, taking the function $\ln v_i'(\cdot)$ of both sides, the retirement age R_i must solve

$$\begin{aligned} \ln v_i'(N_i^{\#}) &= \ln(v_i'(N^*(R_i)) \qquad (25) \\ &= (1-\alpha)\ln(\lambda_0) + (1-\alpha)(\rho-r)R_i + \ln(W_i(R_i)) - \ln(M_i(R_i)) \end{aligned}$$

It is quite reasonable to assume that $(1-\alpha)(\rho-r)t + \ln(W_i(t)) - \ln(M_i(t))$ is single peaked, since wages will tend to go up fast with age at first and slower later on, while the disutility of work will go up slowly with age first, then faster later on. The assumption of single-peakedness to $\ln(v_i'(N^*(t)))$ implies that the worker can be indifferent to working versus not working at most twice: once when he or she starts working and once when he or she retires. It is straightforward to add in the effects of an endogenous inception of work, but for clarity, we will assume in the main text that time 0 is after the inception of work for anyone in the household who will in fact work in the future and deal with an endogenous inception of work only in footnotes.[2]

The structure involved in saying that it is a multiplicative aversion to work parameter that evolves over time makes $N_i^{\#}$ constant. Also, given that worker i is alive, equation (25), guarantees that the optimal retirement age is not affected by the death of a spouse. This is a reflection of (a) the assumption of fair annuity and life insurance markets and (b) the additive separability of g in N_1 and N_2. As for the other terms in equation (25), it is important to distinguish between evolution over time according to an undisturbed optimal plan and comparative-statics changes. We will analyze the comparative statics of wealth effect using a complete differential, trusting that this use of the symbol d will not be confused with its use in the integration differential dt. Wealth effects operate through λ_0 and can change the optimal retirement age. The total differential of equation (25) needed to find out how much is

[2]Since the inception of work is not our focus, even then, we will abstract from the effects of endogenous education on the inception of work.

$$0 = (1-\alpha)d\ln(\lambda_0) + (1-\alpha)(\rho - r)dR_i + \frac{W_i'(R_i)}{W_i(R_i)}dR_i - \frac{M_i'(R_i)}{M_i(R_i)}dR_i. \quad (26)$$

Solving for dR_i,

$$dR_i = \frac{(1-\alpha)d\ln(\lambda_0)}{\frac{M_i'(R_i)}{M_i(R_i)} - \frac{W_i'(R_i)}{W_i(R_i)} - (1-\alpha)(\rho - r)}. \quad (27)$$

The second-order condition implies that the denominator is positive around the optimal retirement date. For the sake of interpretation, it is helpful to use

$$B_0 = B(0) = \lambda_0^{-(1-\alpha)}\psi(\nu_0)$$

from (7) in differential form

$$d\ln B_0 = -(1-\alpha)d\ln\lambda_0.$$

(The household equivalent scale for those alive at time zero $\psi(\nu_0)$ will not be affected by the wealth shock.) Then

$$\frac{dR_i}{d\ln(B_0)} = \frac{-1}{\frac{M_i'(R_i)}{M_i(R_i)} - \frac{W_i'(R_i)}{W_i(R_i)} - (1-\alpha)(\rho - r)}. \quad (28)$$

3.2 The Effect of Wealth Shocks on Initial Baseline Consumption and the Optimal Retirement Age

What remains in order to find the effect of wealth shocks on the optimal retirement age is to relate initial baseline consumption B_0 to total resources. The starting place for this is the budget constraint (2). Let us add the expected present value of government transfers into initial financial wealth to get a combined concept of nonhuman wealth, $\mathcal{A}$:

$$\mathcal{A} = A_0 + \sum_{\nu\in\Gamma}\int_0^\infty e^{-rt}p(\nu,t)\Pi(\nu,t)\,dt. \quad (29)$$

