University of Michigan Retirement Research Center

Working Paper

WP 2004-080

Decaying Asymmetric Information and Adverse Selection in Annuities

David McCarthy

MRRC

Project #: UM03-S2

Decaying Asymmetric Information and Adverse Selection in Annuities

David McCarthy
Tanaka Business School, Imperial College

June 2004

Michigan Retirement Research Center
University of Michigan
P.O. Box 1248
Ann Arbor, MI 48104

Acknowledgements

This work was supported by a grant from the Social Security Administration through the Michigan Retirement Research Center (Grant # 10-P96362-5). The opinions and conclusions are solely those of the authors and should not be considered as representing the opinions or policy of the Social Security Administration or any agency of the Federal Government.

Decaying Asymmetric Information and Adverse Selection in Annuities

David McCarthy

Abstract

This paper develops an equilibrium model of the annuities market where agents have private information about their mortality, and where the predictive value of this information decays over time. The paper shows that in this case, insurance companies will observe a duration-related trend in the mortality of annuitants under certain conditions. This effect is tested for using a Cox proportional hazards methodology and data from the South African annuities market, which since the early 1990's has permitted phased withdrawals of retirement savings instead of mandating pure annuitisation. Evidence is equivocal: substantial differences are found between the duration-related mortality trends of different insurance companies, data problems seem to have some effect, and factors outside the model which might change the results cannot be excluded. However, the presence of a strong duration-related trend cannot be decisively rejected. The observed trend indicates that mortality at earlier policy durations is better than at later durations by the equivalent of about 6 years of age, although data factors cannot be precluded as a cause of this trend.

Authors' Acknowledgements

The author would like to thank the Social Security Administration for financial support which enabled this research, grant USM03-S2. The author would like to thank Ales Cerny, Francis Breedon and other members of the Finance Group of the Tanaka Business School, for valuable comments on an earlier draft, Olivia S. Mitchell for valuable discussions and support in this research. Also, the author is very grateful to several members of staff at South African life offices who provided data. They cannot be individually named without identifying their employers, who have asked to remain anonymous. This paper is a first draft: comments are very welcome.

1. Introduction

Private annuity markets are an important area of study for economists. Annuities are relevant to many different aspects of economic behaviour - including the demand for insurance, asymmetric information, bequest motives, and investment and consumption behaviour. The shift in many countries from defined benefit to defined contribution pension plans, which will continue to reduce the proportion of assets held by future retirees that is already annuitised, gives greater importance to the study of private annuity markets. Yaari (1965) pointed out the tremendous insurance benefits that annuities offer purchasers. These benefits are so large that simple models predict they should be prepared to sacrifice a large fraction of their wealth in order to purchase annuities. Yet, in virtually every country, individuals need to be forced to purchase annuities. Various explanations have been offered for this. These include the fact that individuals are able to insure themselves against living too long because they are married or because they have children who will support them in old age (Brown and Poterba, 2000), the presence of bequest motives (Bernheim, 1991), the fact that a large fraction of wealth is pre-annuitised anyway (Mitchell *et al.*, 1999), and precautionary motives that might cause individuals to prefer liquid assets over annuities, such as the possibility of health shocks (Brown, 2001). A summary of this literature can be found in McCarthy and Neuberger (2003).

One other factor that is often cited to explain low annuity purchase rates is that asymmetric information causes a failure in the annuities market. The theory states that individuals have private knowledge about their longevity. Individuals who perceive that annuities are better value - *i.e.* those who believe that they will live for a long time - will purchase more annuities than those who do not - which will cause annuities to be more expensive than they would be if everyone had no knowledge about their longevity or if insurance companies were able to measure the information that people had about their own longevity absolutely perfectly. Various studies (e.g. McCarthy and Mitchell, 2002) have documented the lower mortality of annuity purchasers relative to the general population in virtually every annuity market. However, commentators remain divided about the extent to which this difference is the result of asymmetric information in the annuities market or merely a reflection of the different characteristics of annuity purchasers that are, at least in principle, observable, such as wealth and employment status. This distinction may have important implications for economic policy on mandating annuity purchase from individual account-type pension schemes.

This study uses a new data set and a novel technique to test for the presence of adverse selection in the South African annuities market. The South African annuities market is an interesting case study for reasons which will be discussed below. Section 2 will discuss the theory underlying the method used to test for adverse selection, section 3 will discuss the data used in more detail and section 4 will present results. Section 5 will be a conclusion.

2. Theory

Several techniques have been used to test for adverse selection in insurance and annuities markets. Three papers are Cawley and Philipson (1999), Finkelstein and Poterba (2002) and Mitchell and McCarthy (2002). The method adopted here bears closest resemblance to Finkelstein and Poterba (2002). Cawley and Philipson (1999) use data from the US life insurance market. They test for various implications of a Rothschild-Stiglitz-type separating equilibrium in the life insurance market: a positive relationship between self-perceived risk and the price of insurance, the absence of bulk discounts, a negative relationship between risk and quantity of insurance purchased and the prediction that individuals should hold only one insurance contract. They find convincing evidence that none of these predictions hold in their data and conclude that the Rothschild-Stiglitz theory does not apply to the US life insurance market. Finkelstein and Poterba (1999) test for adverse selection in the UK annuities market. They demonstrate that purchasers of different annuity products in the UK have different mortality risk profiles, and show that these differences are priced into the annuity products. They take this as evidence that there is a separating equilibrium in the UK life annuities market, by product type. McCarthy and Mitchell (2002) illustrate that purchasers of annuities have different mortality profiles to the general population in most countries where annuities are sold, and that there are regular patterns to these differences across countries. Neither of the two papers that deal with annuities markets is able to demonstrate that the difference between the mortality profiles of purchasers of different annuity types (or of purchasers and non-purchasers) is the result of asymmetric information or of factors that may be correlated with unobserved factors underlying the decision to purchase an annuity, such as risk aversion, wealth, and the presence of a bequest motive.

This paper uses a different methodology to test for asymmetric information. The effect underlying the test is the decline in the predictive value of a given set of information over time. This decline is probably best understood by an example: the level of the stock market today tells one a great deal about the level of the stock market tomorrow, but it tells one almost nothing about the level of the stock market in 20 years. A similar example is the information that insurance companies collect during underwriting about the likely mortality of individuals who apply for life insurance. Individuals who pass a certain health standard are permitted to buy insurance at standard rates, other individuals are not. The predictive power of this information on mortality rates is initially high but declines over time. In the case of insurance, one can actually observe the effect: individuals who have recently purchased their insurance policies have lighter mortality than those who are identical in every other respect save for the fact that they purchased their insurance policies earlier. An example of this effect, taken from real data collected from life insurance companies in the United Kingdom, is shown in Figure 1. It shows the mortality of life insurance purchasers aged 50 who purchased their insurance policies at age 50, 46-49 and before age 46. The procession of fitted mortality is then shown as the different individuals age. Initially, recent purchasers of life insurance have mortality very much lighter than the average for all those who are their age who have purchased insurance, but that this difference gets smaller as individuals age. One explanation for this effect is due to the decline in the value of the information that the insurance company collected about the individual's state of health at the time of purchase, for explaining the health status of the individual many years later. Of course, it is also possible that some other factor correlated with mortality and the decision to purchase insurance declines in a similar way. For instance, it might be that wealthy individuals purchase insurance and that if one is wealthy when one purchases insurance it says relatively little about whether one will be wealthy in the future. Also, another factor may be operating in the life insurance market: because individuals do not commit to purchase insurance indefinitely, but can choose to lapse their policies if they so wish, it may be that individuals with lower probabilities of dying have less need for insurance and so selectively lapse their policies. Of course, one needs to posit some factor that explains why they only realise this after they have purchased the policy as opposed to before!

However, the example raises an interesting question: if we make the assumption that the private information one had about one's mortality at a certain point becomes less and less useful at predicting one's mortality as one ages, what implication does this have for the observed pattern of mortality in a pool of annuitants? In the next few paragraphs, we formalise some of the ideas discussed here and answer this question.

To model this effect, let us assume that we have a continuum of individuals. Individuals differ from each other in two ways: some people are healthy (type h) and some are unhealthy (type u), and each individual has a fixed parameter θ which affects how willing they are to annuitise their assets, which will be discussed later. Other than this, individuals are initially assumed to be identical to each other.

Assume that healthy individuals can either become unhealthy (with probability λ) or die (with probability q_h) in each period. Similarly, unhealthy individuals can either become healthy (with probability η) or die (with probability q_u) in each period. Assume that these probabilities are independent of θ, and that they remain constant until all individuals in a given cohort are dead. An illustration of this model is shown in Figure 2. We could introduce a time trend in the mortality probabilities to mimic the effect of population ageing, but this would add unnecessary complications.

Assume that individuals know their own type but that this is private information: outsiders cannot observe what type they are. Each individual's θ is also assumed to be private information.

Under certain very mild conditions discussed in appendix A, in this model there exists a steady state proportion of individuals who are healthy, which, once reached, will not change from year to year. Let us assume that these mild conditions hold, and that the proportion of individuals who are healthy is at the steady state, p. This is simply equivalent to saying that, over time, the proportion of individuals in our population who are classified as healthy or unhealthy (relative to their peers) does not change. This is slightly artificial, but given that the emphasis of the model is on relative health within a given population, rather than on some absolute standard of health, it is not too onerous.

Into this environment, we introduce a market for life annuities which pay constant units of consumption until the death of the individual, and we offer a one-off option to purchase a life annuity to all population members. The parameter θ affects each individual's demand for annuities. For convenience, suppose that θ represents the fixed cost of purchasing an annuity - it could equally represent a bequest motive, or the extent to which the individual's wealth is pre-annuitised. Assume that the distribution of θ in the population of healthy and unhealthy people is identical, and that each individual's θ does not change over time. Let θ be defined over the range $[0, \theta_{max}]$, and let the distribution of θ be represented by the density function p_θ. The fact that θ is private information implies that individuals do not reveal their health type to annuity companies when they purchase annuities.

Let a_h denote the present value of a life annuity for an individual who is in the 'healthy' state and let a_u denote the present value of an annuity for an individual who is currently 'unhealthy'. Note that these values do not depend on the age of the individual as our individuals are assumed not to age. Assume for convenience that the risk free interest rate is constant equal to r, and that the annuity payments are made at the end of each time period rather than continuously.

To derive formulae for the expected discounted present value of annuities for individuals who are currently healthy (a_h) and unhealthy (a_u), we note that if the healthy individual is alive at the end of the first period (probability, $1-q_h$), we need to pay him an annuity payment of 1. If the individual is healthy (probability, $1-\lambda-q_h$), the value a_h will be sufficient to buy out all future annuity payments. If the individual is unhealthy, (probability λ) then the value a_u will be sufficient to buy out all future payments, while if the individual is dead, (probability q_h) then the value of all future payments will be 0.

This reasoning, and similar reasoning for the case of the individual who is currently unhealthy, yields a set of simultaneous equations in a_u and a_h:

$$(1+r)a_h = 1\cdot(1-q_h)+a_h\cdot(1-\lambda-q_h)+a_u\cdot\lambda+0\cdot q_h \qquad (1)$$

$$(1+r)a_u = 1\cdot(1-q_u)+a_u\cdot(1-\eta-q_u)+a_h\cdot\eta+0\cdot q_u$$

These equations can be solved to yield the following equations for a_u and a_h:

$$a_h = \frac{(1-q_u)\lambda+(1-q_h)(\eta+q_u+r)}{(\lambda+q_h+r)(\eta+q_u+r)-\eta\lambda} \qquad (2)$$

$$a_u = \frac{(1-q_h)\eta+(1-q_u)(\lambda+q_h+r)}{(\lambda+q_h+r)(\eta+q_u+r)-\eta\lambda}$$

By taking simple differences, it will be seen that $a_h > a_r \Leftrightarrow q_h < q_u$ as we would expect.

The implication of this is that the expected discounted present value of a level annuity in the hands of a currently healthy individual is worth more than the same annuity in the hands of a currently unhealthy individual.

