

Cognition and Wealth: The Importance of Probabilistic Thinking

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Project #: UM00-04

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June 2001

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Acknowledgements

This work was supported by a grant from the Social Security Administration through the Michigan Retirement Research Center (Grant # 10-P-98358-5). The opinions and conclusions are solely those of the authors and should not be considered as representing the opinions or policy of the Social Security Administration or any agency of the Federal Government.

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Abstract

This paper utilizes a large set of subjective probability questions from the Health and Retirement Survey to construct an index measuring the precision of probabilistic beliefs (PPB) and relates this index to household choices about the riskiness of their portfolios and the rate of growth of their net worth. A theory of uncertainty aversion based on repeated sampling is proposed that resolves the Ellsberg Paradox within a conventional expected utility model. In this theory, uncertainty aversion is implied by risk aversion. This theory is then used to propose a link between an individual's degree of uncertainty and his propensity to give "focal" answers of "0", "50_50" or "100" or "exact" answers to survey questions and the validity of this interpretation is tested empirically. Finally, an index of the precision of probabilistic thinking is constructed by calculating the fraction of probability questions to which each HRS respondent gives a non-focal answer. This index is shown to have a statistically and economically significant positive effect on the fraction of risky assets in household portfolios and on the rate of growth of these assets longitudinally. These results suggest that there is systematic variation in the competence of individuals to manage investment accounts that should be considered in designing policies to create individual retirement accounts in the Social Security system.

Authors' Acknowledgements

The research herein was performed pursuant to a grant from the U.S. Social Security Administration (SSA) to the Michigan Retirement Research Center (MRRC project number UM 00-04). The opinions and conclusions are solely those of the authors and should not be construed as representing the opinions or policy of SSA or any agency of the Federal Government of the MRRC. Additional support was received by the second author from NIA grant P01 AG10179. Data from the Health and Retirement Study used in this paper were funded by the National Institute of Aging, grant number AG09740, with additional support from SSA. We are grateful for research assistance by Jody Schimmel, Helena Stolyarova and, especially, to Gabor Kezdi who make a number of important suggestions and contributions. We are also grateful for comments on earlier versions of this paper by workshop participants at the University of Michigan, the University of Bergen, Northwestern University, TMR Conference-Paris, University of Chicago, Washington University, and the University of Maryland, the NBER Workshop on Uncertainty, Conference of Retirement Research Consortium and discussions with Gary Becker, Eric French, Jim Heckman, Chuck Manski, Olivia Mitchell, Sherwin Rosen, Mark Rosenzweig, Matthew Shapiro, Jim Smith, Yoram Weiss, and Doug Wolf.

1. Introduction

This paper is motivated by proposed reforms of Social Security which expand the domain of household choice and by corresponding trends in the private sector away from defined benefit pension plans toward defined contribution plans, including growth in tax-sheltered IRA's and 401k plans which allow individual choice in contribution rates and in the timing and magnitude of payouts. Whether such reforms will be beneficial or harmful depend on how well individuals and households plan their finances. Subjective probabilities are a key ingredient in any economic model of optimal financial planning, yet little is known about the capacity of individuals to utilize probabilistic thinking in ways that would enable them to exploit the advantages of expanded choice. In this paper, we utilize a large battery of subjective probability questions that have been administered in the Health and Retirement Study (HRS) to investigate how probabilistic thinking affects portfolio choices and net worth.

Proponents of expanded choice argue that household welfare will be improved because plan characteristics can be better matched to individual preferences and circumstances. In the case of pension plans, it is argued that higher returns available on investment portfolios will enable households to have better and more secure retirements than can be afforded by pay-as-you-go tax and transfer programs such as Social Security, defined benefit plans or fixed income securities. For example, Poterba and Wise (1998) note that estimates of the long term average annual rate of return on a diversified portfolio of stocks range from 8.1 to 9.6 percent compared with returns of about 4.6 percent for a portfolio of long term bonds or 3.8 percent for a portfolio of short term Treasury bills. Compounded over a lifetime of savings, differences in expected returns of this magnitude create very large variations in household resources available for retirement or bequests. While the higher expected returns from stocks may be offset by higher risk of losses, simulated portfolios suggest that the value at retirement of a portfolio of stocks accumulated over a lifetime may stochastically dominate a bond portfolio.¹ Moreover, flexibility about whether and when to annuitize their portfolio after retirement, together with an expanded range of choice of the features of health insurance,

¹ See Poterba and Wise (1998) and references therein.

allow households to reduce their exposure to poor health. Poor health affects household income and wealth primarily through medical expenditures and (mostly uninsurable) losses in labor income (Smith 1999b). Even insurance against inflation risk, a traditional advantage of Social Security benefits, is now attainable for households who choose to hold indexed Treasury bonds.