Also, since the additive separability of g and existence of fair annuities markets guarantee that both labor income and the job-induced consumption from each worker in the household depend only on whether that worker is alive, define probabilities of each worker individually being alive:

$$q_1(t) = p(1,1,t) + p(1,0,t)$$

and

$$q_2(t) = p(1,1,t) + p(0,1,t).$$

Then, using the fact that $C = B + J_1 + J_2$, and that $J_i = N_i = 0$ after retirement,

$$\mathcal{A} = \sum_{\nu\in\Gamma}\int_0^\infty e^{-rt}p(\nu,t)B(\nu,t)\,dt + \sum_i \int_0^{R_i} e^{-rt}q_i(t)[J_i(t) - W_i(t)N_i(t)]dt. \quad (30)$$

Now, let's calculate the total differentials for each term. Equation (7) implies that

$$dB(t) = B(t)d\ln B_0, \quad (31)$$

and therefore

$$d\sum_{\nu\in\Gamma}\int_0^\infty e^{-rt}p(\nu,t)B(\nu,t)\,dt = \left\{\sum_{\nu\in\Gamma}\int_0^\infty e^{-rt}p(\nu,t)B(\nu,t)\,dt\right\}d\ln B_0 \quad (32)$$

Equations (13) and (14) imply that

$$dJ_i(t) = \alpha W_i\,dN_i + J_i(t)\,d\ln B_0 \quad (33)$$

Equations (17) and (21) imply that, except in the right around the moment of retirement (or right around the moment when work begins),

$$dN_i(t) = -N_i(t)\eta(N_i(t))\,d\ln B_0 \quad (34)$$

where the Frisch labor supply elasticity η is defined by

$$\eta(N_i(t)) = \begin{cases} \frac{v_i'(N_i(t))}{N_i(t)v_i''(N_i(t))} & \text{if } N_i = 0 \\ 0 & \text{if } N_i = 0. \end{cases} \quad (35)$$

Note that for wealth shocks, $dW_i(t) = dM_i(t) = 0$.

By Leibniz's rule and equations (24), (33) and (34),

$$
\begin{aligned}
d\int_0^{R_i} e^{-rt}q_i(t) \quad & [J_i(t) - W_i(t)N_i(t)]dt \\
&= \left\{\int_0^{R_i} e^{-rt}q_i(t)[J_i(t) + (1-\alpha)W_i(t)N_i(t)\eta(N_i(t))]dt\right\} d\ln B_0 \\
&\quad -(1-\alpha)e^{-rR_i}q_i(R_i)W_i(R_i)N_i^{\#}\, dR_i \qquad (36)
\end{aligned}
$$

Let $\mathcal{H}_i$ be human wealth—the expected present value of labor income—from person i:

$$
\mathcal{H}_i = \int_0^{R_i} e^{-rt}q_i(t)W_i(t)N_i(t)\, dt. \qquad (37)
$$

Also, define η_i as the weighted average of the Frisch labor supply elasticity:

$$
\eta_i = \frac{\int_0^{R_i} e^{-rt}q_i(t)W_i(t)N_i(t)\eta(N_i(t))dt}{\int_0^{R_i} e^{-rt}q_i(t)W_i(t)N_i(t)dt}. \qquad (38)
$$

Then using (28), equation (36) becomes

$$
\begin{aligned}
\frac{d\int_0^{R_i} e^{-rt}q_i(t)[J_i(t) - W_i(t)N_i(t)]dt}{d\ln B_0} &= \frac{(1-\alpha)e^{-rR_i}q_i(R_i)W_i(R_i)N_i^{\#}}{\frac{M_i'(R_i)}{M_i(R_i)} - \frac{W_i'(R_i)}{W_i(R_i)} - (1-\alpha)(\rho - r)} \\
&\quad + \int_0^{R_i} e^{-rt}q_i(t)J_i(t)\, dt + (1-\alpha)\eta_i\mathcal{H}_i \qquad (39)
\end{aligned}
$$