Now, if we make the assumption that the insurance company cannot observe the total quantity of annuities that an individual has purchased then a separating equilibrium in the annuities market, along the lines of Rothschild and Stiglitz (1976), is impossible. This is because their result depends on the insurance company being able to observe the value of the loss and the quantity of insurance purchased. Following Abel (1986), we therefore must have a pooling equilibrium. Each insurance company must charge a single price for annuities, and assuming a competitive market, each must charge the value that will ensure that it makes no profit. If we assume that the proportion of individuals of a given cohort who purchase annuities who are in healthy is $x_{h,0}$, and that the proportion of individuals of a given cohort who purchase annuities who are in poor health is $x_{u,0} = 1 - x_{h,0}$, then the single price charged by the insurance company must be:

$$a = x_{h,0}a_h + (1 - x_{h,0})a_h = a_{u,0} + x_{h,0}(a_h - a_u) \geq a_u .$$

Similarly, it can be shown that: (3)

$$a = (1 - x_{u,0})a_h + x_{u,0}a_u = a_h - x_{u,0}(a_h - a_u) \leq a_h .$$

Now we present an annuity demand model for each type of individual. Let the expected discounted total utility of a healthy individual who optimises consumption at each future time point, conditional on an equilibrium annuity price *a*, non-annuitised wealth *w* and amount of annuity purchased, *α*, be denoted:

$$V_h(w, \alpha \mid a) = \max_{\{C_i\}} E \sum_{i=0}^{\infty} \beta^i (p_{hh}^i u(C_{h,i}) + p_{hu}^i u(C_{u,i})) , \qquad (4)$$

where p_{hh}^i is the probability that an individual healthy at time 0 is still healthy at time *i*, and similarly for p_{hu}^i, and $C_{h,i}$ is optimal consumption if individual is healthy at time *i*, and $C_{u,i}$ is optimal consumption if the individual is unhealthy at time *i*, and *β* is the individual's discount factor. When individuals decide how much to consume, they take into account three state variables: the amount of wealth they have on hand (*w*), the amount of annuity income they receive (*α*) and their state of health. Consumption in each period is constrained to be less than wealth in that period. All the individual's wealth is invested in an asset that pays a risk-free return of *r* per period. Hence, the individual's wealth at time *i*, denoted W_i, follows the following process:

$$W_{i+1} = (W_i - C_i)(1 + r) + \alpha . \qquad (5)$$

At time 0, the individual exchanges $\alpha a + \theta$ units of wealth for an annuity that pays an annual payment of α. If the individual chooses not to annuitise any wealth, then they do not pay the fixed cost *θ*. The optimal values of *α* for individuals in different states are therefore given by:

$$\hat{\alpha}_h(a, \theta) = \arg\max_{\alpha} V_h(w_0 - \alpha a - \theta 1_{\alpha > 0}, \alpha \mid a) \text{, and}$$

$$\hat{\alpha}_u(a,\theta) = \arg\max_{\alpha} V_u(w_0 - \alpha a - \theta 1_{\alpha>0}, \alpha \mid a). \quad (6)$$

We assume that $\hat{\alpha}_h(a,\theta_{\max}) = 0$: in other words, there is at least one healthy individual for whom purchasing an annuity is so expensive that the optimal purchase amount is 0. Further, we know from Yaari (1965) that $\hat{\alpha}_h(a,0) = W_0 / a$: given annuities that are at least fairly priced, individuals will annuitise all their wealth in simple models like this if there are no transactions costs.

Model solution

Our agents choose consumption according to the following program:

$$V_h(w_0, \alpha \mid a) = \max_{\{C_i\}} E \sum_{i=0}^{\infty} \beta^i (p_{hh}^i u(C_{h,i}) + p_{hu}^i u(C_{u,i})), \quad (7)$$

$$V_u(w_0, \alpha \mid a) = \max_{\{C_i\}} E \sum_{i=0}^{\infty} \beta^i (p_{uh}^i u(C_{h,i}) + p_{uu}^i u(C_{u,i}))$$

Remembering that the level of *a* is given, the Bellman equations for the agents in each state are:

$$V_u(w,\alpha) = \max_{C_u(w,\alpha)} u(C_u(w,\alpha)) + \beta(1 - q_u - \eta) V_u((w - C_u(w,\alpha))(1+r) + \alpha, \alpha)$$
$$+ \beta\eta V_h((w - C_u(w,\alpha))(1+r) + \alpha, \alpha). \quad (8)$$

$$V_h(w,\alpha) = \max_{C_h(w,\alpha)} u(C_h(w,\alpha)) + \beta(1 - q_h - \lambda) V_h((w - C_h(w,\alpha))(1+r) + \alpha,)$$
$$+ \beta\lambda V_u((w - C_u(w,\alpha))(1+r) + \alpha, \alpha).$$

Differentiating with respect to $C_u(w,\alpha)$ and $C_h(w,\alpha)$ gives the first order conditions of each equation, and setting these equal to 0 for a maximum yields the following two equations:

$$u'(C_u(w,\alpha)) = \beta(1 - q_u - \eta)(1+r) V_u'((w - C_u(w,\alpha))(1+r) + \alpha, \alpha)$$
$$+ \beta\eta(1+r) V_h'((w - C_u(w,\alpha))(1+r) + \alpha, \alpha) \quad (9)$$

$$u'(C_h(w,\alpha)) = \beta(1 - q_h - \lambda)(1+r) V_h'((w - C_h(w,\alpha))(1+r) + \alpha, \alpha)$$
$$+ \beta\lambda(1+r) V_u'((w - C_h(w,\alpha))(1+r) + \alpha, \alpha)$$

However, using the envelope theorem to differentiate the Bellman equations w.r.t. w yields:

$$\begin{aligned} V_u'(w,\alpha) &= \beta(1-q_u-\eta)(1+r)V_u'((w-C_u(w,\alpha))(1+r)+\alpha,\alpha) \\ &\quad + \beta\eta(1+r)V_h'((w-C_u(w,\alpha))(1+r)+\alpha,\alpha) \\ V_h'(w,\alpha) &= \beta(1-q_h-\lambda)(1+r)V_h'((w-C_h(w,\alpha))(1+r)+\alpha,\alpha) \\ &\quad + \beta\lambda(1+r)V_u'((w-C_h(w,\alpha))(1+r)+\alpha,\alpha) \end{aligned} \tag{10}$$

Combining these equations with the first order conditions yields:

$$\begin{aligned} u'(C_u(w,\alpha)) &= V_u'(w,\alpha) \\ u'(C_h(w,\alpha)) &= V_h'(w,\alpha)\,. \end{aligned} \tag{11}$$

Since these hold for all levels of wealth, these imply that:

$$\begin{aligned} u'(C_u((w-C_u(w,\alpha))(1+r)+\alpha,\alpha)) &= V_u'((w-C_u(w,\alpha))(1+r)+\alpha,\alpha) \\ u'(C_u((w-C_h(w,\alpha))(1+r)+\alpha,\alpha)) &= V_u'((w-C_h(w,\alpha))(1+r)+\alpha,\alpha) \\ u'(C_h((w-C_u(w,\alpha))(1+r)+\alpha,\alpha)) &= V_h'((w-C_u(w,\alpha))(1+r)+\alpha,\alpha) \\ u'(C_h((w-C_h(w,\alpha))(1+r)+\alpha,\alpha)) &= V_h'((w-C_h(w,\alpha))(1+r)+\alpha,\alpha) \end{aligned} \tag{12}$$

Substituting these four identities into the first order conditions yields the Euler equations that a solution to the dynamic programming problem must satisfy:

$$\begin{aligned} u'(C_u(w,\alpha)) &= \beta(1-q_u-\eta)(1+r)u'(C_u((w-C_u(w,\alpha))(1+r)+\alpha,\alpha)) \\ &\quad + \beta\eta(1+r)u'(C_h((w-C_u(w,\alpha))(1+r)+\alpha,\alpha)) \\ u'(C_h(w,\alpha)) &= \beta(1-q_h-\lambda)(1+r)u'(C_h((w-C_h(w,\alpha))(1+r)+\alpha,\alpha)) \\ &\quad + \beta\lambda(1+r)u'(C_u((w-C_h(w,\alpha))(1+r)+\alpha,\alpha)) \end{aligned} \tag{13}$$

This is a pair of simultaneous equations in the functions $C_u(w,\alpha)$ and $C_h(w,\alpha)$. If the value of β is less than $\min(\frac{1}{(1-q_u)(1+r)},\frac{1}{(1-q_h)(1+r)})$ then the value function will be finite and the conditions of the verification theorem will hold[1]. Hence a solution to (11) will be a solution to the overall problem. Unfortunately, it is well known that no analytic solution for a problem of this type exists owing to the borrowing constraint on wealth and the infinite time horizon[2]. Merton (1969) and Samuelson (1969) have demonstrated analytic solutions to similar problems (with only one health state) where the time horizon is finite and there are no borrowing constraints.

Some insight about the properties of these functions can be derived from considering a pair of equations similar to (13), but where individuals do not switch from type to type, that is, where $\lambda = \eta = 0$:

$$u'(C_u(w,\alpha)) = \beta(1-q_u)(1+r)u'(C_u((w-C_u(w,\alpha))(1+r)+\alpha,\alpha)) \qquad (14)$$
$$u'(C_h(w,\alpha)) = \beta(1-q_h)(1+r)u'(C_h((w-C_h(w,\alpha))(1+r)+\alpha,\alpha))$$

Here we have two independent problems. Since $\beta < \min(\frac{1}{(1-q_u)(1+r)}, \frac{1}{(1-q_h)(1+r)})$, the verification theorem will hold for both problems. It is relatively easy to see that, for all levels of *w* and *α*, $C_u(w,\alpha) > C_h(w,\alpha)$ owing to the fact that $q_h < q_u$, which implies that the individual in state *u* discounts the future more heavily than the individual in state *h*. Since the functions $C_u(w,\alpha)$ and $C_h(w,\alpha)$ change continuously with λ and η, we can

---

1 See the first chapter of Fleming and Mete Soner (1991) for details.

2 If the discount rates are allowed to differ by state of health, and $\beta_h = [(1+r)(1-q_h)]^{-1}$ and $\beta_u = [(1+r)(1-q_u)]^{-1} < \beta_h$ then it can be verified that a solution to the above equations for all concave utility functions is given by:

$$C_u(w,\alpha) = C_h(w,\alpha) = (rw+\alpha)(1+r)^{-1}.$$

Even though the verification theorem does not hold in this case (the value functions will be infinite), this is a solution to the above problem as can be seen by solving a finite-horizon problem and allowing the time to tend to infinity. The rest of this section assumes that this approach has been adopted. In this model, $\hat{\alpha}_h(a,\theta) = \hat{\alpha}_u(a,\theta) \forall \alpha, \theta$. This implies that, $x_{h,0}$, the proportion of individuals who choose to purchase annuities who are healthy, is equal to *p*, the steady-state proportion of healthy individuals in the population. The equilibrium annuity price in this model is therefore $a = pa_h + (1-p)a_u$. Also, $x_{0,t} = p$ implies that $x_{h,t} = p \forall t$, from the definition of *p*. Therefore there will be no duration-related trend in the average observed mortality of annuity purchasers, as we would expect.

see that $C_u(w,\alpha) > C_h(w,\alpha)$ for at least a neighbourhood of (λ,η) around (0,0). Intuitively, we can see that the two equations $C_u(w,\alpha)$ and $C_h(w,\alpha)$ must become more similar as λ and η increase, and since if $\lambda = \lambda_{\max} = 1 - q_h$ and $\eta = \eta_{\max} = 1 - q_u$ (in other words, individuals alternate between different states in each period) then $C_u(w,\alpha)$ is still greater than $C_h(w,\alpha)$ for all w and α, owing to the higher mortality probability in state u.

Similarly, in the main problem, we must have $C_u(w,\alpha) > C_h(w,\alpha)$ for all admissible values of λ and η: unhealthy individuals consume more that all levels of wealth and annuitisation than healthy individuals, as they have higher mortality probabilities and there are no bequest motives. Anything else would result in a contradiction. From equation (11), and the concavity of the utility function u this implies that $V_u^{'}(w,\alpha) = u'(C_u(w,\alpha)) < u'(C_h(w,\alpha)) = V_h^{'}(w,\alpha)$: healthy individuals value an extra unit of wealth more highly than unhealthy individuals.

From this we can extract the conclusion we need. Two effects will affect annuity purchase behaviour: the wealth effect and the substitution effect. In a previous section we have shown that the wealth effect is unambiguous: healthy people will value a given annuity at a given price more highly than unhealthy people. The result we have shown here about the consumption functions of healthy and unhealthy people shows that unhealthy individuals will value a level annuity weakly less than healthy individuals because they would prefer an annuity with more steeply decreasing payments, and they may have more difficulty than healthy people (because of borrowing constraints) in altering their consumption pattern sufficiently to undo the undesirable payment pattern in the level annuity with a given present value. Therefore both effects are unambiguous: in the absence of constraints on the amount of annuities that can be purchased, healthy individuals will purchase more level annuities than unhealthy individuals[3].

