

# **Life-cycle Asset Allocation with Annuity Markets: Is Longevity Insurance a Good Deal?**

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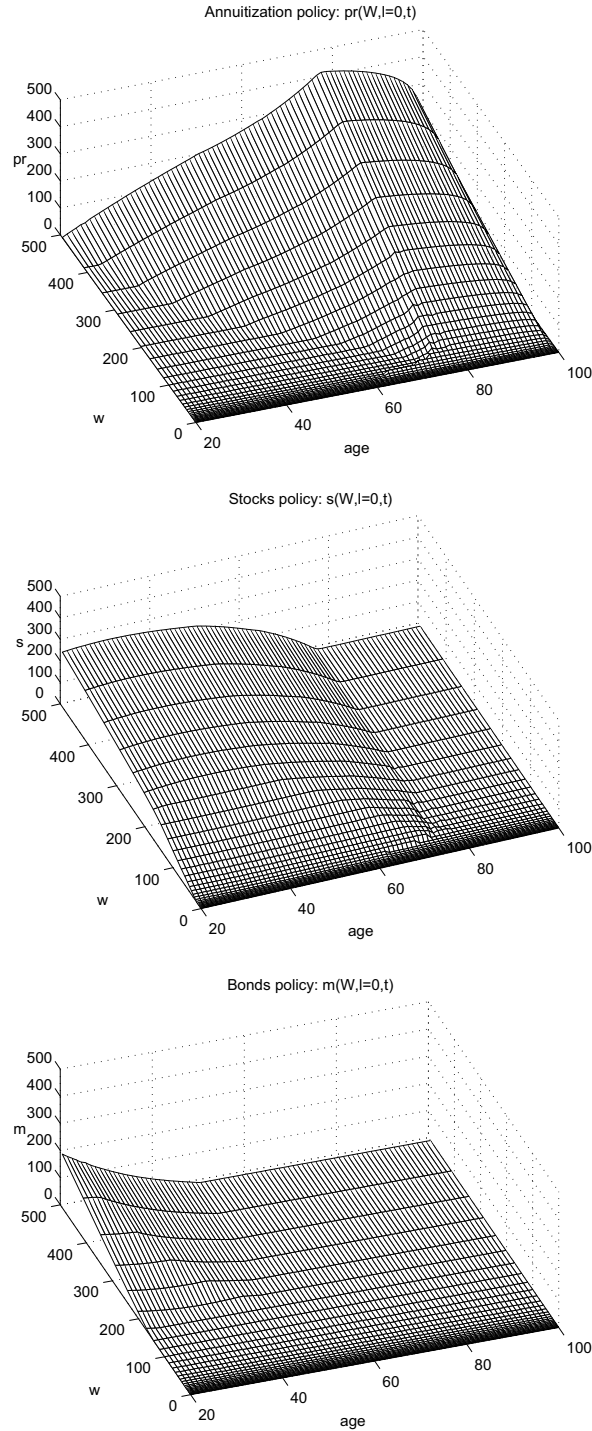
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## **Abstract**

We derive the optimal portfolio choice over the life-cycle for households facing labor income, capital market, and mortality risk. In addition to stocks and bonds, households also have access to incomplete annuity markets offering a hedge against mortality risk. We show that a considerable fraction of wealth should be annuitized to skim the return enhancing mortality credit. The remaining liquid wealth (stocks and bonds) is used to hedge labor income risk during work life, to earn the equity premium, and to ensure estate for the heirs. Furthermore, we assess the importance of common explanations for limited participation in annuity markets.