Although expanded choice offers many important potential benefits to households, critics argue that large segments of the population will fail to make choices that exploit these potential benefits and, consequently, the expansion of choice will expose these segments of the population to greater risks of poverty and exacerbate the already very large inequalities in wealth among older households. Some of the reasons given for these worries fit within the conventional life cycle model of expected utility maximization. For example, in the presence of incomplete insurance and annuity markets, risk aversion might lead low income households to choose less risky portfolios with lower expected returns than higher income households. There may also be variation in taste parameters, such as time preference, such that persons with high rates of time preference save at low rates (and also choose lower investments in human capital and less healthy lifestyles), which leave them with poor health and few resources in old age.

Those who emphasize “equality of opportunity” may view such outcomes as an acceptable consequence of the exercise of free choice. Those who value “equality of outcomes” stress the value of placing constraints on choice *ex ante*, both to protect others from the bad consequence of their actions and to protect themselves from higher taxes to fund redistributive programs needed to offset these consequences.

This paper utilizes a large battery of subjective probability questions that have been administered to a sample of over 20,000 individuals in the 1998 wave and earlier waves of the Health and Retirement Study (HRS) and its companion study, Asset and Health Dynamics of the Oldest Old (AHEAD). Our goal is to develop a measure of competence in probabilistic thinking which is based on the degree to which an individual’s probabilistic beliefs are precise or imprecise and examines the empirical relationship between this measure and measures of asset accumulation and portfolio composition. We provide a theoretical justification for our approach using a simple model in which people with imprecise probability beliefs behave in a more risk averse

manner than those with more precise beliefs. Our model represents a possible resolution of the “Ellsberg Paradox” (Ellsberg 1961), which alleges that individuals display “uncertainty aversion.” Unlike most models of uncertainty aversion, our model is compatible with rationality as defined by the axioms underlying the Ramsey-Savage theory of personal probabilities (Ramsey 1926, Savage 1954), Bayesian statistical theory and the Von Neuman-Morgenstern subjective expected utility (SEU) model.² In addition to providing a link between probabilistic thinking and uncertainty aversion, the model also provides a framework which could be used to study how individuals may acquire more precise information about probabilities, how they may increase the precision of their beliefs through experimentation, and how information acquisition is related to preference parameters such as risk aversion and time preference. These further implications are not pursued in this paper.

The plan of the paper is as follows. Section 2 provides a discussion of the subjective probability questions in the HRS and some background about the elicitation of data on expectations in a survey context. The distribution of responses to these probability questions suggests that there may be considerable heterogeneity among respondents in the precision their probability beliefs and/or their competence in probabilistic thinking. In Section 3, we discuss the idea of uncertainty aversion in the context of the Ellsberg paradox. In Section 4 we develop a rational model of uncertainty aversion. In Section 5, we consider how survey responses to probability questions are related to the precision of probability beliefs. Section 6 presents an econometric analysis relating measures of imprecise probability beliefs to the share of risky assets in household portfolios and to the growth rate of household assets. A summary and conclusions are presented in Section 7.

² After formulating our model, we discovered that Scheeweiss (1999) independently proposed essentially the same resolution of the Ellsberg Paradox. He does not, however, discuss the implications of the model for learning or apply it empirically.

2. Subjective Probability Questions in the HRS

This paper makes use of a unique body of data on large number of subjective probability questions asked to respondents in several longitudinal waves of the Health and Retirement Study.³ In this paper, we analyze questions asked in the 1998 wave which surveyed a national probability sample of over 22,000 respondents representing the U.S. population over age 50, as well as questions asked to subsets of these persons in other waves.⁴ These probability questions cover a wide range of topics ranging from personal life expectancy and date of retirement to beliefs about the rate of inflation, future Social Security policy and other macro-level variables.

The format of the probability questions is presented in Table 1. There is an introduction, which states that the respondent will be asked some questions about how likely they think various events might be. For each question, the respondent is told to give a number between 0 and 100 where “0” means “no chance at all” and “100” means the event is absolutely sure to happen. In several waves of the survey, a “warm up” question is asked about the chance that tomorrow will be sunny,⁵ followed by a list of questions on a variety of topics. Several examples of probability questions are given in Table 1, classified by whether they are about general events (e.g., social security generosity or rate of inflation), events with personal knowledge (e.g., survival to age 75, income will keep up with inflation) or events subject to personal control (e.g., leaving an inheritance or working at age 62). In HRS-1998, there were seventeen probability questions. Some questions were asked to a subset of respondents (e.g., the probability of working at age 62 was only asked of those less than age 62). Others contain additional sub-questions, depending on the answer (e.g., if the probability of leaving an inheritance

³ See Dominitz and Manski (1999) for a discussion of the history of methods of eliciting expectations data in surveys.