Finally, totally differentiating (30) using (32) and (39),

$$
\begin{aligned}
\frac{d\mathcal{A}}{d\ln B_0} &= \sum_{\nu\in\Gamma}\int_0^{\infty} e^{-rt}p(\nu,t)B(\nu,t)\, dt + \sum_i \int_0^{R_i} e^{-rt}q_i(t)[J_i(t) - W_i(t)N_i(t)] \\
&\quad + \sum_i \left\{[1 + (1-\alpha)\eta_i]\mathcal{H}_i + \frac{(1-\alpha)e^{-rR_i}q_i(R_i)W_i(R_i)N_i^{\#}}{\frac{M_i'(R_i)}{M_i(R_i)} - \frac{W_i'(R_i)}{W_i(R_i)} - (1-\alpha)(\rho - r)}\right\} \\
&= \mathcal{A} + \sum_i [1 + (1-\alpha)\eta_i]\mathcal{H}_i \\
&\quad + \sum_i \frac{(1-\alpha)e^{-rR_i}q_i(R_i)W_i(R_i)N_i^{\#}}{\frac{M_i'(R_i)}{M_i(R_i)} - \frac{W_i'(R_i)}{W_i(R_i)} - (1-\alpha)(\rho - r)}. \qquad (40)
\end{aligned}
$$

Taking the reciprocal of both sides of (40) and using (28),

$$\begin{aligned}\frac{d\ln B_0}{d\mathcal{A}} &= \frac{1}{\mathcal{A}+\sum_i[1+(1-\alpha)\eta_i]\mathcal{H}_i+\sum_i \frac{(1-\alpha)e^{-rR_i}q_i(R_i)W_i(R_i)N_i^{\#}}{\frac{M_i'(R_i)}{M_i(R_i)}-\frac{W_i'(R_i)}{W_i(R_i)}-(1-\alpha)(\rho-r)}} \\ &= \frac{1}{\mathcal{A}+\sum_i\left\{[1+(1-\alpha)\eta_i]\mathcal{H}_i-\frac{dR_i}{d\ln(B_0)}(1-\alpha)e^{-rR_i}q_i(R_i)W_i(R_i)N_i^{\#}\right\}}\end{aligned}$$

and

$$\frac{dR_j}{d\mathcal{A}} = \frac{\frac{dR_j}{d\ln(B_0)}}{\mathcal{A}+\sum_i\left\{[1+(1-\alpha)\eta_i]\mathcal{H}_i-\frac{dR_i}{d\ln(B_0)}(1-\alpha)e^{-rR_i}q_i(R_i)W_i(R_i)N_i^{\#}\right\}}, \tag{41}$$

where

$$\frac{dR_i}{d\ln(B_0)} = \frac{-1}{\frac{M_i'(R_i)}{M_i(R_i)}-\frac{W_i'(R_i)}{W_i(R_i)}-(1-\alpha)(\rho-r)}.$$

4 Hours Contraints Imposed by Employers

One feature of the foregoing model that may be unrealistic is the assumption that workers can freely choose their hours. As the polar opposite case, suppose instead that throughout a worker's life, the only options for work hours are zero and a fixed number of weekly hours $\bar{N}_i$ imposed by the employer as a condition of employment. (The constrained number of hours may differ from worker to worker depending on the type of jobs they tend to hold.) How does this modify the equations above?

To begin with, optimal consumption given N_1 and N_2 is unaffected. Also, the maximization subproblem for optimal labor hours is unchanged except for the constraint. Subproblem (16) becomes

$$\max_{N_i\in\{0,\bar{N}_i\}} \quad W_iN_i-\lambda_0^{-(1-\alpha)}e^{-(1-\alpha)(\rho-r)t}\chi(N_i)M_i(t)[F_i+v_i(N_i)]. \tag{42}$$

Thus, the individual will work if

$$W_i\bar{N}_i - \lambda_0^{-(1-\alpha)} e^{-(1-\alpha)(\rho-r)t} M_i(t)[F_i + v_i(\bar{N}_i)]$$

is positive and be indifferent to working and not working when this criterion function is zero. At the moment of planned retirement, the individual will be indifferent between working and not working, so (23) becomes

$$\begin{aligned} W_i(R_i)\bar{N}_i &= \lambda_0^{-(1-\alpha)} e^{-(1-\alpha)(\rho-r)R_i} M_i(R_i)[F_i + v_i(\bar{N}_i)] \\ &= \frac{B_0}{\psi(\nu_0)} e^{(\rho-r)R_i} M_i(R_i)[F_i + v_i(\bar{N}_i)]. \end{aligned} \tag{43}$$