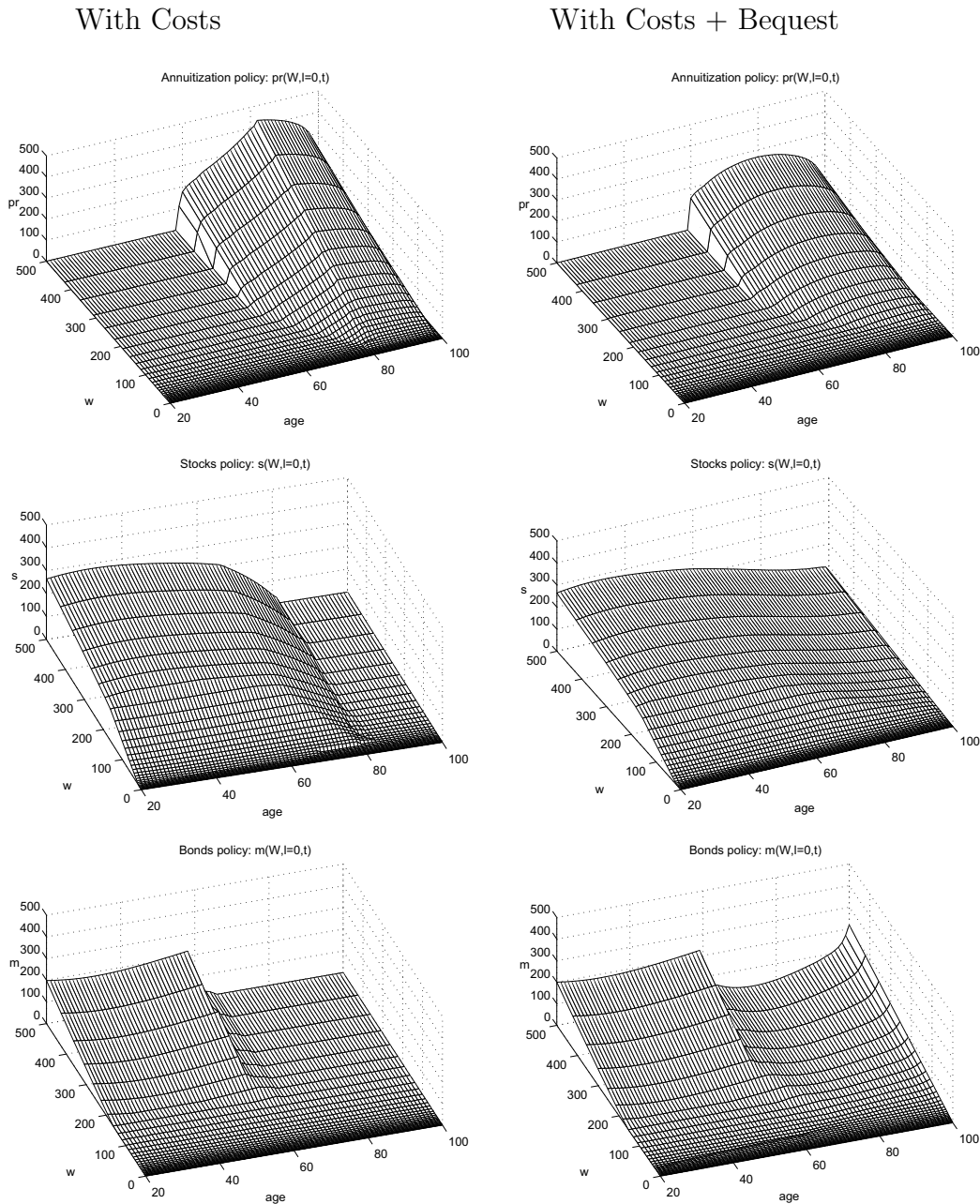
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Figure 1