---

[3] Brown (2003) examines the issue of annuity demand with mortality heterogeneity and finds that poorer people find the insurance element of annuities to be greater than richer people, to some extent canceling out the wealth effect. In this paper we are assuming that wealth is the same across mortality types. The description here may need to be made more precise, especially if wealth is allowed to differ.

However, we assume that agents are constrained in the amount of annuities they can purchase. We assume that they cannot spend more than their total wealth buying annuities, and we assume that they cannot sell annuities rather than buy them. This has the implication that we may only observe the difference in annuity purchases of healthy and unhealthy individuals for some of our population. If, however, we assume that θ range is large enough to ensure that at least one healthy individual will annuitise their entire wealth and that at least one healthy individual will annuitise none of it (as we do in a previous section), then we will observe at least one value of θ for which a healthy individual will purchase annuities and an unhealthy individual will not. This implies that, in equilibrium, $x_{h,0} > p$, the steady state proportion of healthy individuals in the population.

We have therefore demonstrated that there exists a pooling equilibrium in this annuities market; that the equilibrium annuity price lies in the range $[a_u; a_h]$, and that, if the distribution of θ and wealth is the same in healthy and unhealthy individuals, $x_{h,0}$, the proportion of individuals who choose to purchase annuities who are healthy, is greater than *p*, the steady-state proportion of healthy individuals.

Given that this is the case, what will the insurance company observe to the mortality of individuals who have purchased annuities in this case? Let $x_{h,1}$ be the proportion of individuals who bought annuities one year ago who are healthy. Following the derivation of the value of *p* in the appendix, we note that:

$$E[x_{h,1} \mid x_{h,0}] \cong \frac{(1-\lambda-q_h)x_{h,0} + \eta(1-x_{h,0})}{x_{h,0}(1-q_h) + (1-x_{h,0})(1-q_u)} \qquad (15)$$

given that the number of individuals who purchased annuities at time 0 is large.

By mild assumptions described in appendix A, there will be a non-oscillating progression of $x_{h,t}$ back to *p*. This implies that the observed mortality of a cohort of annuity purchasers will tend back to the steady state mortality level from a lower level, again provided that there are sufficient annuity purchasers. Obviously, because the population was in the steady state before the option to purchase the annuity was offered, the mortality of the population who chose not to purchase the annuity will start at a higher level and gradually revert to the steady state: the population viewed as a whole will remain in the steady state, but there will be systematic differences between the mortality of purchasers and non-purchasers that will exactly cancel each other out.

Therefore we have demonstrated that the presence of asymmetric information whose value decays in an annuities market in which there is a pooling equilibrium will cause an increase in the observed mortality of a cohort of annuity purchasers, independent of any ageing effect. This effect is caused by the decline in the predictive power of the individual's health status at the time they purchase the annuity, for their health status many years later. The decline in observed mortality happens despite the fact that the annuity purchasers are forward-looking, rational purchasers who are fully informed about the process their mortality follows, and despite the fact that the insurance company charges the equilibrium price for the annuity.

Before applying this model to real data, several observations are important. The first is that if all the asymmetric information about mortality has a predictive value that does not decline over time, then the effect modelled here will not be observed, despite the existence of asymmetric information. This implies that the model can only be regarded as a test for asymmetric information whose predictive value declines over time. In our model, mortality information whose predictive value does not decline over time might be modelled by assuming that individuals remain type h or type u their entire lives. Both types of asymmetric information are reasonable in the context of annuities: asymmetric information about mortality whose predictive value does not decline might be the age at which your parents died, while asymmetric information which does become less valuable might be the fact that you were in hospital with pneumonia at age 55. Secondly, annuity contracts are permanent: individuals cannot withdraw from them, which implies that they do not have the opportunity to adjust their annuity holdings downwards in response to flows of new information, particularly bad news about their health. This makes the annuities market suitable for testing for the presence of asymmetric information whose predictive power declines with time. A third point to note is that this test might pick up other duration-related effects unrelated to asymmetric information. For instance, if, the longer an individual is retired, the more likely they are to die, regardless of age, this may show up in our data and result in a false result.

It is also useful to examine the issue of correlation between the mortality type of an individual (h or u) and the propensity of an individual to purchase annuities (θ). If the distribution of θ differs between healthy and unhealthy individuals, then we can no longer conclude that a positive duration-related trend in annuitant mortality implies that asymmetric information about mortality is present. For instance, if we assume that healthy individuals are more likely to purchase annuities than unhealthy individuals, even if there is no asymmetric information used in insurance purchases (for instance, if individuals themselves were ignorant of their health type) then there would be a duration-related trend of annuitant mortality in our model. This effect has prevented previous studies of asymmetric information in annuities markets from reaching definitive conclusions, and regretfully it limits the generality of our model too,

3. Data

This section will discuss the data used in the study, and the insurance environment in South Africa, where the data comes from. South Africa is a middle-income developing country situated on the southern tip of Africa. The South African Reserve Bank (2003) reports that South Africa's 2002 GDP was approximately US$160 billion (in PPP terms it is around 2.5 times that, by UNDP figures, although estimates of this vary (see UNDP, 2004)). The population of South Africa was 44.7 million according to the most recent census, conducted in October 2001 (Statistics SA, 2003a). This implies that GDP per capita was around $3500 per head per year in 2002, and around $8000 per head per year in PPP terms. According to the South African Reserve Bank (2003), the South African GDP is made up roughly of 12% primary industry such as mining, forestry and agriculture, 24% manufacturing and construction and 64% services such as financial intermediation, wholesale and retail trade and personal services. The life insurance industry in South Africa is very large: insurance premiums to life insurance companies were about US$18 billion in 2002, around 11% of GDP (Life Offices Association of South Africa, 2003). This is divided into individual premiums of around US$10 billion and group premiums of around $8 billion per year.

One of the reasons that life insurance premiums are such a large part of GDP is that South Africa's state old-age benefit is a non-contributory means-tested pension fixed at a very low level (currently around US$100 per month). Individuals who wish to consume more than this level in retirement are forced to make their own provision for this, and the large life insurance sector has grown to meet this need. More information on the state pension system in South Africa can be obtained from the South African Department of Social Development (2003) and from Case and Deaton (1998).

In South Africa, tax incentives are given for saving through pension arrangements, which typically take the form of either a personal pension or IRA (called a 'Retirement Annuity' in South Africa), or an occupational pension, which is either of the defined benefit or defined contribution variety. South Africa began the shift from defined benefit to defined contribution pension plans in the early 1980's, which implies that there are significant numbers of people who have already retired from defined contribution pension schemes.

In South Africa, members of defined contribution pension plans or holders of IRA's are required to purchase an annuity with 2/3 of their accumulated account balance on retirement. The other 1/3 is paid as a lump sum, partly tax free. The range of annuity products whose purchase is permitted is very large indeed. There are two main groups of permitted products: 'traditional' annuities, and 'living annuities'. 'Traditional' annuities include single life and joint-and-survivor annuities (which may be level or escalating), annuities where the annuity payment is guaranteed for the first few years of the contract, regardless of whether the individual lives or dies, annuities which return a portion of the capital on death, impaired-life annuities and with-profits annuities, where the annual annuity payment is linked in some way to the returns on a portfolio of underlying assets. 'Living annuities', on the other hand, are more like phased withdrawals than annuities:

the individual has complete investment choice, there is no transfer of mortality risk, and withdrawals are fairly flexible. In a living annuity, each purchaser's assets are invested in a separate account. The purchaser can choose the asset mix, which may include equities, bonds, cash and even shares of commercial real estate portfolios, and change it frequently. Some providers permit international investments, subject to some restrictions. The purchaser must take a monthly pension from the account, whose annual value must be between 5% and 20% of the accumulated fund value at the last policy anniversary. (Historically, South Africa has had relatively high interest rates: currently, long-term government bonds yield around 8.5% to maturity, their lowest in at least two decades). Each year, the purchaser can decide on a new pension level. When the purchaser dies, the funds remaining in the account do not revert to the insurance company, but form part of his or her estate. The policy cannot be surrendered unless the accumulated funds are transferred to a similar policy with another provider. In a 'living annuity' there is therefore no mortality risk pooling: the obligation is entirely on the purchaser to ensure that he or she outlives his assets. These policies are called annuities in order to attract the favourable tax treatment accorded to annuities in South Africa and in order to be eligible for compulsory purchase by members of DC pension plans. They have been sold in South Africa since the early 1990's. Their introduction was at least confirmed by a government Commission of Inquiry into the South African tax system which commenced in 1994, and which found that the state had little interest in differentiating between products that paid lump sums and products that paid a lifetime income to retirees[4]. This is not surprising given the relatively low level of the state old age pension in South Africa.

The option to purchase such a broad range of products has implications for adverse selection in the traditional annuities market. If a purchaser believes that he or she is relatively more healthy than average, then he or she can elect to purchase a life annuity. However, those who are less healthy than average might prefer to purchase a living annuity, because they will be able to bequeath any assets they are unable to consume

---

[4] The enquiry was chaired by Michael Katz. His complete report can be found online at the South African Treasury website: http://www.treasury.gov.za/documents/katz/default.htm

during their lifetime, or because they may consume assets outside the annuity policy and bequeath assets that remain inside it. We might therefore expect that if the effects of the asymmetric information on mortality are temporary, as modelled in the previous section, we would observe a duration-related increase in mortality rates amongst purchasers of life annuities, after controlling for age, and gender. Controlling for age is necessary because mortality exhibits a predictable increase with age. Women also have lower mortality than men. What we therefore wish to test for is a difference between the mortality of purchasers of the same age and gender who retired and purchased their annuities at different ages. If there is a significant increase in mortality with both duration and age, this could be evidence of asymmetric information about mortality in the annuities market.

To test for this effect, data was collected from two South African insurance companies. The policies collected were all life annuity policies in force at those companies at any point between the beginning of 1997 and the end of 2002 for the one company and the beginning of 1998 and the end of 2002 for the other company. Three filters were applied to the data. Firstly, joint-and-survivor policies, voluntary purchase annuities, or any kind of with-profits annuities were excluded from the analysis for all years. Secondly, annuities where the purchaser was younger than 55 or older than 85 were excluded only in the years where this was the case. This was to restrict the data to the age range of interest, and to restrict the number of variables to estimate. At ages above 85 there was very little data in any case. There were fewer than 5 policies where there were such severe data errors in processing that it was not possible to include them in the analysis in an obvious way. There were also some policies where the date of death was recorded during the investigation period but where the death actually occurred before the investigation period began. Both of these groups of policies were excluded from the analysis.

Table 1 shows various characteristics of the data. These exclusions left 123482 policies in the sample. The total number of deaths observed in the sample over the observation period was 13299, and the total number of life-years observed was 493313.6 years.

Around 68% of the policies were held by males, and around 31% of the policies came from one of the two companies and the remainder from the other. For technical reasons, the data extract only contained the age of each policyholder in whole years and the month, rather than the date of death. For reasons of convenience, it was therefore assumed that the birthday and the policy anniversary occurred at the same time.

An examination of the data by year of inception shows an interesting trend. Treating each policy as a unit of analysis (rather than each policy year), it can be seen that the annual number of annuities sold by these two companies in this category is small, but stable over time. It is difficult to obtain accurate figures about the size of the entire South African annuities market, and therefore it is not clear whether this implies that the market for life annuities is growing or contracting. Given the change in policy, it would be interesting if the proportion of retirees who annuitised their savings remained constant at a relatively high level, given the unwillingness of people elsewhere in the world to annuitise their assets. A study of the effects of the policy change on annuitisation behaviour would be extremely interesting.

Table 2 shows summary statistics for this data by policy-year, rather than by policy as was shown in Table 1. In this data set, each policy has one record for each policy year or part-policy year that it remains in the investigation.