Taking logarithms and collecting all of the constants on the left-hand-side of the equation,

$$\ln(\bar{N}_i) - \ln(F_i + v_i(\bar{N}_i)) + \ln(\psi(\nu_0)) = \ln(B_0) + (\rho - r)R_i + \ln(M_i(R_i)) - \ln(W_i(R_i)). \tag{44}$$

Clearly, as resources vary, the total derivative of (45) implies that

$$\frac{dR_i}{d\ln(B_0)} = \frac{-1}{\frac{M_i'(R_i)}{M_i(R_i)} - \frac{W_i'(R_i)}{W_i(R_i)} - (1-\alpha)(\rho - r)}. \tag{45}$$

The form of (45) is identical to the form of equation (28) that holds in the absence of hours constraints. Without hours constraints, hours right before retirement are always $N_i^{\#}$. With hours constraints, hours right before retirement are always $\bar{N}_i$.

Finally, with hours constraints, job-induced consumption at the moment right before planned retirement is

$$\begin{aligned} J_i(R_i) &= \frac{B_0}{\psi(\nu_0)} e^{-(1-\alpha)(\rho-r)R_i} g_i(\bar{N}_i, R_i) \\ &= \alpha \frac{B_0}{\psi(\nu_0)} e^{-(1-\alpha)(\rho-r)R_i} M_i(R_i)[F_i + v_i(\bar{N}_i)] \\ &= \alpha W_i(R_i)\bar{N}_i. \end{aligned} \tag{46}$$

Following through the rest of the calculations for the retirement elasticity with $dN_i = 0$ up until retirement,

$$\frac{dR_j}{d\mathcal{A}} = \frac{\frac{dR_j}{d\ln(B_0)}}{\mathcal{A} + \sum_i \left\{\mathcal{H}_i - \frac{dR_i}{d\ln(B_0)}(1-\alpha)e^{-rR_i}q_i(R_i)W_i(R_i)\bar{N}_i\right\}}, \tag{47}$$

where $\frac{dR_i}{d\ln(B_0)}$ is given in (44) above.

5 A Numerical Illustration

Even the simplest numerical example that makes a serious attempt at being realistic is quite instructive. In this section, we calculate the retirement elasticity for particular parameter values of a single-person household facing employer-imposed hours constraints. Given our analytic model outlined above, these calculations are simple to carry out. We focus here simply on a clear description of the calibration assumptions and a discussion of the results. We consider how the age of retirement increases in response to a 10 percent decrease in the generosity of Social Security benefits.

5.1 Calibration

For simplicity, consider an agent who would retire at age 65 in the absence of any intervention, and we assume that both the real interest rate r and utility discount rate are equal to 3 percent per year. Because of the employer-imposed hours constraints, annual labor income is proportional to the real wage. Without loss of generality, we can normalize the annual labor income to 1 at age 65. Before retirement, we assume a quadratic wage profile hitting its maximum at age 55 and falling at a rate of 5 percent per year at age 65. Denoting the agent's age by a,

$$W\bar{N} = 1.25 - .0025(a-55)^2$$

Initial financial and pension wealth is represented in ratio to the wage at the age a_0 at which the agent finds out about a change to the Social Security rules. This allows a closer relationship with the way the data appear, but it should be kept in mind that current annual labor income is a moving yardstick. We consider values of 0 and 5 years for the financial and pension wealth to labor income ratio.

The current Social Security rules make it realistic to assume that Social Security payments are constant in real terms and that the adjustments for early or late retirement do not change the annuity value of the Social Security payments. That is, benefits are indexed for changes in the cost of living. Moreover, within the relevant range, changing retirement age leads to an actuarially-fair adjustment in the annuity. We express the level of Social Security payments in terms of the replacement rate relative to the final annual wage income for someone who retires at 65. We consider a replacement rate of 0.4 and 0.7 to represent the effects of Social Security for those with higher and lower income.