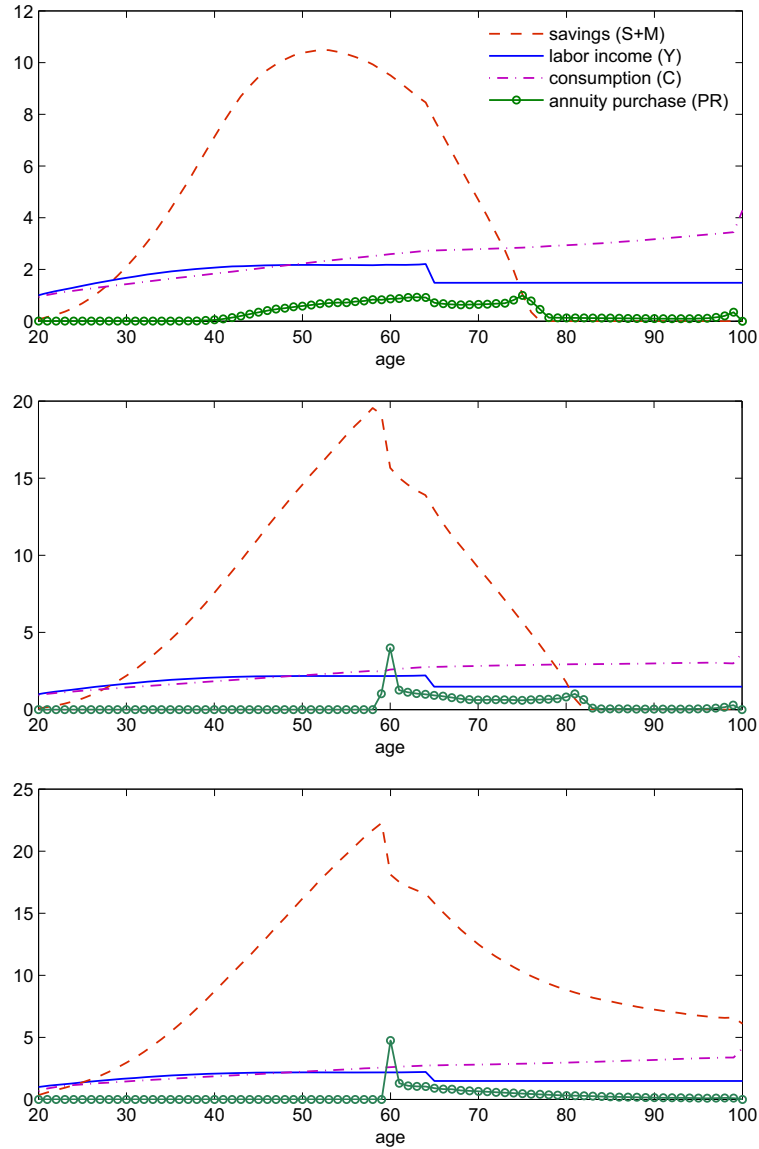
**Figure 1: Optimal Asset Allocation: Stylized Case.** For this case, we assume a female with maximum life-span age 20 - 100, no initial endowment, no administration costs for annuities, no mortality asymmetries,  $RRA = 5$ ,  $EIS = 1/5$ , and a zero-bequest motive ( $k = 0$ ). The figure depicts the optimal holdings of annuities  $pr(w, l, t)$  (upper graph), stocks  $s(w, l, t)$  (middle graph), and bonds  $m(w, l, t)$  (lower graph) depending on the age and wealth on hand  $w$ . The state variable level of annuity payouts is set to  $l = 0$ .

Figure 2



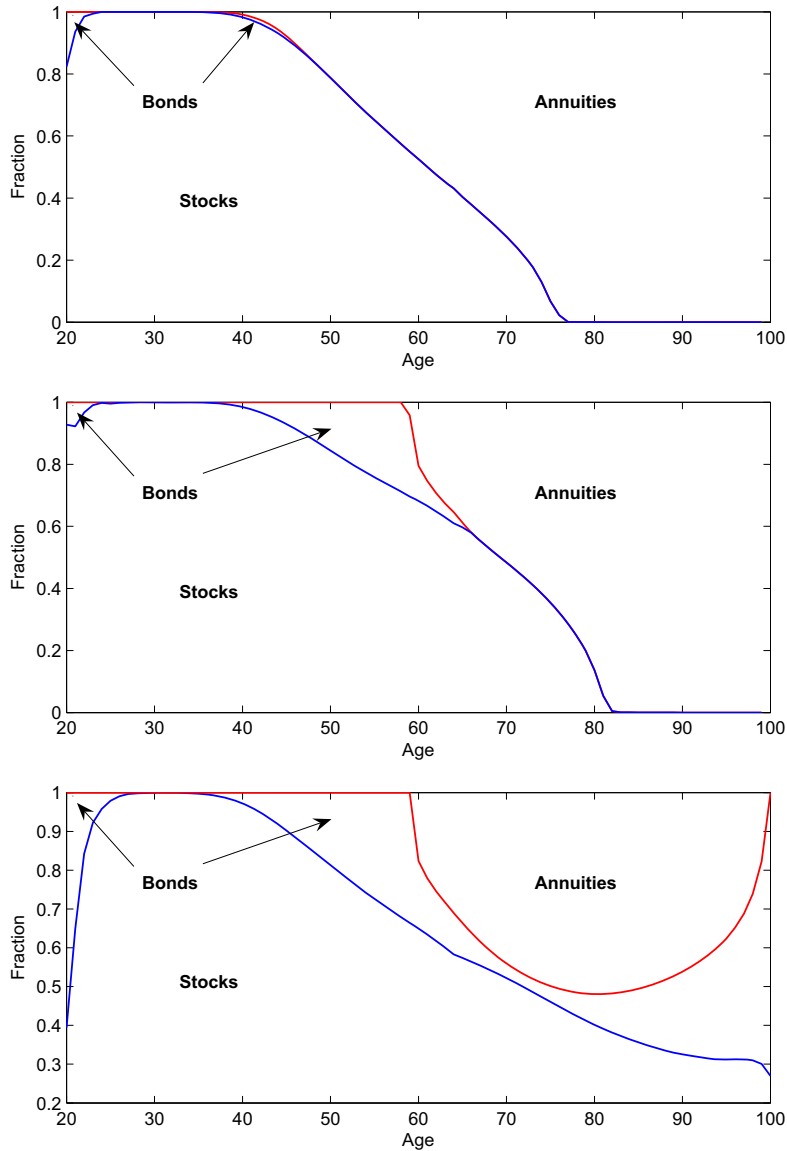
**Figure 2: Optimal Asset Allocation with Costs (Left Column) and Optimal Asset Allocation with Costs and a Bequest Motive (Right Column).** The figure depicts the optimal holdings of annuities  $pr(w, l, t)$  (upper panel), stocks  $s(w, l, t)$  (middle panel), and bonds  $m(w, l, t)$  (lower panel) depending on the age and wealth on hand  $w$ . Left column: for this case, we assume administration costs for annuities  $\delta = 0.073$ , mortality asymmetries (2000 Population Basic vs. 1996 US Annuity 2000 Basic mortality table), and a zero-bequest motive ( $k = 0$ ). Right column: for this case, we additionally assume a bequest motive of ( $k = 2$ ). The state variable level of annuity payouts is set to  $l = 0$ . As in the stylized case, this optimization assumes a female with maximum life-span age 20 - 100, no initial endowment, and  $RRA = 5$ ,  $EIS = 1/5$ .