A conceptual representation of the structure of our data is illustrated in Figure 3. Duration cells are shown on the vertical axis and age cells are shown on the horizontal axis. Individuals, shown as diagonal lines, enter the analysis either when the investigation period begins, or when they purchase a policy. They leave the investigation either when they die (marked with a cross) or the investigation ends (marked with a circle). With the passage of time, they move diagonally through increasing age and duration cells. Owing to the fact that birthdays are not recorded in our data, and our assumption that policy anniversaries occur exactly on birthdays, each individual (marked by the diagonal arrows) exits age and duration cells simultaneously. This assumption will have no effect on our final analysis beyond making the estimated mortality at each age applicable to people who are, on average, half a year older assuming that birthdays are evenly distributed across the calendar year. This effect is unimportant for our purposes. For example, in the figure, individual A purchases a policy aged *x* exactly at the beginning of the investigation and survives all the way through to the end. The dashed line marked A′ shows the effect of our assumption about age: in our data, individual A is indistinguishable from individual A′, even though he is actually nearly 1 year younger than A′ (although he purchases his policy at the same time, and, like A′, survives to the end of the investigation). Individual B purchases a policy aged $x+2$ mid-way through the investigation and survives until the end. Individual C was present in the analysis at the beginning of the investigation, aged $x+1$, but died in the middle of his second policy year, aged $x+2$.

Figure 4a shows a histogram of the number of life-years in the data set separated by age and duration. It can be seen from the graph that the age distribution of policies at inception is highly lumpy, with most annuities being purchased at the round ages 55, 60, 65 and 70. The modal age at inception in this data is 55 years old. This lumpy retirement pattern has remained constant for at least 10 years in this data, as can be seen from the length of the parallel diagonal peaks in the data displayed. In the second panel, labelled Figure 4b, the graph has been rotated to illustrate the effect of the lumpy inception age on the age-duration structure of the data. In the shorter term, these peaks are caused by individuals aging and policy duration increasing simultaneously as in the theoretical representation. The peaks are slightly more pronounced than they would be if the actual birthday and the actual policy anniversary had been used as opposed to approximations described above.

Figure 5 shows the logarithm of the crude death rate in each duration-age cell, calculated as the number of deaths observed in each cell divided by the number of life-years in each cell. The increase in mortality rates with age shows up clearly as a decline in the negative logarithm of the crude death rate; however, it is less clear that there is any effect by duration. However, the graph masks the fact that in many of the cells there is not very much data and therefore the crude death rates are very variable in many cells. We therefore use an econometric method to test for any duration-related trend in mortality.

4. Regression Analysis

A more precise estimate of the effect of increasing age and duration on mortality can be obtained by using the Cox proportional hazards regression methodology, described in Cox (1972). This fits a model to the hazard rate $h(t,x)$, where $h(t,x)$ is given by the following:

$$h(t,x) = \frac{f(t,x)}{1-F(t,x)}. \tag{16}$$

In this case, $F(t,x)$ is the survival distribution function of individuals, and $f(t,x)$ the corresponding density function. The Cox proportional hazards model assumes that all hazards are proportional to a base-line hazard rate, $h_0(t)$, in the following way:

$$h(t,x) = h_0(t)\exp(\beta_1 x_1 + ... + \beta_n x_n). \tag{17}$$

The model, as its name and formula suggest, assumes that the hazard rates of different populations are proportional to one another, or, equivalently, that the survival functions form a family of Lehmann alternatives (in other words that they are powers of one another), an assumption which can be tested.[5] The method can be implemented with both tied failure times (as we will have in this data owing to the fact that dates of death are only recorded monthly), and time-varying covariates. The method does not explicitly estimate the baseline hazard function $h_0(t)$, although this can be estimated by other methods. This has the implication that an omitted category is not strictly necessary, but that time trends in hazard ratios need to be interpreted carefully.

As a further approximation, we treat age and duration as constants between the time that they change, effectively assuming that these covariates change discretely rather than continuously. Note also that the data is censored: each individual spends a maximum of

[5] See Miller (1981) for more information about the Cox proportional hazards model.

1 year in each duration-age cell before either moving on to the next cell, dying, or the investigation being terminated, as shown in Figure 3.

We use sets of dummy variables for age and duration. To lower the number of categories, we lump all policies with duration greater than 15 into one category, and use this as the omitted category. The omitted age category is age 71. We also have dummies for gender (we omit females), for company and for calendar year (1996 is omitted) to capture any effects related to these variables. To control for wealth, which affects mortality, we include the annual annuity payment in 2002 South African Rands. The annual payments of annuities purchased in earlier years were grossed up using the South African Consumer Price Index, available from Statistics South Africa (2003b)[6]. Escalating annuities were first deflated by their annual escalation, and then grossed up by inflation.[7]

STATA has implemented the Cox proportional hazards estimation procedure for data with discretely time-varying covariates. The results do not appear to be sensitive to the method assumed for dealing with tied failure times in the data. The hazard ratios for dummy variables show the estimated ratio between the hazard rate when the variable is 1 to when the variable is 0.

First results are shown in Table 2. The χ^2 test shows that the model as a whole is very significant, as we would expect with such a large amount of data. The estimated hazard ratio for males is 2.110209 and highly significant. This shows, as expected, that males have much higher mortality than females: in this data set the hazard rate of men is estimated to be double that of women. The logarithm of the amount of the annual income is also extremely significant. A 2.78-times increase in the annual income from an annuity policy lowers the hazard rate by a factor of 0.9524708, as we would expect: richer people who can afford bigger annuities have lower mortality, as in other countries. There is also

[6] Before 1975, base effects make reasonable inflation values difficult to estimate from the Statistics South Africa CPI data release. For the few policies that were purchased before 1975 in our data, we assumed inflation until the year 1975 was 5% per year. This is likely to be a reasonable approximation.
[7] This understates the value of these policies, although the effect is unlikely to be significant.

a significant difference between the hazard rates of policyholders of company 1 and company 2. Company 1 policyholders have a hazard rate lower by a factor of 0.9409329. This may be because the companies appeal to different segments of the South African insurance market, it may be a reflection of regional differences or it may be the result of other factors. It is easier to understand the results for age and duration by visualising them than by looking at lists of numbers. Figure 6 shows the estimated hazard ratios by age, along with their confidence intervals. As expected, hazard ratios increase exponentially with age.

The results by duration, illustrated in Figure 7, show an increasing trend with age, with a peak at duration 10 and a slight fall thereafter. The hazard ratio increases from roughly 0.7 at very short durations to 1.2 at 10 years, the largest hazard ratio, before declining back to one by 15 years. This effect is large relative to the increase in mortality by age: it represents the effect on mortality of ageing about 6 years. If this trend does represent the effect of temporary asymmetric information on mortality, it indicates that asymmetric information is a significant and material feature of the annuities market in South Africa. As the confidence intervals show, a hypothesis test would clearly reject the hypothesis that hazard ratios at all ages are equal. This increasing mortality with duration may indicate the presence of adverse selection due to asymmetric information whose predictive value declines with time, or there may be some other explanations. In any case, we need to check that the assumptions of the Cox regression model hold before reaching any conclusion.

STATA recommends that two tests are performed to check the validity of the assumption that the hazard rates of different categories of individuals are indeed proportional over time – a global test for the entire data set and a specific test for each variable. The results are shown in Table 3, and indicate that the assumption of proportional hazards does not hold in these data: globally, the hypothesis that the data meets the assumption is overwhelmingly rejected. Variable-by-variable tests show that, in particular, the company variable and the amounts variable are at fault. This means that the hazard rates of individuals who purchase policies from the two companies have a different pattern in the data (either over age or duration, or both) and that the same is true of individuals who purchase large and small annuities. Weaker deviations from the proportional hazards assumption are found for males and females, and the null hypothesis that the proportional hazards assumption holds is rejected for a few of the duration and age variables also (although we would expect one variable in 20 to fail the hypothesis test even if the null were true). An obvious solution to this problem is to run the model separately for different companies and for large and small amounts separately. The results are shown in Tables 5 and 6, and graphically in Figures 6a-d.

The results show striking differences between the hazard rate change by duration, but not by age, for the different companies. One company shows hazard ratios that, except for the first year, are essentially level, while the other company shows hazard ratios that increase gradually with duration until the 10th year, when they start to fall again: a large number of annuity holders at this company seem to die around their 10th policy anniversaries. Interestingly, this company sells mainly 10-year guaranteed policies. At least the peak of the hazard ratios around 10 years seems to indicate that there are some problems with mortality data collection: if the annuity holder dies before the guarantee term expires, the company will continue payment until the end of the guarantee term and apparently recorded the date of death, in a few cases, as the end of the guarantee term rather than the true date of death[8]. This would cause a consequent lightening of apparent mortality before this time, inducing a duration-related positive slope in the hazard rate curve. However, the peak at 10 years seems to be too low to justify the entire slope of the curve with duration, indicating that there may be other effects operating. There do not seem to be tremendous differences by amounts between the two companies, although the data set is so large that the slight differences that are present may explain the results of the statistical tests. There do not seem to be terribly large differences by age in any of the four subgroups of the data, as can be seen in Table 7, which plots the fitted hazard ratios by age for all four subgroups of the data. The Schoenfeld residual tests are unable to reject the assumption that the proportional hazards model holds in these data when

[8] When asked about this, a representative of the company mentioned that there had in fact been some cases where they had permitted this to happen, owing to staff shortages in the department where policy movements were processed.

companies and amounts are modelled separately, except for smaller policies in company 1, where the test indicate that a split by gender is required. This was done, but the results did not differ very much from current results, and so these are not reported. The null hypothesis that the proportional hazard model assumptions hold in these sub-data groups could not be rejected and so no further split was performed.

It is difficult to understand these differences in mortality patterns by company, and several hypotheses can be postulated. Firstly, the companies may sell a different mix of variable and living annuities. This would not matter if the annuity market was frictionless and policyholders routinely exercised their open-market options on retirement. Evidence from the UK (Association of British Insurers, 2000) shows that relatively few policyholders shop around for annuities: most purchase their annuities from the insurance company that administered their retirement policy or pension fund before retirement. In South Africa, although pension fund and retirement annuity members do have an open market option at retirement (living annuity holders also have the right to change insurance companies after retirement), the behaviour pattern of retiring individuals is likely to be similar to that in the UK. This may cause selection-related effects to differ between companies if each company has a different business mix of living and conventional annuities. Secondly, the business each company receives may differ on some other axis that is correlated with the decision to purchase a conventional or a living annuity. A final explanation might be that one of the companies started selling living annuities a long time before the other one, which is unlikely given the competitive nature of the South African insurance market in the product development area. One common factor underlying all these explanations is that the open market option in South Africa does not seem to be preventing market segmentation by insurance company, assuming that the duration-related change in hazard rates is not entirely the result of data effects in the one company. This further implies that a study of the effects of asymmetric information in any annuities market requires data from more than one insurance company.

This policy change is of great importance for the study of mandatory annuitisation. How large is the relative market share of living and conventional annuities? Has the policy change had any effect on the mortality rates in the market for conventional annuities? How have individuals who purchased living annuities allocated their funds? How have they withdrawn their funds? Should this change be reversed or imitated in other countries? There is clearly room for much further research.

5. Conclusion

This paper has presented an equilibrium model of the annuities market under asymmetric information about mortality. An implication of this model, under some conditions, is that if the asymmetric information used to make an annuity purchase at the equilibrium annuity price is temporary rather than permanent, as evidence from life insurance data suggests it may be, then insurance companies should observe a duration-related increase in the mortality of individuals who choose to purchase annuities. This model was tested using South African annuity data. A policy change in South Africa in the early 1990's allowed individuals to purchase phased-withdrawal type policies with accumulated defined contribution and IRA-type balances, rather than just life annuities, as was the case before this time. However, large numbers of policyholders continued to purchase traditional life annuities, which allowed this model to be tested. The results were ambiguous. There were significant differences between the durational pattern of mortality hazard ratios between the two companies in the study, which was at least partly the result of data problems. The difference between the companies, if real, suggests that South African retirees are not taking advantage of the open-market option at retirement, and that the South African annuities market, like the UK market, is not in equilibrium in this sense. If the duration-related trend in the mortality of one company's annuitants is not an artefact of data processing errors, then this may be interpreted as evidence in favour of a temporary component in asymmetric information about mortality in the annuities market. A rough calculation would suggest that this asymmetric information improves mortality by the equivalent of around 6 years at short policy durations.

However, there are other possible explanations for this effect, and an explanation is required for the difference in mortality trends between the two companies.

In any case, the effects of the policy change in South Africa have great relevance for any country examining mandatory annuitisation provisions, and merit a great deal of further study. In particular, actual and duration-related differences in mortality between purchasers of living annuities and conventional annuities in South Africa should be tested for, as well as an examination of data sets from UK or US annuity providers, to test for duration-related trends in these data sets would be interesting extensions to this work.

6. References

Association of British Insurers. (2000). *Annuities: The Consumer Experience.* London.

Abel, A.B. (1986), "Capital Accumulation and Uncertain Lifetimes with Adverse Selection", *Econometrica*, 54, pp. 1079-97.