Another important element of the calibration is to calibrate mortality. Here it is important to realize that mortality probabilities matter primarily in the calculation of the values of annuities. In order to have a compact and tractable representation of mortality, we looked for an expression for survival probabilities that was the sum or difference of two exponential functions that did a good job of matching annuity values as seen from the perspective of the years leading up to retirement. Given our assumptions of a 3 percent real interest rate and focus on an agent retiring at age 65 to begin with, the following parameterization works meets our criteria:

$$q \propto e^{.0035(a-92)} - e^{.0535(a-92)}$$

where q is the survival probability, a is age as above and $\propto$ is the symbol for "is proportional to." In terms of the earlier equations, this proportionality implies

$$q(t) = \frac{e^{.0035(a_0+t-92)} - e^{.0535(a_0+t-92)}}{e^{.0035(a_0-92)} - e^{.0535(a_0-92)}},$$

where a_0 is the agent's age at time zero. It is clear that this calibration of mortality has the counterfactual implication that everyone dies by age 92, but it implies somewhat lower than actual mortality at ages earlier on (but after 65) so that the annuity values for preretirement ages are on track. These annuity values are within 1 percent of those implied by life tables (for men and women combined) for each age from 50 to 64. Moreover, the mortality probabilities leading up to age 65 are close enough that the probability of dying between any age later than 50 years old and age 65 is within 2 percent of what the life tables imply. Given the accuracy of the annuity values and the mortality probabilities before age 65, close study of the relevant equations indicates that the divergence of the mortality probabilities from the life tables after age 65 will not affect our calculated retirement elasticities in any important way.

The assumption of employer-imposed hours constraints makes it unnecessary to know the path of the multiplicative disutility of work M other than the value of M'/M at the initial age of retirement, 65. We consider a wide range of possible values for

$$m = \frac{M'(R)}{M(R)},$$

the growth rate of the disutility of work.

5.2 Results

The results for the various cases are presented in Table 1 through Table 4. Overall, for those with a modest rate of growth of the disutility of work, the effect on the retirement age of a 10 percent decrease in the generosity of Social Security payments is on the order of half a year for those close to retirement and on the order of a quarter of a year for those about 50 years old when they find out about the policy change. These changes are substantial, though not overwhelming.

The tables illustrate the effect of varying key parameters.

- Obviously, the wealthier is an individual or the lower the replacement rate, the smaller the effect on retirement age of the cut in benefits. Social Security benefits are a smaller fraction of such individuals' total wealth, so a change in their value has a smaller impact on decisions. (Compare Tables 1 and 3 or Tables 2 and 4.)

- The older an individual is when the change in policy is announced, the bigger the change in retirement age. Older individuals have less time over which to spread the increased saving required to smooth consumption before and after retirement with the lower level of benefits. (Read down the tables.)

- The more rapidly the disutility of work is increasing, the less sensitive is retirement age to the decrease in benefits. For those individuals with rapidly increasing disutility of work, this consideration damps the financial consideration of working longer. Increasing disutility of work is analogous to decreasing health status, which would operate similarly. (Read across the tables.)

To the extent that our assumption that the wage will be declining at 5 percent per year at age 65 is inaccurate, the wide range of values for m allow one to partially adjust for this. Abstracting from the modest effects on the measurement of the value of human capital if the wage path is different, one can adjust for a lower rate of decline in the wage at age 65 by looking at a column labeled with a lower value of m. Similarly, one can adjust for a higher rate of decline in the wage at age 65 by looking at the column labeled with a higher value of m.

Without explicit modeling, one can see intuititively that for an agent who is liquidity constrained up to a certain age, the age at which the liquidity constraint finally loosens is relevant, rather than the age at which the policy change is announced. Thus, liquidity constraints may make the entries for higher ages relevant. Unless there is pension wealth, the tables with zero wealth at that age are the appropriate ones. This reasoning is why we have started the tables at age 50, since we suspect that many households are liquidity constrained until about age 50, but it would be easy to extend the tables to earlier ages.