Figure 3



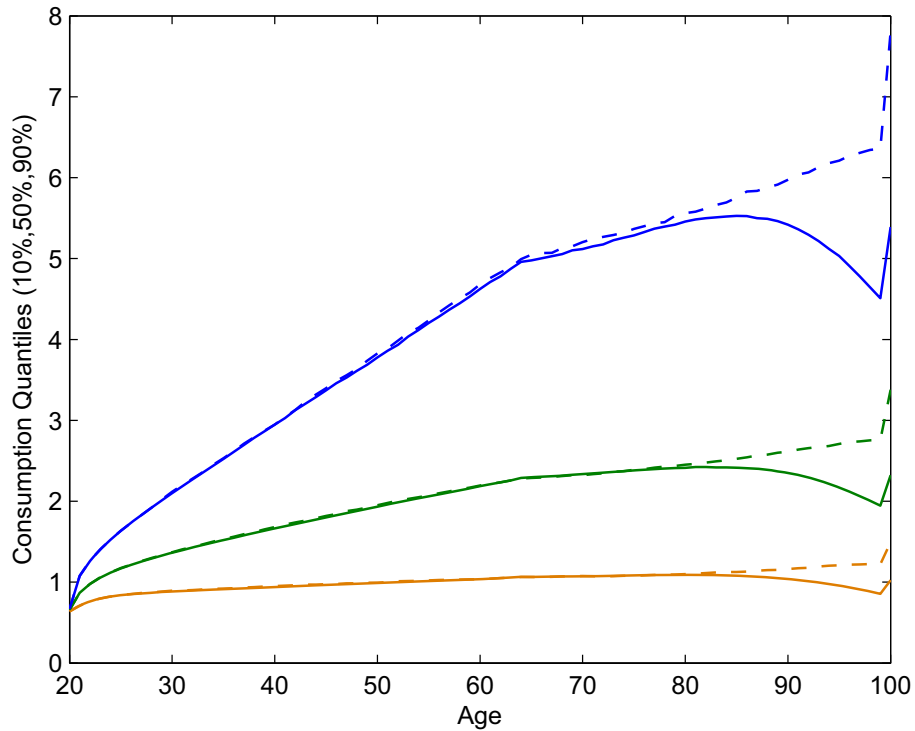
**Figure 3: Life-Cycle Profiles: Stylized Case (Top), with Costs (Middle), and with Costs and Bequest (Bottom).** All three graphs depict expected consumption, labor income, savings, and annuity purchases over the life-cycle. Stylized case: for this case, we assume a female with maximum life-span age 20 - 100, no initial endowment, no administration costs for annuities, no mortality asymmetries,  $RRA = 5$ ,  $EIS = 1/5$ , and a zero-bequest motive ( $k = 0$ ). Case with costs: here, we additionally consider administration costs  $\delta = 0.073$  for annuities and mortality asymmetries (2000 Population Basic vs. 1996 US Annuity 2000 Basic mortality table). Case with costs and bequest: for this case, we add a bequest motive of ( $k = 2$ ) to the case with costs. The expected values are computed by simulating 100,000 life-cycle paths based on the optimal policies derived by the numerical optimization.