⁴ The initial HRS sample, consisting of 12,654 persons born in 1931-41, was first surveyed in 1992 when respondents were 51-61 years of age and has been resurveyed in 1994, 1996 and 1998. The AHEAD survey, consisting of 8,221 persons born before 1924, were first surveyed in 1993 when they were 70 years of age and up and were followed up in 1995. Beginning with 1998, the AHEAD survey of persons born before 1924 has been integrated into the HRS which also incorporated a new cohort born in 1924-30 who were entering their 70's and another new cohort born in 1942-47 who were entering their 50's. Thus, the 1998 wave of the HRS represents the entire U.S. population over age 50. See Juster and Suzman (1995), Soldo, et. al. (1997) and Willis (1999) for more detailed descriptions of the HRS and AHEAD studies.

⁵ Basset and Lumsdaine (1999a) have used this question to investigate whether some individuals are persistently “optimistic” or “pessimistic”. They find that persons who give high probabilities that the

is larger than zero, the respondent is asked about the probability of leaving inheritances larger than \$10,000 and, subject to that probability being positive, the probability of leaving more than \$100,000).

Analysis of the subjective probability questions has demonstrated that they contain considerable useful information. For example, on average, subjective probabilities of life expectancy match life tables surprisingly well and co-vary with variables such as smoking, drinking, health conditions or education in ways that would be expected from studies of actual mortality (Hurd and McGarry, 1995). As another example, Smith (1999a) finds that average values of expected bequest probabilities behave in sensible ways and appear to provide genuine information about behavior.

Although the subjective probability responses in HRS seem to “work well” when averaged across respondents, individual responses appear to contain considerable noise and are often heaped on “focal values” of “0”, “50” and “100”.⁶ The degree of heaping is apparent in Figure 2, which presents histograms of the answers to the six probability questions listed in Table 1. A comprehensive tabulation of focal answers to all probability questions in HRS-1998 is presented in Table 2. On average, only 5% of respondents refused to answer the probability questions. However, 52% of questions were heaped on either “0” or “100” and an additional 15% were heaped on “50”.

Probabilities and probabilistic thinking are crucial ingredients of economic models of decision making about asset accumulation and portfolio choice. In this paper, we hypothesize that a persistent tendency to give focal answers to probability questions indicates that a respondent has imprecise beliefs about the true value of a probability. Further, we hypothesize that imprecise beliefs about probabilities will lead an individual to make conservative financial choices which yield lower wealth and lower rates of wealth accumulation than otherwise similar individuals who have more precise probability beliefs. In the next two sections, we explore a more formal theoretical

weather will be sunny tomorrow tend to give higher probabilities of “good” outcomes on other topics. This question was omitted from HRS-1998, but has been put back into HRS-2000 which is currently in the field.

⁶ Several researchers who have analyzed the HRS data emphasize the large number of focal answers and show that the likelihood of such answers is correlated with education and cognitive measures (see, especially, Hurd, McFadden and Gan (1998)). In addition, Bassett and Lumsdaine (1999b) find other evidence of noise and unobserved heterogeneity in answers to these questions.

framework to understand why imprecise probabilistic thinking might lead to such a relationship.

3. Probabilistic Thinking, Imprecise Beliefs, and Uncertainty Aversion

Despite much criticism, subjective expected utility (SEU) theory remains the dominant framework for economic models of decision making under uncertainty. (See Starmer 2000, for a survey of alternatives to SEU theory). SEU theory assumes that individuals decide on a given course of action by choosing that action which yields the highest expected utility. For example, consider a decision between actions A and B by individual i . The expected utilities associated with these actions are

$$(1) \quad EU_{ia} = \sum_{j=1}^{\Omega_a} p_{ij}^a U_{ij}^a \quad \text{and} \quad EU_{ib} = \sum_{j=1}^{\Omega_b} p_{ij}^b U_{ij}^b,$$

where there are Ω_a discrete states of the world that might occur if action A is chosen and Ω_b states if B is chosen. p_{ij}^a represents i 's beliefs about the probability that the j th state of the world will occur and U_{ij}^a is the utility i believes that he will receive in that state. Similarly, p_{ij}^b and U_{ij}^b represent i 's beliefs about probabilities and utilities when B is chosen. The individual's choice of action is given by the decision rule

$$(2) \quad \text{Choose } A \text{ if } EU_i^a > EU_i^b; \text{ Otherwise Choose } B.$$

Taken literally, the SEU model presumes that individuals competently perform some extremely demanding tasks before making any given decision. The individual must be capable of imagining a large number of states of the world. Within each state, the individual solves an optimization problem in which he maximizes his (possibly state-dependent) utility subject to a set of constraints embodying his income and wealth endowments, prices, state of health and all other data relevant to the optimization problem. The outcome of this optimization yields the level of utility associated with that state. Finally, the SEU model assumes that the individual has a coherent and well-defined set of beliefs about the probabilities that each state will occur.