6 Discussion

Some of the findings from the theory are (1) the theoretical distinction between the magnitude of the retirement elasticity and the magnitude of the ordinary extensive and intensive labor supply elasticities, (2) the importance of the degree of age-dependence of the disutility of work for the retirement elasticity and (3) the importance of focusing the calibration of the degree of nonseparability between consumption and labor on the behavior of consumption at retirement rather than on the elasticity of intertemporal substitution per se, and (4) the dependence of the effect of wealth shocks on how long before retirement the household finds out about the wealth shock.

This model provides a precise context for analyzing the impact of wealth on retirement. It can be used to study how individuals will change the timing of their retirement in response to announced changes in Social Security (e.g., from a shift to individual plans, changes in the normal retirement age, or changes in the effective rate of taxation of labor earnings once Social Security payments have already begun). The effects of policy changes on the average retirement age depend critically on one unknown parameter-the rate of increase of the disutility of work with age.

If there is any significant effect of policy changes on the average retirement age, policy analysis based on the faulty assumption that retirement ages would not change (or would change very little) could be badly misleading. Taking full account of the retirement elasticity is especially important when the number of people nearing retirement but not yet retired is large. This is exactly the situation the impending retirement of the Baby Boom generation creates.

For example, suppose that cuts in Social Security benefits cause people to choose to retire later than they otherwise would and that this additional work generates additional tax revenue. Then the size of cuts needed to restore solvency to Social Security might be smaller, but two new issues arise.

1. An important part of the extra tax revenue from the later retirement induced by the Social Security policy change might show up as an increase in general income tax revenue, both personal and corporate. Logically speaking, this extra income tax revenue should be credited to the Social Security policy change, but giving due credit in this case would require solid evidence of the magnitude of the retirement elasticity.

2. Whatever revenue benefits accrue from the later retirement are just as large when the benefit cuts apply only to those who have not yet retired as when they are applied to retirees. In other words, what under an analysis ignoring the retirement elasticity would be considered $1 billion in benefit cuts that will apply to those not yet retired may have a budgetary effect considerably larger than $1 billion, while $1 billion of benefit cuts on those already retired would have only $1 billion worth of budgetary effect.

The second point, about the retirement elasticity mattering more for one group (the not-yet-retired) than another (the already retired), applies to other distinctions as well. In particular, the retirement elasticity should be higher for those in good health than for those in poor health. This point suggests the budgetary (and welfare) benefits of concentrating cuts on the more able-bodied, and exempting those for whom later retirement would represent a greater hardship. This policy would involve making the disability-linked component (as opposed to the across-the board component) to Social Security more important.

As another example of how the retirement elasticity could matter, consider a policy change that would reward later retirement with higher monthly Social Security payments at what under the standard analysis would be considered more than an actuarially fair rate. This might make sense budgetarily if all revenue effects (including general income tax revenue effects) of later retirement were taken into account. The size of the welfare gains from this efficiency-enhancing measure would be greater the greater the retirement elasticity. Not only are such welfare gains worthwhile in and of themselves, their inclusion in a needed package of reforms could be important in helping to generate political support for that package of reforms.

Among those not yet retired, the dependence of the retirement elasticity on the age at which people find out about a policy change can also have important policy implications. For example, if those near retirement respond to an announced cut primarily by planning to retire later, while those far from retirement respond primarily by cutting consumption, the larger retirement elasticity for those near retirement means that they generate more extra income-tax revenue than those who are further away from retirement. This effect could provide an important budgetary issue that calls into question the usual presumption that those near retirement should always be exempted from any future benefit cuts.

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Table 1.
Response of Retirement Age to a 10 Percent Reduction in the Value of Social Security Benefits:
Low Income, Low Wealth Household