Bernheim, B.D. (1991). "How Strong Are Bequest Motives? Evidence Based on Estimates of the Demand for Life Insurance and Annuities", *Journal of Political Economy*, 99, pp. 899-927.

Brown, J.R. (2001). "Private Pensions, Mortality Risk, and the Decision to Annuitize", *Journal of Public Economics*, 82, pp. 29-62.

Brown, J.R. (2003). "Redistribution and Insurance: Mandatory Annuitization with Mortality Heterogeneity", *Journal of Risk and Insurance*, 70, pp. 17-41.

Brown, J.R. and James M. Poterba. (2000). "Joint Life Annuities and Annuity Demand by Married Couples", *Journal of Risk and Insurance*, 67, pp. 527-53.

Case, Anne and Angus Deaton, 1998. "Large Cash Transfers to the Elderly in South Africa", *Economic Journal*, Vol. 108, no. 450, September, pp 1330-1361.

Cawley, J. and Tomas Philipson. (1999). "An Empirical Examination of Information Barriers to Trade in Insurance", *American Economic Review*, 89, pp. 827-46.

Cox, D. R. (1972). "Regression Models and Life Tables", *Journal of the Royal Statistical Society B*, 34, pp 187-202.

Finkelstein, A. and James M. Poterba. (2002). "Selection Effects in the United Kingdom Individual Annuities Market", *Economic Journal*, 112, pp. 28-50.

Fleming, Wendell H. and Mete Soner, H. (1991), *Controlled Markov Processes and Viscosity Solutions.* Springer Verlag. New York.

Life Offices Association. (2003), *Annual Review 2003*, http://www.loa.co.za/Downloads/AnnualReview-2003.pdf

McCarthy, David G. and Anthony Neuberger. (2003). *Pensions Policy: Evidence on Aspects of Savings Behaviour and Capital Markets*, Centre for Economic Policy Research, London.

Merton, Robert C. (1969). "Lifetime Portfolio Selection under Uncertainty: the Continuous-Time Case", *Review of Economics and Statistics*, 51 (3): 247-257.

Miller, Rupert G. (1981). *Survival Analysis*, John Wiley and Sons, New York.

Mitchell, O.S. and McCarthy, D. (2002), "Estimating International Adverse Selection in Annuities", *North American Actuarial Journal*, 6, pp. 38-54.

Mitchell, O.S., Poterba, J.M., Brown, J.R. and Warshawsky, M. (1999). "New Evidence on the Money's Worth of Individual Annuities", *American Economic Review*, 89, pp. 1299-318.

Rothschild, M. and Stiglitz, J.E. (1976). "Equilibrium in Competitive Insurance

Markets: An Essay on the Economics of Imperfect Information", *Quarterly Journal of Economics,* 90, pp. 630-49.

Samuelson, Paul. (1969). "Lifetime Portfolio Selection by Dynamic Stochastic Programming", *Review of Economics and Statistics*, 51 (3): 239-246.

South African Reserve Bank. (2003). *Quarterly Bulletin*, No 230, December, http://www.reservebank.co.za/

South African Department of Social Development. (2003) *You and Your Grants*, http://www.welfare.gov.za/Documents/2003/April/ugrant.htm

Statistics South Africa. (2003a) *2001 Census in Brief*, http://www.statssa.gov.za/SpecialProjects/Census2001/HTML/CinBrief/CIB2001.pdf

Statistics South Africa. (2003b). *Historical CPI*, http://www.statssa.gov.za/Products/Releases/CPI_Historical/CPIHistoricalCurrent.pdf

United Nations Development Programme. (2004). *Human Development Report 2003*, http://www.undp.org/hdr2003/

Yaari, M. (1965). "Uncertain Lifetime, Life Insurance and the Theory of the Consumer", *Review of Economic Studies*, 32, pp. 137-50.

Table 1: Data description by policy

Variable	Number	Mean	Min	Max
Company	123482	.3066763	0	1
Male	123482	.6780098	0	1
Amount	123482	13659.43	100	842922
Log(Amount)	123482	8.9367	4.60517	13.64463
YOI<=1989	123482	0.235208	0	1
YOI=1990	123482	0.044322	0	1
YOI=1991	123482	0.050979	0	1
YOI=1992	123482	0.063864	0	1
YOI=1993	123482	0.067872	0	1
YOI=1994	123482	0.056024	0	1
YOI=1995	123482	0.051238	0	1
YOI=1996	123482	0.057409	0	1
YOI=1997	123482	0.059952	0	1
YOI=1998	123482	0.044938	0	1
YOI=1999	123482	0.062066	0	1
YOI=2000	123482	0.066625	0	1
YOI=2001	123482	0.069759	0	1
YOI=2002	123482	0.069743	0	1

Note: YOI = Year of Inception

Table 2: Summary statistics for data by policy-year

Variable	Obs	Mean	Std. Dev.	Min	Max
company	584399	.3226871	.4675045	0	1
y	584399	.0227567	.1491271	0	1
age55	584399	.02532	.1570954	0	1
age56	584399	.0287184	.1670141	0	1
age57	584399	.0311671	.1737692	0	1
age58	584399	.0328936	.1783583	0	1
age59	584399	.035664	.1854511	0	1
age60	584399	.0426113	.2019793	0	1
age61	584399	.0444405	.2060719	0	1
age62	584399	.0453748	.208125	0	1
age63	584399	.0454912	.208379	0	1
age64	584399	.0465367	.2106445	0	1
age65	584399	.0577054	.2331858	0	1
age66	584399	.0554467	.2288503	0	1
age67	584399	.0520911	.222211	0	1
age68	584399	.0491753	.2162341	0	1
age69	584399	.0519217	.2218692	0	1
age70	584399	.0487578	.2153614	0	1
age71	584399	.0444285	.2060454	0	1
age72	584399	.0401472	.1963046	0	1
age73	584399	.0361602	.1866888	0	1
age74	584399	.0324145	.1770984	0	1
age75	584399	.0287817	.1671926	0	1
age76	584399	.0251729	.1566501	0	1
age77	584399	.0215059	.1450634	0	1
age78	584399	.0179997	.1329501	0	1
age79	584399	.0147006	.1203515	0	1
age80	584399	.0120808	.1092468	0	1
age81	584399	.0099384	.0991951	0	1
age82	584399	.008128	.0897884	0	1
age83	584399	.0067385	.0818117	0	1
age84	584399	.0049658	.0702932	0	1
age85	584399	.0035216	.0592383	0	1
male	584399	.6776483	.4673772	0	1
dur0	584399	.0796836	.2708029	0	1
dur1	584399	.0760114	.2650166	0	1
dur2	584399	.0716634	.2579299	0	1
dur3	584399	.0700686	.2552628	0	1
dur4	584399	.0706931	.2563118	0	1
dur5	584399	.0737185	.2613124	0	1
dur6	584399	.071162	.2570955	0	1
dur7	584399	.0675669	.2510014	0	1
dur8	584399	.0641189	.2449648	0	1
dur9	584399	.0589939	.2356136	0	1
dur10	584399	.0513878	.2207877	0	1
dur11	584399	.0435045	.2039901	0	1
dur12	584399	.0375976	.1902211	0	1
dur13	584399	.0323341	.1768859	0	1
dur14	584399	.0275154	.1635799	0	1
dur15	584399	.1039803	.3052353	0	1
yoi1989	584399	.2703855	.4441593	0	1
yoi1990	584399	.0526062	.2232462	0	1
yoi1991	584399	.0617284	.2406618	0	1
yoi1992	584399	.0779998	.2681715	0	1
yoi1993	584399	.0833198	.2763652	0	1
yoi1994	584399	.0691189	.2536564	0	1
yoi1995	584399	.0628201	.2426393	0	1
yoi1996	584399	.0708369	.2565524	0	1
yoi1997	584399	.0716394	.2578901	0	1
yoi1998	584399	.0445603	.2063365	0	1
yoi1999	584399	.0502448	.2184498	0	1
yoi2000	584399	.0410079	.1983087	0	1

```
     yoi2001 |    584399    .0289956    .1677942          0          1
     yoi2002 |    584399    .0147365    .1204963          0          1
-------------+--------------------------------------------------------
    year1997 |    584399    .0818721    .2741701          0          1
    year1998 |    584399    .2415165     .428003          0          1
    year1999 |    584399    .1543346    .3612695          0          1
    year2000 |    584399    .1645297    .3707559          0          1
    year2001 |    584399    .1736742    .3788294          0          1
    year2002 |    584399    .1840729    .3875439          0          1
-------------+--------------------------------------------------------
```

Note:

There is one observation for each policy in each policy year.
`company` is a dummy variable which equals 1 if the policy is from company 1.
`y` is a dummy variable which equals 1 if the individual died during that policy year.
`agexx` is a dummy variable which equals 1 if the individual was aged xx at the most recent policy anniversary.
`durxx` is a dummy variable which equals 1 if the policy was in force for xx years at the most recent policy anniversary.
`yoixxxx` is a dummy variable which equals 1 if the policy's year of inception was xxxx.
`yearxxxx` is a dummy variable which equals 1 if the last policy anniversary occurred during year xxxx.

Table 3: Results of Cox proportional hazards regression: pooled data

```
Cox regression -- Breslow method for ties

No. of policies =       123482                  Number of records =  584399
No. of death    =        13299
Time at risk    =  493313.5787
                                                LR chi2(48)      =   5683.83
Log likelihood  =   -148596.93                  Prob > chi2      =    0.0000
```

_t	Haz. Ratio	Std. Err.	z	P>|z|	[95% Conf.	Interval]
age55	.2401499	.0228687	-14.98	0.000	.1992624	.2894272
age56	.3501225	.0347489	-10.57	0.000	.2882306	.4253045
age57	.4001802	.0355173	-10.32	0.000	.3362859	.4762144
age58	.482163	.0388201	-9.06	0.000	.4117766	.5645808
age59	.4964443	.0378093	-9.19	0.000	.427605	.5763658
age60	.4864291	.0347815	-10.08	0.000	.4228199	.5596077
age61	.4779248	.0330424	-10.68	0.000	.417359	.5472796
age62	.5203848	.0345588	-9.84	0.000	.4568738	.5927247
age63	.5874915	.0372548	-8.39	0.000	.5188288	.6652412
age64	.6285476	.0383329	-7.61	0.000	.5577332	.7083533
age65	.6118148	.035721	-8.42	0.000	.5456602	.6859897
age66	.6174261	.0363611	-8.19	0.000	.5501188	.6929685
age67	.771436	.0430872	-4.65	0.000	.6914448	.8606811
age68	.721267	.0417891	-5.64	0.000	.6438413	.8080037
age69	.7507758	.0424268	-5.07	0.000	.6720604	.8387107
age70	.923903	.0505007	-1.45	0.148	.8300411	1.028379
age72	.9892879	.055638	-0.19	0.848	.8860347	1.104574
age73	1.018257	.0582746	0.32	0.752	.9102135	1.139125
age74	1.090094	.0624911	1.50	0.132	.9742443	1.21972
age75	1.205599	.0683116	3.30	0.001	1.078877	1.347204
age76	1.350394	.0784378	5.17	0.000	1.205086	1.513222
age77	1.53865	.0913153	7.26	0.000	1.369693	1.72845
age78	1.683749	.1019663	8.60	0.000	1.495303	1.895943
age79	1.640693	.1066155	7.62	0.000	1.44449	1.863546
age80	1.921335	.1278393	9.81	0.000	1.686425	2.188967
age81	2.128317	.1492232	10.77	0.000	1.855051	2.441838
age82	2.310241	.1682181	11.50	0.000	2.002986	2.66463
age83	2.737774	.2024463	13.62	0.000	2.368399	3.164755
age84	2.622566	.2154364	11.74	0.000	2.232557	3.080706
age85	2.796203	.2590348	11.10	0.000	2.331927	3.352913
dur0	.847155	.0486254	-2.89	0.004	.7570163	.9480267
dur1	.7025083	.0412242	-6.02	0.000	.6261836	.7881361
dur2	.7652807	.0424732	-4.82	0.000	.6864027	.853223
dur3	.8569505	.0453124	-2.92	0.004	.7725868	.9505264
dur4	.8951728	.0450786	-2.20	0.028	.8110406	.9880323
dur5	.9209735	.044181	-1.72	0.086	.8383265	1.011768
dur6	.9208739	.0431245	-1.76	0.078	.8401143	1.009397
dur7	.9405746	.0431713	-1.33	0.182	.8596547	1.029111
dur8	.9510207	.0431089	-1.11	0.268	.8701733	1.03938
dur9	1.142047	.0497339	3.05	0.002	1.048615	1.243805
dur10	1.204748	.0527269	4.26	0.000	1.105714	1.312653
dur11	1.157054	.0516326	3.27	0.001	1.060155	1.262809
dur12	1.016671	.0478078	0.35	0.725	.9271576	1.114826
dur13	1.084668	.0509072	1.73	0.083	.9893431	1.189177
dur14	.9802528	.0489024	-0.40	0.689	.8889426	1.080942
lamount	.9524708	.0082101	-5.65	0.000	.9365145	.968699
company	.9409329	.0181405	-3.16	0.002	.9060414	.977168
male	2.110209	.049059	32.12	0.000	2.016212	2.208587