Figure 4



**Figure 4: Expected Asset Allocation: Stylized Case (Top), with Costs (Middle), and with Costs and Bequest (Bottom).** All three graphs depict the expected stock, bond, and annuity holdings. Stylized case: for this case, we assume a female with maximum life-span age 20 - 100, no initial endowment, no administration costs for annuities, no mortality asymmetries,  $RRA = 5$ ,  $EIS = 1/5$ , and a zero-bequest motive ( $k = 0$ ). Case with costs: here, we additionally consider administration costs  $\delta = 0.073$  for annuities and mortality asymmetries (2000 Population Basic vs. 1996 US Annuity 2000 Basic mortality table). Case with costs and bequest: for this case, we add a bequest motive of ( $k = 2$ ) to the case with costs. The expected values are computed by simulating 100,000 life-cycle paths based on the optimal policies derived by the numerical optimization.

Figure 5



**Figure 5: Consumption Percentiles (10th,50th, and 90th) with and without Annuities.** The dashed (solid) lines reflect the case with (without) annuities. The blue, green, and orange lines reflect the 90th, 50th, and 10th percentile respectively. The calculations are based on 100,000 Monte-Carlo simulations. We assume the case with bequest and costs.

**Table I**  
**Expected Optimal Annuitization Strategy**

Table I reports the expected optimal annuitization policy for three alternative cases. The first, stylized case assumes a female with maximum life-span age 20 - 100, no initial endowment, no administration costs for annuities, no mortality asymmetries,  $RRA = 5$ ,  $EIS = 1/5$ , and a zero-bequest motive ( $k = 0$ ). The second case introduces administration costs for annuities  $\delta = 0.073$  and mortality asymmetries (2000 Population Basic vs. 1996 US Annuity 2000 Basic mortality table). The third case additionally considers a bequest motive ( $k = 2$ ).  $PR/W$  is the fraction of financial wealth used to purchase new annuities at a certain age.  $L/Y$  reports the magnitude of annuity payouts  $L$  relative to the labor income  $Y$ .  $L/(C - Y)$  depicts how much of the consumption  $C$  not taken from the labor income is backed by annuity payouts. We compute the expected values based on 100,000 Monte-Carlo simulations by using the optimal policy functions  $s(w, l, t)$  (stocks),  $m(w, l, t)$  (bonds),  $pr(w, l, t)$  (annuities), and  $c(w, l, t)$  (consumption). All figures are reported as percentages.

Age	Stylized case			Expected Optimal Annuitization			With costs and bequest		
	$PR/W$	$L/Y$	$L/(C - Y)$	$PR/W$	$L/Y$	$L/(C - Y)$	$PR/W$	$L/Y$	$L/(C - Y)$
25	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0
40	0.4	0.1	0.1	0	0	0	0	0	0
45	2.3	2.0	12.9	0	0	0	0	0	0
50	3.7	7.8	40.4	0	0	0	0	0	0
55	4.5	16.7	75.1	0	0	0	0	0	0
60	5.6	28.7	116.9	15.4	2.9	38.1	16.5	0	0
65	5.1	63.9	59.9	4.5	41.2	33.8	3.7	41.2	33.4
70	6.2	81.2	75.9	3.8	58.9	48.8	3.2	58.6	48.6
75	15.9	103.3	96.7	5.2	75.1	62.5	2.6	74.1	61.9
80	2.7	123.1	109.2	11.7	96.8	82.1	2.0	87.8	72.2
85	2.4	129.9	107.8	1.0	118.3	103.2	1.4	100.3	77.9
90	2.0	138.3	106.2	0.8	120.6	102.9	1.0	111.9	81.8
95	1.9	149.5	105.6	0.9	124.0	103.1	0.8	123.8	85.3