m	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
age	Increase in retirement age (years)										
50	0.36	0.32	0.29	0.26	0.24	0.22	0.20	0.19	0.17	0.16	0.15
51	0.38	0.34	0.30	0.27	0.25	0.23	0.21	0.20	0.18	0.17	0.16
52	0.40	0.35	0.32	0.29	0.26	0.24	0.22	0.21	0.19	0.18	0.17
53	0.41	0.37	0.33	0.30	0.28	0.25	0.24	0.22	0.21	0.19	0.18
54	0.43	0.39	0.35	0.32	0.29	0.27	0.25	0.23	0.22	0.21	0.20
55	0.46	0.41	0.37	0.34	0.31	0.29	0.27	0.25	0.23	0.22	0.21
56	0.48	0.43	0.39	0.35	0.33	0.30	0.28	0.26	0.25	0.23	0.22
57	0.50	0.45	0.41	0.38	0.35	0.32	0.30	0.28	0.27	0.25	0.24
58	0.52	0.47	0.43	0.40	0.37	0.34	0.32	0.30	0.28	0.27	0.25
59	0.55	0.50	0.46	0.42	0.39	0.36	0.34	0.32	0.30	0.29	0.27
60	0.58	0.53	0.48	0.45	0.42	0.39	0.36	0.34	0.32	0.31	0.29
61	0.60	0.55	0.51	0.47	0.44	0.41	0.39	0.37	0.35	0.33	0.31
62	0.63	0.58	0.54	0.50	0.47	0.44	0.42	0.39	0.37	0.35	0.34
63	0.66	0.61	0.57	0.53	0.50	0.47	0.44	0.42	0.40	0.38	0.36
64	0.69	0.64	0.60	0.56	0.53	0.50	0.47	0.45	0.43	0.41	0.39
65	0.72	0.67	0.63	0.59	0.56	0.53	0.50	0.48	0.46	0.44	0.42

Notes: Replacement rate is 0.7. Wealth/labor-income ratio is 0.
$m = M'(R)/M(R)$, the growth rate of the disutility of work at retirement age.

Table 2.
Response of Retirement Age to a 10 Percent Reduction in the Value of Social Security Benefits: Low Income, High Wealth Household

m	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
age					Increase in retirement age (years)						
50	0.30	0.26	0.23	0.21	0.19	0.17	0.16	0.15	0.14	0.13	0.12
51	0.31	0.27	0.24	0.22	0.20	0.18	0.17	0.15	0.14	0.14	0.13
52	0.32	0.28	0.25	0.23	0.21	0.19	0.17	0.16	0.15	0.14	0.13
53	0.34	0.30	0.26	0.24	0.22	0.20	0.18	0.17	0.16	0.15	0.14
54	0.35	0.31	0.28	0.25	0.23	0.21	0.19	0.18	0.17	0.16	0.15
55	0.37	0.33	0.29	0.26	0.24	0.22	0.20	0.19	0.18	0.17	0.16
56	0.39	0.34	0.31	0.28	0.25	0.23	0.22	0.20	0.19	0.18	0.17
57	0.40	0.36	0.32	0.29	0.27	0.25	0.23	0.21	0.20	0.19	0.18
58	0.42	0.38	0.34	0.31	0.28	0.26	0.24	0.23	0.21	0.20	0.19
59	0.44	0.40	0.36	0.33	0.30	0.28	0.26	0.24	0.23	0.21	0.20
60	0.47	0.42	0.38	0.35	0.32	0.30	0.27	0.26	0.24	0.23	0.22
61	0.49	0.44	0.40	0.37	0.34	0.31	0.29	0.27	0.26	0.24	0.23
62	0.52	0.47	0.42	0.39	0.36	0.34	0.31	0.29	0.28	0.26	0.25
63	0.54	0.49	0.45	0.41	0.38	0.36	0.34	0.31	0.30	0.28	0.27
64	0.57	0.52	0.48	0.44	0.41	0.38	0.36	0.34	0.32	0.30	0.29
65	0.60	0.55	0.51	0.47	0.44	0.41	0.38	0.36	0.34	0.33	0.31

Notes: Replacement rate is 0.7. Wealth/labor-income ratio is 5.
$m = M'(R)/M(R)$, the growth rate of the disutility of work at retirement age.