Table 4: Tests of proportional hazards assumption: pooled data

Test of proportional hazards assumption

	rho	chi2	df	Prob>chi2
global test		102.54	48	0.0000
age55	-0.01215	1.86	1	0.1728
age56	0.00896	1.05	1	0.3048
age57	-0.00424	0.24	1	0.6248
age58	-0.00788	0.82	1	0.3637
age59	0.00814	0.89	1	0.3457
age60	0.00254	0.09	1	0.7685
age61	0.00862	0.99	1	0.3204
age62	-0.00415	0.23	1	0.6327
age63	0.01244	2.06	1	0.1514
age64	0.00098	0.01	1	0.9097
age65	0.00832	0.94	1	0.3331
age66	-0.00022	0.00	1	0.9798
age67	0.00291	0.11	1	0.7367
age68	-0.01899	4.80	1	0.0285
age69	0.00538	0.39	1	0.5347
age70	-0.00620	0.52	1	0.4720
age72	-0.00929	1.15	1	0.2835
age73	0.00261	0.09	1	0.7643
age74	-0.00496	0.33	1	0.5648
age75	0.00321	0.14	1	0.7113
age76	-0.00362	0.17	1	0.6762
age77	-0.00425	0.24	1	0.6256
age78	-0.00064	0.01	1	0.9416
age79	-0.00260	0.09	1	0.7643
age80	0.00252	0.08	1	0.7722
age81	0.01870	4.63	1	0.0315
age82	-0.00349	0.16	1	0.6868
age83	-0.01054	1.48	1	0.2240
age84	-0.00586	0.45	1	0.5001
age85	0.00049	0.00	1	0.9554
dur0	0.00357	0.17	1	0.6834
dur1	0.00289	0.11	1	0.7402
dur2	-0.00081	0.01	1	0.9254
dur3	-0.00071	0.01	1	0.9346
dur4	-0.01134	1.71	1	0.1909
dur5	-0.01676	3.72	1	0.0539
dur6	-0.00613	0.50	1	0.4799
dur7	-0.01288	2.19	1	0.1390
dur8	-0.01243	2.04	1	0.1528
dur9	0.00262	0.09	1	0.7633
dur10	0.00998	1.32	1	0.2504
dur11	-0.01000	1.33	1	0.2490
dur12	-0.00406	0.22	1	0.6405
dur13	-0.00415	0.23	1	0.6325
dur14	0.02914	11.26	1	0.0008
lamount	0.02371	7.28	1	0.0070
company	0.03255	14.00	1	0.0002
male	0.01501	3.06	1	0.0800

Table 5: Results of Cox proportional hazards model: by company and amount

	Company 0						Company 0					
	Large Policies			Small Policies			Large Policies			Small Policies		
	Coeff	95% CI:L	95% CI:H	Coeff	95% CI:L	95% CI:H	Coeff	95% CI:L	95% CI:H	Coeff	95% CI:L	95% CI:H
age55	0.2604894	0.182595	0.371614	0.253457	0.189761	0.338534	0.1657	0.101654	0.270097	0.257472	0.159126	0.416599
age56	0.4234785	0.292185	0.613769	0.369855	0.27403	0.499187	0.230649	0.140747	0.377976	0.323076	0.191091	0.546222
age57	0.5666574	0.413991	0.775622	0.382781	0.29034	0.504655	0.257115	0.163958	0.403203	0.37972	0.237434	0.607273
age58	0.5522254	0.405946	0.751215	0.506056	0.397316	0.644556	0.348126	0.236799	0.511792	0.431158	0.276802	0.671589
age59	0.4467215	0.329188	0.606219	0.503745	0.3993	0.635509	0.487898	0.348022	0.683992	0.504258	0.338115	0.75204
age60	0.5230432	0.399624	0.684579	0.535401	0.432561	0.662692	0.416174	0.298762	0.579727	0.289549	0.182764	0.458728
age61	0.5328168	0.413583	0.686426	0.525406	0.427643	0.645519	0.290444	0.201025	0.41964	0.423939	0.285812	0.62882
age62	0.5598166	0.438946	0.71397	0.529808	0.432713	0.64869	0.382458	0.273511	0.534802	0.540749	0.376515	0.776619
age63	0.6808336	0.542722	0.854092	0.556546	0.456719	0.678192	0.516895	0.382119	0.699207	0.535545	0.375388	0.764032
age64	0.6707947	0.537596	0.836995	0.670529	0.557124	0.807019	0.395797	0.285207	0.549268	0.610168	0.436375	0.853177
age65	0.6379677	0.515991	0.788778	0.622646	0.518699	0.747424	0.533976	0.404892	0.704213	0.637625	0.46558	0.873245
age66	0.716233	0.582135	0.881221	0.585522	0.484845	0.707103	0.553931	0.42286	0.725631	0.583452	0.421568	0.807498
age67	0.8268247	0.677452	1.009133	0.824719	0.692067	0.982798	0.656563	0.506257	0.851494	0.621568	0.450162	0.858239
age68	0.7972626	0.650018	0.977862	0.684384	0.56696	0.826128	0.627624	0.480612	0.819605	0.78254	0.577755	1.059912
age69	0.8313112	0.682809	1.01211	0.768593	0.640326	0.922554	0.572622	0.437889	0.748811	0.747443	0.553117	1.01004
age70	1.069014	0.885781	1.290151	0.923566	0.771405	1.105742	0.746343	0.58261	0.956089	0.855301	0.634122	1.153627
age72	1.159611	0.958742	1.402564	0.85078	0.697648	1.037525	0.947314	0.744754	1.204967	1.020356	0.759306	1.371154
age73	1.173194	0.967711	1.422308	0.919657	0.750646	1.12672	0.996904	0.780047	1.274047	0.971774	0.71853	1.314275
age74	1.189991	0.97923	1.446114	0.938507	0.760973	1.157459	1.10641	0.871087	1.405306	1.244807	0.927883	1.669978
age75	1.280763	1.057051	1.551821	1.192534	0.969261	1.46724	1.111634	0.876269	1.410218	1.352169	1.012162	1.806393
age76	1.45158	1.193448	1.765544	1.223073	0.979267	1.527578	1.418538	1.120321	1.796136	1.384534	1.025227	1.869765
age77	1.841229	1.513486	2.239944	1.355263	1.073675	1.710701	1.386769	1.081157	1.778768	1.593745	1.179035	2.154322
age78	1.881051	1.533171	2.307866	1.513139	1.188517	1.926426	1.760897	1.382226	2.243309	1.615294	1.18461	2.202561
age79	2.015213	1.628777	2.493332	1.575229	1.205002	2.059207	1.587968	1.224032	2.060111	1.308601	0.931696	1.837978
age80	2.112552	1.681718	2.65376	1.743674	1.299988	2.338789	2.154407	1.684306	2.755718	1.461604	1.034463	2.065117
age81	2.604615	2.061477	3.290853	2.21147	1.625301	3.009041	2.003517	1.534257	2.616303	1.79847	1.260257	2.566538
age82	2.737719	2.129689	3.519341	2.765954	2.005101	3.81552	2.062916	1.568379	2.71339	2.104235	1.485064	2.98156
age83	3.102428	2.382199	4.040408	2.008075	1.334349	3.021972	2.779652	2.138759	3.612593	2.756347	1.949653	3.896821

age84	2.577662	1.884665	3.525477	2.123332	1.32999	3.389905	2.799928	2.12354	3.691757	2.54865	1.750998	3.709665
age85	2.900199	2.041306	4.120477	1.69462	0.892209	3.21868	3.212981	2.394195	4.311781	2.301098	1.477311	3.58425
dur0	0.6580415	0.529298	0.8181	0.666577	0.545608	0.814366	1.250267	0.978127	1.598124	1.563436	1.184702	2.063247
dur1	0.5913321	0.475402	0.735533	0.621763	0.507517	0.761727	0.98099	0.770927	1.248291	0.873392	0.64356	1.185302
dur2	0.6180898	0.502108	0.760862	0.676097	0.558869	0.817914	1.031562	0.816526	1.30323	1.112983	0.83606	1.48163
dur3	0.6520757	0.535565	0.793934	0.843822	0.703059	1.012769	1.142151	0.915032	1.425644	1.111923	0.843215	1.466261
dur4	0.9012215	0.757977	1.071537	0.856865	0.71941	1.020582	0.856137	0.676134	1.084061	1.044436	0.802106	1.359978
dur5	0.8724794	0.738721	1.030458	0.941136	0.797188	1.111075	1.02581	0.82812	1.270693	0.789811	0.60055	1.038717
dur6	0.8749439	0.74419	1.028672	0.889418	0.753179	1.050301	1.214692	0.995337	1.48239	0.862752	0.669578	1.111656
dur7	0.9346873	0.800461	1.091421	0.965139	0.819464	1.136709	0.950789	0.772598	1.170076	0.825899	0.646054	1.05581
dur8	0.8928222	0.764642	1.04249	0.960813	0.815736	1.13169	0.977008	0.7958	1.199478	1.03345	0.826297	1.292536
dur9	1.049005	0.903392	1.218088	1.250038	1.068177	1.462863	1.254715	1.039177	1.514957	0.981989	0.782107	1.232955
dur10	1.248951	1.082578	1.440892	1.253924	1.065266	1.475994	1.094866	0.898883	1.333581	1.098082	0.87406	1.37952
dur11	1.120896	0.966995	1.299292	1.177464	0.991306	1.39858	1.191853	0.99247	1.431292	1.125303	0.892568	1.418722
dur12	1.025712	0.882823	1.19173	1.080735	0.899405	1.298622	0.985627	0.806721	1.204209	0.923581	0.718312	1.18751
dur13	1.013364	0.870759	1.179323	1.169633	0.970469	1.40967	0.996776	0.818119	1.214447	1.217398	0.962938	1.539102
dur14	0.8915051	0.757925	1.048628	1.01823	0.826683	1.25416	1.010833	0.833171	1.226378	1.0588	0.822331	1.363269
lamount	0.9063258	0.867745	0.946622	0.93596	0.880531	0.994878	0.970658	0.921989	1.021897	1.060627	0.963681	1.167325
male	2.298077	2.10452	2.509435	2.214898	2.058962	2.382644	1.779445	1.600307	1.978635	1.945618	1.734385	2.182578
N(Records)	184814			211007			107382			81196		
N(Deaths)	4316			4280			2825			1878		
N(Policies)	38882			46741			21386			16483		
Log Like	-43641.03			-44039.3			-26550.6			-17048.7		
LR Chi2(47)	2070.03			1669.93			1350.37			743.27		

Note:

Small policies are policies whose inflation-adjusted annual payment is below the median amount for the whole data set
95% CI shows the 95% confidence interval for the parameter.