The latter assumption has been called into question at least since Frank Knight (1921) introduced the distinction between "risk" and "uncertainty" where risk refers to a situation in which a specific probability can be attached to a given outcome while

uncertainty refers to a situation in which a probability cannot be specified. For example, examination of a symmetric coin may lead an individual to conclude that heads and tails have equal probability, that there are no other possible outcomes and, consequently, that the probability of heads is one half. Or, a person may have examined actuarial tables to determine his probability of death this year. In contrast to these risks, uncertainty may attach to questions in which there is neither a quantitative model of the random process, such as the simple physical model of a coin described above, nor is there adequate data to form a statistical estimate of the event from past frequencies of occurrence. Examples range from mundane questions about the probability that a new start-up firm will be in business in ten years or that a given marriage match will succeed to large questions such as the probability of nuclear war or the probability that there is intelligent life elsewhere in the universe.

The Ramsey-Savage theory of personal probability appeared to resolve this problem by eliminating the distinction between risk and uncertainty within a Bayesian framework in which individual preferences are assumed to fulfill certain axioms which lead to rational, maximizing behavior (Savage, 1954). According to this theory, the probabilities that enter into decision problems are subjective. Individuals are assumed to have a set of prior beliefs about the probability of any given event given by the distribution function, $f(p)$. The probability of the event is simply given by the expected value of this distribution, $\bar{p} = \int_0^1 pf(p)dp$. The probabilities entering the von Neumann-Morgenstern expected utility function in (1) have this interpretation.

An influential challenge to this resolution was presented by Ellsberg (1961) in an example that has come to be known as the “Ellsberg Paradox”. In one version of the example, subjects are confronted with choices about betting on the outcome of draws from two urns, Urn I and Urn II, each containing red and/or black balls. The subject chooses which urn to play and the color of the ball to bet on. If a ball of the chosen color is drawn, the subject wins \$1; otherwise, he wins nothing. Urn I contains 50 red balls and 50 black balls. Urn II contains 100 red and black balls in unknown proportion.

When asked about which color they preferred to bet on, most subjects indicated indifference regardless of which urn was under consideration. This suggests that subjects

believe that red and black are equiprobable in either urn. When asked which urn they preferred to draw from, some subjects indicated indifference but a majority indicated a preference for Urn I, explaining that they preferred the greater certainty about the probability of drawing a ball of given color. The latter preference clearly violates the Savage axioms.

The paradox that Ellsberg identified, and that subsequent experimental research has verified, is that most people prefer a known risk to an uncertain one of equal expected value. Ellsberg often found uncertainty aversion even in a decidedly non-random sample of the founders of SEU theory. He reports that G. Debreu, R. Schlaiffer and P. Samuelson do not violate the Savage axioms, while J. Marshak and N. Dalkey violate them “cheerfully and even with gusto” and “others sadly but persistently, having looked into their hearts, found conflicts with the axioms and decided, in Samuelson’s phrase, to satisfy their preferences and let the axioms satisfy themselves.” (Ellsberg, pp. 655-56). Interestingly, Ellsberg places Savage himself in the latter group. Among the violators, he writes, “What is at issue might be called the *ambiguity* [italics in original] of this information, a quality depending on the amount, type, reliability and ‘unanimity’ of information, and giving rise to one’s degree of ‘confidence’ in an estimate of relative likelihoods.” (Ellsberg, 1961, p. 657.)

Ellsberg and most of the subsequent literature react to this paradox by partially giving up on the Savage axioms, instead suggesting non-rational preferences that allow for “uncertainty aversion” in ambiguous situations. An example, is the maxmin expected utility (MMEU) function originally proposed by Gilboa and Schmeidler (1989). Intuitively, these preferences suggest that a decision maker who is uncertain about the true probabilities governing the outcomes of his decision may focus special attention to thinking about the consequences of the “worst case scenario” (among all “reasonable” scenarios) and will make conservative choices according to a maxmin criterion.

4. A SEU Model of Uncertainty Aversion and Learning about Probabilities

In this section, we propose an alternative model of uncertainty aversion. Given repeated sampling, individuals are fully rational in the sense that they behave in accord

with the SEU model, but they may have more or less precise beliefs about the “true values” of the probabilities upon which their decisions are based.⁷ Ellsberg’s examples all concern “one shot” bets on a single roll of a die or a single ball drawn from an urn. Our model departs from the Ellsberg framework by assuming that most individuals implicitly form their preferences among uncertain prospects in a context of real world choices whose consequences determine repeated random outcomes.⁸ Obvious examples include the returns from investments in human or physical capital, the choice of a marriage partner, or the returns from a stock held more than one period. In this model, it is easy to show that uncertainty aversion is simply a consequence of risk aversion. In addition, we show that if experimentation or other forms of learning such as job shopping (Johnson 1978) are possible and decisions are reversible, then even risk-averse people may display uncertainty preference.