**Table II**  
**Expected Optimal Asset Allocation for Alternative Cases**

Table II reports the expected asset allocation of the individual's portfolio consisting of stocks, bonds, and annuities. The expected asset allocations are displayed for the age 30, 45, 60, 75. The first, stylized case assumes a female with maximum life-span age 20 - 100, no initial endowment, no administration costs for annuities, no mortality asymmetries,  $RRA = 5$ ,  $EIS = 1/5$ , and a zero-bequest motive ( $k = 0$ ). The second case introduces administration costs for annuities  $\delta = 0.073$  and mortality asymmetries (2000 Population Basic vs. 1996 US Annuity 2000 Basic mortality table). The third case additionally considers a bequest motive ( $k = 2$ ). The remaining cases are variations of the third case with costs and bequest. The value of all annuities the investor holds at a certain age is calculated as the premium the investor would have to pay in order to achieve the same payout stream as with the previously purchased annuities. We compute the expected values based on 100,000 Monte-Carlo simulations by using the optimal policy functions  $s(w, l, t)$  (stocks),  $m(w, l, t)$  (bonds),  $pr(w, l, t)$  (annuities), and  $c(w, l, t)$  (consumption). All figures are reported as percentages.

Case	Stock fraction (%)				Bond fraction (%)				Annuity fraction (%)			
	30	45	60	75	30	45	60	75	30	45	60	75
Stylized case	100.0	91.3	52.4	6.7	0.0	0.9	0.0	0.0	0.0	7.8	47.6	93.3
With costs	100.0	93.0	68.6	35.8	0.0	7.0	11.3	0.0	0.0	0.0	20.1	64.2
With bequest	100.0	90.4	64.8	45.9	0.0	9.6	17.4	4.1	0.0	0.0	17.7	50.0
Males	99.9	89.6	59.8	40.9	0.1	10.4	1.9	7.7	0.0	0.1	38.3	51.5
Bad health	99.9	90.3	65.2	48.6	0.1	9.7	26.6	18.5	0.0	0.0	8.2	32.9
Low IES ( $\psi = 0.1$ )	99.6	92.2	67.8	49.8	0.4	7.8	23.2	12.6	0.0	0.0	9.0	37.5
High IES ( $\psi = 0.3$ )	100.0	88.3	61.6	41.3	0.0	11.7	12.2	0.0	0.0	0.0	26.2	58.7
Low RRA ( $\rho = 2$ )	100.0	100.0	100.0	97.9	0.0	0.0	0.0	2.1	0.0	0.0	0.0	0.0
Low pension income ( $\lambda = 0.5$ )	100.0	86.1	53.9	37.7	0.0	13.9	11.2	3.3	0.0	0.0	35.0	59.0
High pension income ( $\lambda = 1$ )	99.8	94.5	81.6	63.0	0.1	5.4	15.0	6.0	0.0	0.0	3.2	30.8

**Table III**  
**Welfare Analysis: Equivalent Increase in Financial Wealth (Percentage Points)**  
**of Having Access to Annuity Markets**

Table III reports welfare gains in the presence of annuity markets for all cases considered previously. The first, stylized case assumes a female with maximum life-span age 20 - 100, no initial endowment, no administration costs for annuities, no mortality asymmetries,  $RRA = 5$ ,  $EIS = 1/5$ , and a zero-bequest motive ( $k = 0$ ). The second case introduces administration costs for annuities  $\delta = 0.073$  and mortality asymmetries (2000 Population Basic vs. 1996 US Annuity 2000 Basic mortality table). The third case additionally considers a bequest motive ( $k = 2$ ). The remaining cases are variations of the third case with costs and bequest. Welfare gains are computed as the equivalent percentage increase in financial wealth an individual without access to annuity markets would need in order to attain the same expected utility as in the case with annuity markets. The computations are done for age 60, 70, 80, and 90.

<i>Case</i>	<i>Age</i>			
	60	70	80	90
Stylized case	14.41	16.00	23.75	49.83
With costs	9.54	12.79	16.51	31.16
With bequest	5.69	8.43	14.14	30.07
Males	5.35	8.95	18.75	41.31
Bad health	0.96	2.62	6.73	21.74
Low IES ( $\psi = 0.1$ )	0.40	1.18	3.68	14.70
High IES ( $\psi = 0.3$ )	8.34	11.87	21.30	43.80
Low RRA ( $\rho = 2$ )	0.00	0.00	0.42	0.10
Low pension income ( $\lambda = 0.5$ )	6.87	8.75	14.18	30.19
High pension income ( $\lambda = 1$ )	0.90	2.19	7.64	24.38