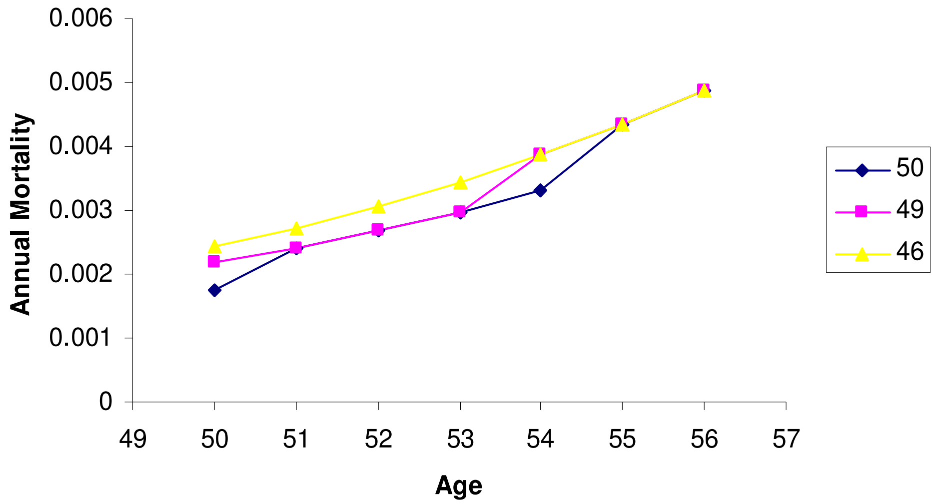
Table 6: Tests of proportional hazards assumption: data by company and amount

	Company 0						Company 1					
	Large Policies			Small Policies			Large Policies			Large Policies		
	rho	chi2	Prob>chi2	rho	chi2	Prob>chi2	rho	chi2	Prob>chi2	rho	chi2	Prob>chi2
age55	0.01343	0.75	0.386	−0.01941	1.49	0.2229	−0.0279	2.09	0.1484	−0.02089	0.8	0.3699
age56	0.0223	2.13	0.144	0.01567	1.03	0.3096	−0.00965	0.26	0.6121	−0.00121	0	0.9582
age57	0.00942	0.38	0.5358	−0.00782	0.26	0.6098	−0.0163	0.74	0.3897	0.00362	0.02	0.8748
age58	0.00718	0.22	0.6414	−0.00793	0.27	0.6011	−0.0013	0	0.9449	−0.03535	2.35	0.1254
age59	0.01876	1.51	0.2189	0.00807	0.28	0.5945	0.01331	0.5	0.4778	−0.01513	0.43	0.51
age60	0.00848	0.31	0.5768	0.02121	1.94	0.1636	−0.02363	1.59	0.2076	−0.02849	1.54	0.2153
age61	0.01604	1.11	0.2927	0.02697	3.14	0.0765	−0.02245	1.42	0.2337	−0.00253	0.01	0.9134
age62	0.02472	2.62	0.1054	0.00805	0.28	0.5973	−0.04379	5.38	0.0204	−0.02593	1.26	0.262
age63	0.04401	8.22	0.0041	0.01889	1.54	0.214	−0.02759	2.15	0.1423	0.00295	0.02	0.8982
age64	0.01431	0.88	0.3476	0.01876	1.5	0.2203	−0.0172	0.84	0.3602	−0.02916	1.63	0.2023
age65	0.02941	3.73	0.0536	0.00617	0.17	0.6841	−0.01366	0.55	0.4602	0.00074	0	0.974
age66	0.02369	2.43	0.1189	0.00574	0.14	0.7079	−0.02885	2.34	0.1261	−0.01142	0.25	0.6185
age67	0.0286	3.54	0.0598	−0.00012	0	0.9936	−0.0138	0.54	0.4609	0.00725	0.1	0.7536
age68	−0.00273	0.03	0.8571	−0.02366	2.38	0.1226	−0.02273	1.45	0.2285	−0.02376	1.07	0.3011
age69	0.03277	4.57	0.0325	−0.01289	0.72	0.3965	−0.00538	0.08	0.7746	0.02109	0.83	0.3633
age70	0.02105	1.92	0.1654	−0.00994	0.42	0.5146	−0.02627	1.99	0.1584	−0.00765	0.11	0.7382
age72	−0.00174	0.01	0.9091	−0.01104	0.52	0.4706	−0.00784	0.18	0.6753	−0.01132	0.24	0.6212
age73	0.01589	1.07	0.3016	−0.00427	0.08	0.7805	−0.01585	0.72	0.3974	0.03118	1.82	0.177
age74	0.01425	0.88	0.3476	−0.00445	0.09	0.77	−0.01468	0.63	0.4292	−0.02199	0.92	0.3374
age75	0.02895	3.62	0.0572	−0.00475	0.1	0.7562	−0.01634	0.75	0.3861	−0.00229	0.01	0.9214
age76	0.0174	1.3	0.2534	−0.01189	0.6	0.4385	−0.01657	0.78	0.3764	−0.01301	0.32	0.5727
age77	0.01541	1.02	0.3117	−0.00285	0.03	0.8527	−0.0365	3.65	0.0559	0.01543	0.44	0.5085
age78	0.0025	0.03	0.8695	−0.03238	4.46	0.0346	0.01746	0.87	0.3501	0.00428	0.03	0.852
age79	0.01924	1.6	0.2059	−0.01269	0.68	0.4084	−0.01858	0.97	0.3236	−0.01482	0.42	0.518
age80	0.01534	1	0.3168	−0.00886	0.34	0.5625	−0.00405	0.05	0.8291	−0.00672	0.08	0.7719
age81	0.0336	4.85	0.0277	0.02316	2.29	0.1298	0.02006	1.14	0.286	−0.00436	0.03	0.8518
age82	0.002	0.02	0.8956	−0.01396	0.84	0.3598	−0.00748	0.16	0.689	0.01088	0.23	0.6344
age83	−0.00425	0.08	0.7799	−0.00402	0.07	0.7923	−0.02506	1.79	0.1806	−0.00369	0.03	0.8731

age84	0.0024	0.02	0.8745	-0.0215	1.96	0.1618	-0.01635	0.76	0.3823	0.006	0.07	0.7956
age85	0.01273	0.7	0.4029	-0.01008	0.44	0.5067	-0.00944	0.25	0.6154	0.00739	0.1	0.751
dur0	0.00307	0.04	0.8432	-0.00823	0.28	0.5947	0.0067	0.13	0.7204	0.03053	1.71	0.1906
dur1	-0.00388	0.07	0.7976	-0.01596	1.07	0.3011	0.01269	0.46	0.4998	0.03672	2.43	0.1187
dur2	-0.00887	0.35	0.5556	-0.02676	3.09	0.0787	0.01503	0.64	0.4226	0.0308	1.79	0.1813
dur3	-0.01778	1.35	0.2461	-0.01583	1.09	0.2975	0.0081	0.18	0.6682	0.02129	0.82	0.3657
dur4	-0.01854	1.46	0.227	-0.02071	1.88	0.1705	0.00341	0.03	0.8558	-0.01284	0.3	0.5828
dur5	-0.02305	2.26	0.1326	-0.02827	3.49	0.0619	-0.02784	2.17	0.1408	0.01608	0.49	0.4853
dur6	-0.00195	0.02	0.8983	-0.02158	1.99	0.1582	-0.01009	0.29	0.5905	0.03775	2.58	0.1082
dur7	-0.01108	0.52	0.4692	-0.03505	5.29	0.0214	-0.02792	2.23	0.1353	0.0429	3.31	0.0689
dur8	-0.0337	4.76	0.0291	-0.02365	2.42	0.1197	0.00081	0	0.9651	0.01716	0.54	0.4644
dur9	-0.00188	0.01	0.9028	-0.01623	1.13	0.2873	0.00387	0.04	0.8344	0.04062	3.04	0.081
dur10	-0.00751	0.24	0.624	-0.0114	0.57	0.4522	0.04851	6.69	0.0097	0.05375	5.22	0.0223
dur11	-0.02382	2.42	0.1196	-0.03034	3.96	0.0467	-0.00607	0.1	0.7472	0.05592	5.8	0.016
dur12	-0.02249	2.15	0.1427	-0.00044	0	0.977	-0.0054	0.08	0.774	0.04654	4.01	0.0453
dur13	-0.02479	2.63	0.1047	0	0	0.9999	-0.01634	0.77	0.3806	0.03506	2.27	0.1322
dur14	0.02493	2.65	0.1032	0.00558	0.13	0.714	0.03116	2.69	0.1008	0.07578	10.93	0.0009
lamount	0.02353	2.41	0.1206	0.04842	10.05	0.0015	-0.00405	0.05	0.8315	0.00129	0	0.9549
male	0.01017	0.46	0.4971	0.05389	12.68	0.0004	0.0067	0.13	0.7187	-0.03137	1.86	0.1725
Global Test (47 df)		51.71	0.2951		73.09	0.0087		55.1	0.1949		47.22	0.4637

Note: The data were further separated by gender for the case Company 1, large policies, but there was not much difference in the duration-hazard profiles.

Figure 1: Temporary mortality effects as a result of selection in life insurance in the UK



Note: The key shows the age at which the policy was purchased. The horizontal axis shows age, and the vertical axis shows the estimated annual mortality in that year. The "lumpy" nature of the data is due to the method of graduation adopted for the life table, where durations 1-4 were included in one category.

Source: Temporary Assurances (Males) Table TM92, Continuous Mortality Investigation (1999)

Figure 2: Stylistic representation of mortality model

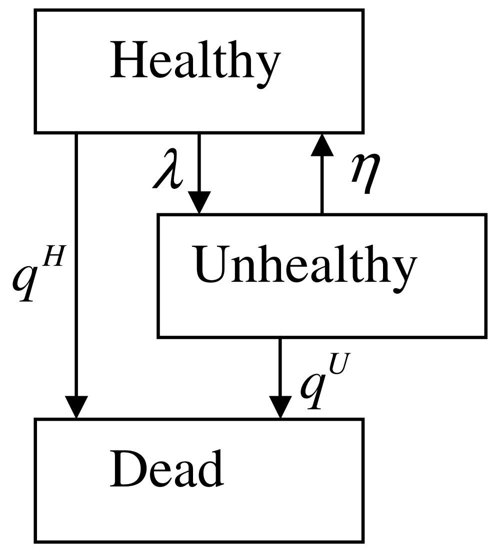


Figure 3: Conceptual representation of data

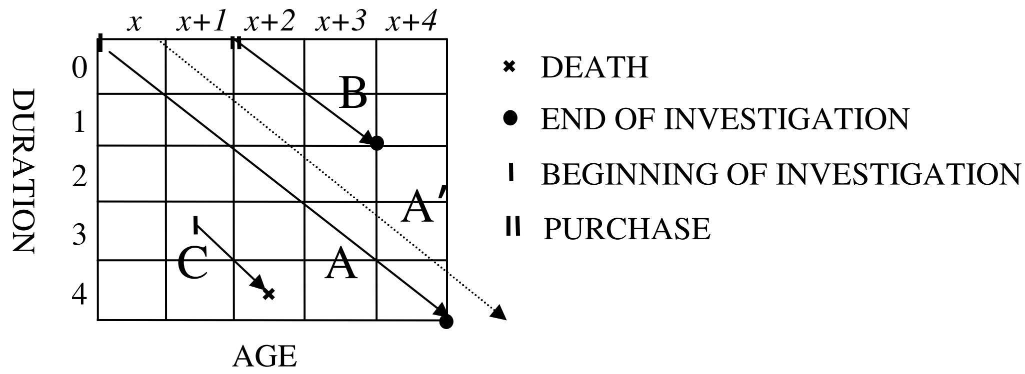


Figure 4a: Histogram of life-years in each age-duration cell

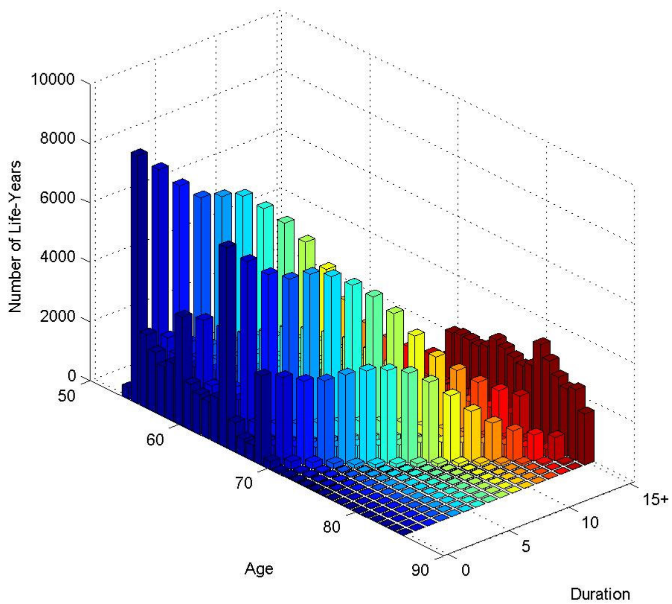


Figure 4b: Histogram of life-years in each age-duration cell (rotated)