The basic idea of our model may be illustrated by continuing with Ellsberg’s urn example, using a modified version in which two balls are drawn instead of one. Assume that there are two urns, each containing red and black balls. By convention, assume that there is a payoff of \$1 each time a red ball is drawn from either urn and a zero payoff if a black ball is chosen.⁹ After a ball is drawn from an urn and its color noted, it is returned to the urn. Urn I contains a known number R red and B black balls. We assume that all individuals agree that $\bar{p} = R/(R + B)$ is the probability of drawing a red ball from Urn I. Urn II contains an uncertain number of red and black balls. . A given person i ’s belief

⁷ An advantage of retaining the SEU model is that we can freely draw upon the entire body of conventional economic theory in our application whereas theories such as the MMEU theory mentioned above are not (yet) fully embedded in a broader economic model. We exploit this advantage later in this section when we show how utility aversion and learning may be related.

⁸ As noted earlier, Scheeweiss (1999) independently proposed essentially the same resolution of the Ellsberg Paradox.

about contents of the urn is given by a subjective probability distribution, $F_i(p)$, where $p = R/(R + B)$ is the proportion of red balls in Urn II. The expected probability of drawing a red ball on a single draw is $E_i(p) = \int_0^1 p dF_i(p) = \bar{p}_i$. To focus on the definition of uncertainty and its implications, we shall assume that all individuals have identical beliefs about the expected proportion of red balls in Urn II so that, for all persons i and j , $\int_0^1 p dF_i(p) = \int_0^1 p dF_j(p) = \bar{p}$. Thus, i 's expected payoff from N draws from either urn is $\bar{p}N$ dollars.

The degree of uncertainty that an individual has about the content of the urn is reflected by how concentrated or diffuse his beliefs are about possible proportions of red balls.¹⁰ For example, if the individual has no uncertainty then all the mass of $F_i(p)$ is concentrated at \bar{p} . As the individual becomes more uncertain, the probability mass is spread across more possible values of p . In analogy to Rothschild and Stiglitz' (1970) definition of increasing risk, we define *increasing uncertainty* in terms of second-order stochastic dominance. That is, for any two distributions, $F_i(p)$ and $F_j(p)$ with the same mean, if $F_i(p)$ second-order stochastically dominates $F_j(p)$ then $F_i(p)$ is less uncertain than $F_j(p)$. Again following Rothchild and Stiglitz, this is equivalent to the statement that person j is more uncertain about p than person i , if the density function that describes

⁹ More generally, we assume throughout this section that returns are a linear function of the number of draws.

¹⁰ Clearly, this concept of uncertainty differs from Knightian uncertainty in the sense that we assume that individuals can and do possess well defined subjective probability beliefs about probabilities even in situations in which objective evidence about probabilities is absent or ambiguous.

his beliefs, $f_j(p)$, is a mean-preserving spread of the density function, $f_i(p)$, that describes i 's beliefs.

Under the Savage axioms, an individual should be indifferent between drawing a single ball from Urn I or Urn II because the distributions of returns from the two urns are identical: win \$1 with probability \bar{p} , win \$0 with probability $\bar{q} = 1 - \bar{p}$. The fact that many people apparently prefer Urn I to Urn II is, as discussed earlier, the source of the Ellsberg Paradox. However, the return distributions are not identical if the individual draws more than one ball and, in the case of multiple draws, there is no paradox implied by a preference for Urn I.

This is illustrated in Table 3 for the case in which an individual chooses to draw n balls either from Urn I, in which the proportion of red balls is known to be \bar{p} , or from Urn II, in which the proportion is uncertain but the expected value is $E(p) = \bar{p}$. As shown in the first column of the table, the distribution of returns from Urn I are \$ n with probability \bar{p}^n , \$0 with probability \bar{q}^n , and between \$1 and \$ $n-1$ with probability $1 - \bar{p}^n - \bar{q}^n$. The corresponding distribution for Urn II is shown in the second column.

Since \bar{p}^n and \bar{q}^n are convex functions, respectively, of \bar{p} and \bar{q} , Jensen's inequality

implies that $E(p^n) = \int_0^1 p^n dF(p) > \bar{p}^n$ and $E(q^n) = \int_0^1 q^n dF(p) > \bar{q}^n$. Hence, it is easy to

see from the final column of Table 3 that the subjective distribution of returns of n draws from Urn II is a mean-preserving spread of the distribution of returns from Urn I. It follows that decision to draw n balls from Urn I is less risky than from Urn II and, therefore, a risk averse individual will prefer Urn I to Urn II.

More generally, increasing uncertainty about the contents of the urn, holding the expected proportion of red balls constant, implies increasing the riskiness of the distribution of returns from multiple draws from the urn. That is, let $F_1(p)$ and $F_2(p)$ be subjective distributions of belief about the contents of two urns with identical means, \bar{p} . Assume that F_2 is more uncertain than F_1 according to the definition of increasing uncertainty given earlier. Let $G_1(y|n)$ and $G_2(y|n)$ be the distributions of returns, y , from n draws from the respective urns. Given that F_2 is more uncertain than F_1 , it follows immediately from the argument in the preceding paragraph that G_2 is riskier than G_1 .