Table 3.
Response of Retirement Age to a 10 Percent Reduction in the Value of Social Security Benefits: High Income, Low Wealth Household

m	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
age	Increase in retirement age (years)										
50	0.23	0.20	0.18	0.16	0.15	0.14	0.13	0.12	0.11	0.10	0.10
51	0.24	0.21	0.19	0.17	0.16	0.14	0.13	0.13	0.12	0.11	0.10
52	0.25	0.22	0.20	0.18	0.17	0.15	0.14	0.13	0.12	0.12	0.11
53	0.26	0.23	0.21	0.19	0.18	0.16	0.15	0.14	0.13	0.13	0.12
54	0.27	0.25	0.22	0.20	0.19	0.17	0.16	0.15	0.14	0.13	0.13
55	0.29	0.26	0.24	0.22	0.20	0.19	0.17	0.16	0.15	0.14	0.14
56	0.30	0.27	0.25	0.23	0.21	0.20	0.19	0.17	0.16	0.16	0.15
57	0.32	0.29	0.27	0.25	0.23	0.21	0.20	0.19	0.18	0.17	0.16
58	0.34	0.31	0.28	0.26	0.25	0.23	0.22	0.20	0.19	0.18	0.17
59	0.36	0.33	0.30	0.28	0.26	0.25	0.23	0.22	0.21	0.20	0.19
60	0.38	0.35	0.32	0.30	0.28	0.27	0.25	0.24	0.23	0.22	0.21
61	0.40	0.37	0.34	0.32	0.30	0.29	0.27	0.26	0.25	0.24	0.22
62	0.42	0.39	0.37	0.35	0.33	0.31	0.29	0.28	0.27	0.26	0.25
63	0.44	0.41	0.39	0.37	0.35	0.34	0.32	0.31	0.29	0.28	0.27
64	0.46	0.44	0.42	0.40	0.38	0.36	0.35	0.33	0.32	0.31	0.30
65	0.49	0.46	0.44	0.42	0.41	0.39	0.38	0.36	0.35	0.34	0.33

Notes: Replacement rate is 0.4. Wealth/labor-income ratio is 0.
$m = M'(R)/M(R)$, the growth rate of the disutility of work at retirement age.

Table 4.
Response of Retirement Age to a 10 Percent Reduction in the Value of Social Security Benefits: High Income, High Wealth Household

m	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
age	Increase in retirement age (years)										
50	0.18	0.16	0.14	0.13	0.12	0.11	0.10	0.09	0.09	0.08	0.08
51	0.19	0.17	0.15	0.13	0.12	0.11	0.10	0.10	0.09	0.08	0.08
52	0.20	0.18	0.16	0.14	0.13	0.12	0.11	0.10	0.09	0.09	0.08
53	0.21	0.18	0.16	0.15	0.14	0.12	0.11	0.11	0.10	0.09	0.09
54	0.22	0.19	0.17	0.16	0.14	0.13	0.12	0.11	0.11	0.10	0.09
55	0.23	0.20	0.18	0.17	0.15	0.14	0.13	0.12	0.11	0.11	0.10
56	0.24	0.21	0.19	0.17	0.16	0.15	0.14	0.13	0.12	0.11	0.11
57	0.25	0.23	0.20	0.19	0.17	0.16	0.15	0.14	0.13	0.12	0.11
58	0.27	0.24	0.22	0.20	0.18	0.17	0.16	0.15	0.14	0.13	0.12
59	0.28	0.25	0.23	0.21	0.19	0.18	0.17	0.16	0.15	0.14	0.13
60	0.30	0.27	0.24	0.22	0.21	0.19	0.18	0.17	0.16	0.15	0.14
61	0.31	0.28	0.26	0.24	0.22	0.21	0.19	0.18	0.17	0.16	0.16
62	0.33	0.30	0.28	0.26	0.24	0.22	0.21	0.20	0.19	0.18	0.17
63	0.35	0.32	0.30	0.28	0.26	0.24	0.23	0.21	0.20	0.19	0.18
64	0.37	0.34	0.32	0.30	0.28	0.26	0.25	0.23	0.22	0.21	0.20
65	0.39	0.36	0.34	0.32	0.30	0.28	0.27	0.25	0.24	0.23	0.22

Notes: Replacement rate is 0.4. Wealth/labor-income ratio is 5.
$m = M'(R)/M(R)$, the growth rate of the disutility of work at retirement age.