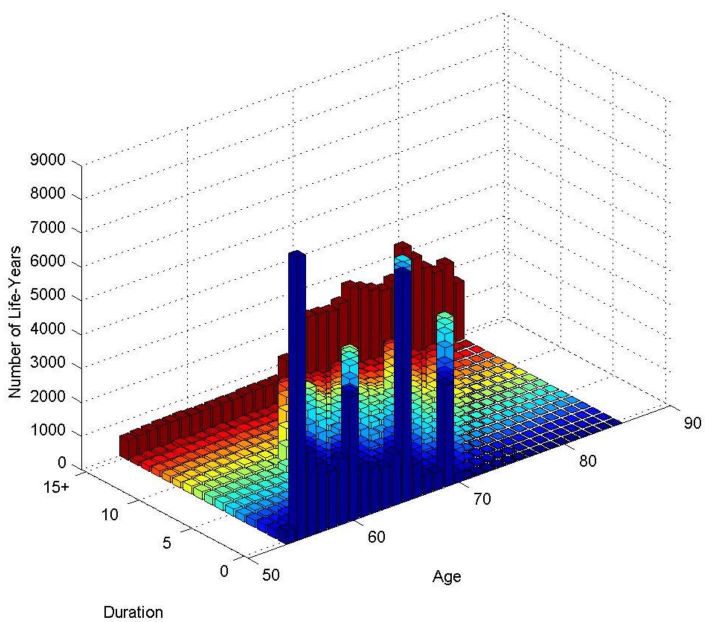


Figure 5: Negative logarithm of crude death rate in each age-duration cell

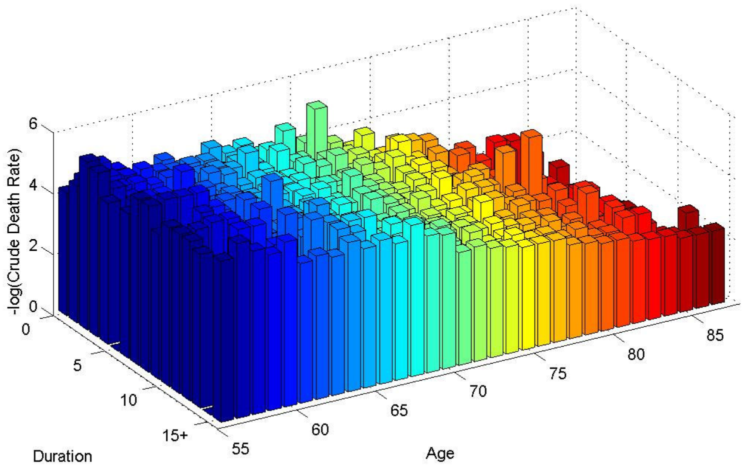


Figure 6: Hazard ratios by age: fitted Cox proportional hazards model

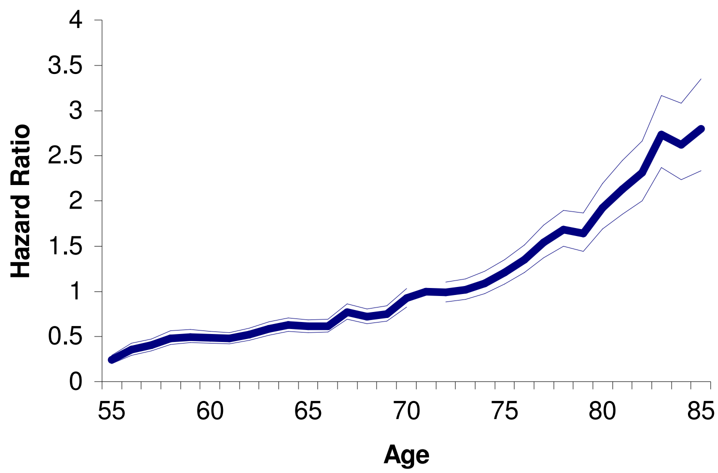


Figure 7: Hazard ratios by duration, all data: fitted Cox proportional hazards model

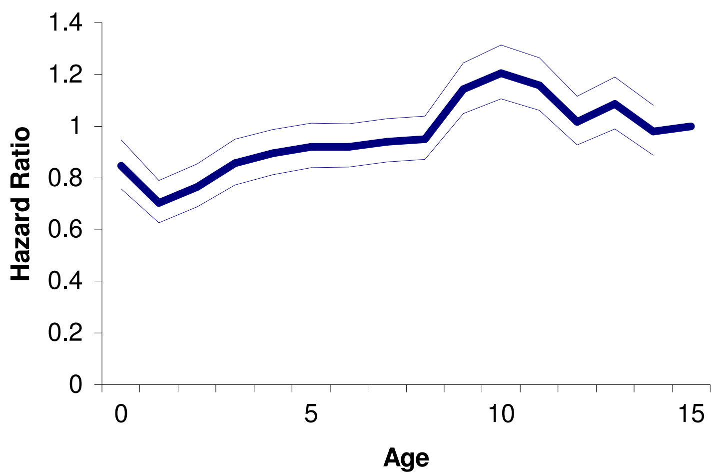


Figure 6a: Hazard ratios by duration: Company 0, small policies

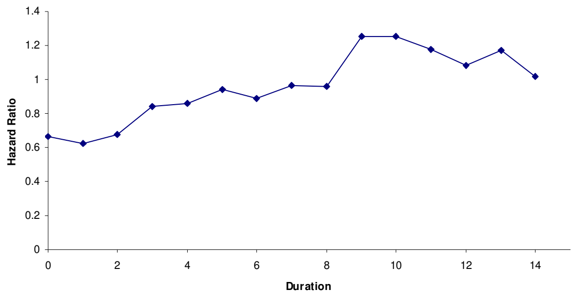


Figure 6b: Hazard ratios by duration: Company 0, large policies

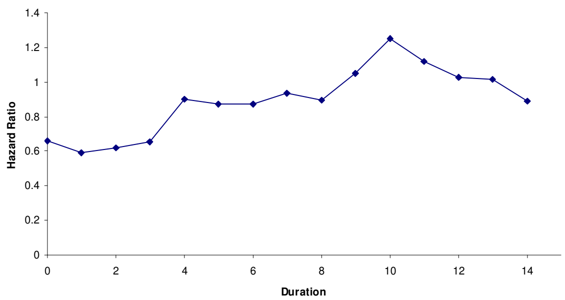


Figure 6c: Hazard ratios by duration: Company 1, small policies

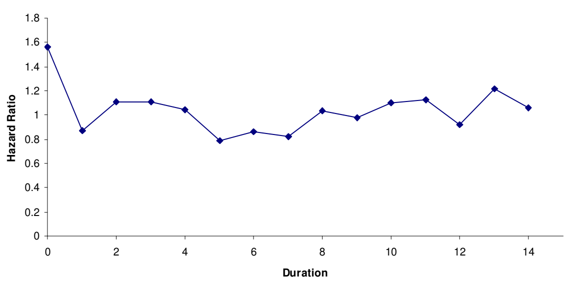


Figure 6d: Hazard ratios by duration: Company 1, large policies

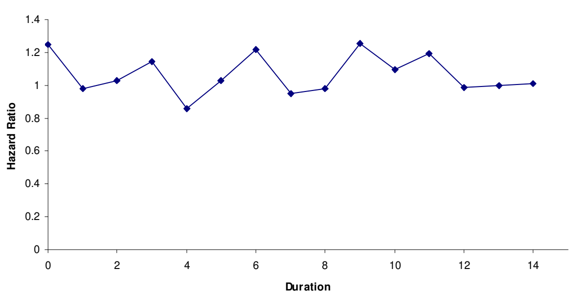
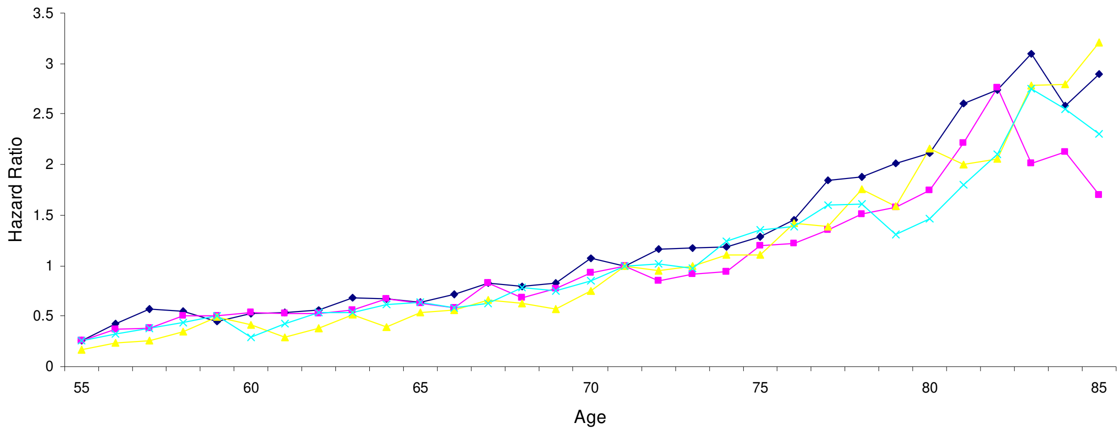


Figure 7: Fitted hazard ratios by age, by company and policy size



Appendix A

This appendix derives the steady-state properties of the Markov chain mortality model.

Given the associated probabilities of changing states,

$$E[H_{t+1} \mid H_t, U_t] = (1-\lambda-q_h)H_t + \eta U_t \text{, and similarly,} \quad \text{(A1)}$$
$$E[U_{t+1} \mid H_t, U_t] = (1-\eta-q_u)U_t + \lambda H_t$$

If the numbers in Ht and Ut are large, then, using the delta method to the second order in H and U gives:

$$E[x_{t+1} \mid H_t, U_t] = \frac{(1-\lambda-q_h)x_t + \eta(1-x_t)}{x_t(1-q_h)+(1-x_t)(1-q_u)} + \tfrac{1}{2}(\sigma^2_{H_{t+1}})(\sigma^2_{U_{t+1}})\frac{E[H_{t+1}-U_{t+1}]}{(E[H_{t+1}+U_{t+1}])^5} + R \quad \text{(A2)}$$

where

$$\sigma^2_{H_{t+1}} = H_t(1-\lambda-q_h)(\lambda+q_h) + U_t(1-\eta)\eta$$
$$\sigma^2_{U_{t+1}} = U_t(1-\eta-q_u)(\eta+q_u) + H_t(1-\lambda)\lambda \quad \text{(A3)}$$

Now, $-E[H_{t+1}+U_{t+1}] < E[H_{t+1}-U_{t+1}] < E[H_{t+1}+U_{t+1}]$, and
$0 < \sigma^2_{H_{t+1}} < E[H_{t+1}] < E[H_{t+1}+U_{t+1}]$ and $0 < \sigma^2_{U_{t+1}} < E[H_{t+1}+U_{t+1}]$. (A4)

This implies that the second-order term is at most of order $(E[H_{t+1}+U_{t+1}])^{-2}$, which implies that for large Ht + Ut, the following holds:

$$E[x_{t+1} \mid H_t, U_t] \cong \frac{(1-\lambda-q_h)x_t + \eta(1-x_t)}{x_t(1-q_h)+(1-x_t)(1-q_u)} \quad \text{(A5)}$$

Therefore, we have an (approximate) transition function for $E[x_{t+1} \mid x_t]$, for large Ht+Ut.

The two conditions for there to be a steady progression to the long-run steady state are:

$E[x_{t+1} \mid x_t = 0] > 0$, and

$\frac{d}{dx_t} E[x_{t+1} \mid x_t] > 0$

If $E[x_{t+1} \mid x_t = 0] < 0$, then the system will diverge, and if $\frac{d}{dx_t} E[x_{t+1} \mid x_t] < 0$ then there will be an oscillating convergence to the steady state. The first condition clearly holds in our model, because η and q_u are probabilities, while the second condition holds if and only if:

$$\frac{1-\lambda-q_h}{1-q_h} > \frac{\eta}{1-q_u}. \tag{A6}$$

Restated in terms of conditional probabilities, this is:

$$\Pr(X_{t+1} = H \mid X_t = H \;\&\; X_{t+1} \neq D) > \Pr(X_{t+1} = H \mid X_t = U \;\&\; X_{t+1} \neq D).$$

If we are willing to assume that individuals don't change health status too readily then this condition is very likely to hold because $(1-\lambda-q_h)$ is likely to be large, η is likely to be small, and $(1-q_h) \cong (1-q_u)$ since both are small. The steady state of the model is then easily derived as:

$$\rho_H = \frac{-(\eta+\lambda+q^H-q^U)+\sqrt{(\eta+\lambda+q^H-q^U)^2+4\eta(q^U-q^H)}}{2(q^U-q^H)}$$