The urn model presented in this section may be used as a metaphor for a life cycle model of decision-making under uncertainty by a risk averse individual. At the beginning of the life cycle, the individual faces an investment decision that is embodied in the choice between alternative urns. The returns from the investment in each period of life are determined a draw (with replacement) from the chosen urn. For simplicity, assume that there are only two urns, Urn I and Urn II, only two periods of life, young and old, and no discounting. A red ball drawn from either urn pays a return of \$100,000 and a black ball yields a return of \$50,000. Urn I is known to contain 50 red balls and 50 black balls so that the subjective probability of drawing a ball of either color on a single draw is one half. Urn II is known to contain either 100 red balls or 100 black balls. While the contents of the urn are uncertain, the individual believes either of the possibilities to be equally likely. Thus, the subjective expected return from either urn is \$75,000 per period or a lifetime return of \$150,000.

This setup accommodates two types of life cycle decision. In one, the individual may make an irreversible investment, in effect making a lifetime bet. Examples might include the decision to go to college or putting savings into a retirement fund that is cashed in at the end of life. In this case, the distribution of returns from Urn I is \$200,000 with probability one-fourth, \$150,000 with probability one-half, and \$100,000 with probability one-fourth. The distribution of returns from Urn II are \$200,000 with probability one-half and \$100,000 with probability one-half. Since returns from Urn II are riskier, a risk-averse person would prefer Urn I.

Other kinds of decisions involve more flexible choices in which initial experimentation and learning takes place before permanent decisions are made. For example, Johnson (1978) presents a model of job shopping by young workers who are confident about how well they will do in some jobs but uncertain about their suitability for other jobs. If a worker may switch jobs should he learn that he is unsuited for his first choice, Johnson shows that it will be optimal for the worker to choose the job about which he is most uncertain if the expected wage on the two jobs is the same.

We also obtain “uncertainty preference” in our two period urn model if the initial choice of urn is reversible even for decision-makers who are risk averse. Specifically, assume that an individual may choose either Urn I or Urn II in the first period, draw one ball, and then decide to continue with that urn or switch to the other urn in the second period. An initial choice of Urn I yields the same distribution of returns as permanent choice of Urn I. That is, no matter what color ball is drawn in the first period, the probability of a red ball on the second draw is one-half whichever urn is chosen in period 2. In contrast, if Urn II is chosen in the first period, and a red ball is drawn, then

obtaining a red ball is a sure thing if the individual continues with Urn II. If a black ball is drawn first, it pays to switch to Urn I on the second draw because the individual has learned that the probability of drawing a red ball from Urn II is zero while it is one-half if Urn I is chosen.¹¹ Note that it is optimal for the individual to choose the more uncertain urn no matter how risk averse he may be. The reason is that the initial choice of Urn II provides valuable information at zero cost because the expected return from a single draw from either urn is the same.

Uncertainty aversion may help to explain the “equity premium puzzle” (Mehra and Prescott, 1985). The puzzle arises from the fact that the historical seven percent difference in the returns on stocks and bonds observed in the U.S. and other markets is too large to be consistent with conventional economic theory. Specifically, the equity premium is much larger than would be required to compensate a marginal investor who has a “plausible” degree of risk aversion for “objective” measures of equity risk based on historical data. According to the theory presented in this section, if investors are more uncertain about the degree of risk associated with equities relative to bonds, they will require an additional “uncertainty premium” to compensate for the additional risk that increased uncertainty implies. Since subjective uncertainty about the true degree of risk is not reflected in the historical data from which objective measures of portfolio risk are calculated, the observed equity premium may appear to be too large.

¹¹ In our example, we have assumed the maximum possible degree of uncertainty about the true nature of Urn II. It either contains all red balls or all black balls. As a consequence, the amount of information that can be revealed through experimentation is also maximized. The true nature of the urn is fully revealed by a single draw! To take a less extreme example, assume that the individual’s belief about the contents of Urn II is that any number of red balls between zero and one hundred is equally likely (i.e., $F(p)$ is uniform) and he is a Bayesian learner. In this case, it is easy to show that the subjective expected probability of a red ball on the second draw is two-thirds, conditional on a red ball on the first draw, and one-third, conditional on an initial draw of a black ball. Even though a single draw does not eliminate uncertainty, it remains true that it is optimal for the individual to choose Urn II in the first period.

If there is heterogeneity across people in the degree of their subjective uncertainty, there will tend to be a separating equilibrium in which those with the greatest subjective uncertainty choose investments with “known” risks while those with less subjective uncertainty choose types of investment whose risks are less well understood. The latter group of people, on average, may be expected to obtain higher rates of return from their investments. However, the degree of uncertainty should not be considered as necessarily a fixed trait. For example, by giving them a direct stake in asset performance, the expansion of 401k plans may induce younger workers to learn more about the performance of alternative types of assets and tend to reduce the degree of uncertainty among these cohorts relative to older cohorts, many of whom have never held stocks.

In this section, we established a theoretical connection between the precision of probabilistic beliefs and decisions about risky alternatives. In order to move toward an empirical specification of this relationship, in the next section we attempt to relate the degree of precision about probabilistic beliefs to survey responses to probability questions.

5. Precision of Probability Beliefs and Survey Responses

As we discussed in Section 2, survey responses to subjective probability questions in the HRS tend to behave quite reasonably when averaged across individuals, but are quite noisy with considerable heaping on “focal” answers of “0”, “50” and “100”. We have hypothesized that heaping is associated with respondents’ ambiguity or uncertainty about true probabilities and a correspondingly diffuse distribution of their prior beliefs. In this section, we first develop a simple formal model of the relationship between the information that a respondent has about the probability of a given outcome

and the shape of the density function of his prior beliefs. We then propose a specific testable hypothesis about how answers to survey questions about subjective probabilities are related to the prior density and, finally, present empirical evidence that responses in the HRS are consistent with this model.

Assume that the information that an individual has about the likelihood that a given discrete outcome will occur is given by the probit function,

$$(3) \quad p = \Pr(I > 0 | x, \delta) = \Pr(x\beta + \delta > u) = F(x\beta + \delta)$$

where I is an index function,

$$(4) \quad I = x\beta + \delta - u .$$

In this function, x is a vector of variables that determine the likelihood of an outcome, δ is a normally distributed variable with mean zero and variance σ_δ^2 which reflects the individual's uncertainty about the true value of the index, and u is a standard normal random variable. For example, if p is the individual's belief about the probability of leaving an inheritance, x would include variables such as age, sex, income, wealth, health history, marital status, number of children and other variables that are predictive of the value of the estate at death, the date of death and the existence of heirs. The effects of these variables, given by the probit coefficients β , might be those that would be estimated by in a scientific study of bequest behavior or they might simply reflect personal beliefs that are not supported by scientific analysis.¹² In addition, of course, the individual may possess personal information about the likelihood of leaving a bequest that would not be known to an outside observer.

¹² See Camerer (1995) for survey of "calibration studies" which attempt to determine how well or poorly subjective probabilities elicited in surveys correspond to "objective" probabilities based on evidence.

The random variable δ indicates the range of the individual's uncertainty about the true probability. If $\sigma_\delta^2 = 0$, the individual has sharp priors which are identical to the predicted survival probabilities that would be produced by an actuarial analysis given by $p^* = F(x\beta)$ where $F()$ is the normal cdf. We shall call p^* the "true personal probability."¹³ As σ_δ^2 increases, the individual's beliefs about personal probabilities become more and more diffuse. The precision of a given person's beliefs may depend on his or her education, cognitive ability and experience in observing and making decisions in various domains. Thus, let $\sigma_\delta = \sigma_\delta(z)$ where z is vector of variables determining the precision of beliefs. Note that some elements of z may overlap with elements of x . For example, increases in education tend to increase the true personal probability of survival and also decrease uncertainty about the "true" probability.

Letting $\sigma_e = \sqrt{1 + \sigma_\delta^2}$, $F()$ denote the standard normal c.d.f., and following Lillard and Willis (1978), the c.d.f. of the prior distribution of subjective probabilities associated with (4) is

$$(5) \quad G(p) = F\left(\frac{\sigma_e}{\sigma_\delta} F^{-1}(p) - \frac{\sigma_e}{\sigma_\delta} x\beta\right),$$

and the density function is

$$(7) \quad g(p) = \frac{\sigma_e f\left(\frac{\sigma_e}{\sigma_\delta} F^{-1}(p) - \frac{\sigma_e}{\sigma_\delta} x\beta\right)}{\sigma_\delta f(F^{-1}(p))}.$$

¹³ The term "true personal probability" simply refers to the subjective probability belief that an individual would have if he had no uncertainty. A separate question that we do not address in this paper is the degree to which such personal probabilities coincide with "objective probabilities" based on expert opinion, scientific research, or cognitive processing of personal experience according to a Bayesian model.

A matrix of density functions corresponding to different values of true personal probabilities, measured by p^* on the horizontal axis, and different degrees of uncertainty, measured by σ_δ on the vertical axis, is illustrated in Figure 2. The graph contained in each cell in the matrix depicts the density function in (7) corresponding to a given pair of values of $p^* = F(x\beta)$ and σ_δ . The bottom row of density functions illustrates (almost completely) precise probability beliefs with $\sigma_\delta = .01$ for nine values of $x\beta$ ranging between -2 and 2 and corresponding values of p^* ranging between $.022$ and $.978$. For each value of $x\beta$, there is a column of nine graphs associated with increasing values of σ_δ up to a maximum of $\sigma_\delta = 100$, reflecting ever increasing uncertainty about the true value of the subjective probability.

The effect of increasing uncertainty on the shape of the density function depends on the value of $x\beta$. Consider first the case of $p^* = F(0) = .5$. As σ_δ increases over the range $0 < \sigma_\delta < 1$, the density function has a symmetric, unimodal shape whose variance grows as σ_δ increases. When $\sigma_\delta = 1$, the density function becomes uniform. For values of $\sigma_\delta > 1$, the density function becomes U-shaped with the density increasingly concentrated near the extremes of $p = 0$ and $p = 1$. Now consider the case of $p^* = F(-0.25) = 0.401$. For values of $0 < \sigma_\delta \leq 0.75$, the density function is a right-skewed unimodal function with a mode near 0.4 for small values of σ_δ . As σ_δ increases to 0.75 , the mode decreases but remains well above zero. At $\sigma_\delta = 1$, the density function becomes J-shaped with its single mode near zero. As σ_δ increases above one, the density function takes on an asymmetric U-shape with the larger mode

near zero and the smaller mode near one. As the degree of uncertainty continues to increase, the U-shaped density functions become more and more symmetric near zero and one so that, for large values of σ_δ , the modes near zero and one are approximately equal. This pattern is repeated for smaller values of p^* , but the range of σ_δ over which the function is unimodal shrinks and the range over which it is J-shaped or U-shaped increases. These patterns are repeated in mirror image for values of $p^* = F(x\beta) > 0.5$.

We now wish to address the following question. How do responses to survey questions in HRS about subjective probabilities differ across individuals who have varying degrees of uncertainty about true probabilities? That is, assuming that density functions such as those depicted in Figure 3 are in the minds of respondents, how do they respond to survey questions about subjective probabilities like those discussed in Section 2? One possible hypothesis is that all individuals give a fully Bayesian response and report the expected value of their prior density. In this case, no matter how diffuse their

priors, they would return an exact answer, $\bar{p} = E(p) = \int_0^1 g(p)dp$.¹⁴ This would be

inconsistent with evidence of heaping on focal values presented in Section 2.

¹⁴ In the probit model presented in this section, \bar{p} is a decreasing function of σ_δ holding $x\beta$ constant, with \bar{p} approaching 0.5 as σ_δ approaches infinity. That is, as uncertainty grows, the Bayesian prior probability approaches 50-50. This is not a general implication of Bayesian models with uncertainty. For

example, assume the prior density p is given by a beta distribution, $g(p) = \frac{1}{B(a,b)} p^{a-1}(1-p)^{b-1}$

where $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1} dt = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ with expected value $E(p) = \frac{a}{a+b}$ and

variance $\sigma_\delta^2 = \frac{ab}{(a+b)^2(a+b+1)}$ which is a decreasing function of a and b . As Heckman and Willis

(1977) show, when covariates are introduced into this model with the parameterization $a = \exp(x\beta_a)$ and

An alternative hypothesis, which we shall call the “modal choice hypothesis,” is consistent with heaping. According to this hypothesis, respondents respond by reporting that probability which is most likely among all possible values of their true subjective probability. Specifically, a respondent would choose an exact (i.e., non-focal) value given by the mode of $g(p)$ when his prior distribution has a unimodal “triangular” shape, a modal value of “0” or “100” when the distribution is J-shaped or U-shaped with one mode much larger than the other, and a value of “50” if the individual is not sure which mode is larger.

According to the modal choice hypothesis, subjective probability responses tend to have a systematic pattern as p^* varies. This pattern, which can be discerned in the matrix of density functions shown in Figure 2, is drawn explicitly in Figure 3 where the (p^*, σ_δ) plane is divided into four areas corresponding to those combinations of p^* and σ_δ in which the respondent gives (a) an exact answer, (b) a focal answer of “0”, (c) a focal answer of “100”, or (d) a focal answer of “50”. The area in which an exact answer is given has an upper boundary given by an inverted U-shaped curve which attains a maximum at $p^* = 0.5$ and $\sigma_\delta = 1$ where it is tangent to the U-shaped lower boundary of the area in which a 50-50 answer is given. At the point of tangency, the prior distribution is uniform; for $\sigma_\delta < 1$, the prior densities are unimodal, and for $\sigma_\delta > 1$ they are bimodal with modes of equal size near zero and one. As p^* deviates in either direction from 0.5,

$b = \exp(x\beta_b)$, the expected value of the probability is given by the logistic function $\bar{p} = 1/(1 + e^{-x\beta})$ where $\beta = \beta_a - \beta_b$. In this model, it is easy to hold \bar{p} constant while increasing the amount of uncertainty. However, the pattern of shapes displayed by a matrix of beta density functions associated with differing levels of \bar{p} and σ_δ is very similar to that displayed in the heterogeneous probit model in Figure 2.

the values of σ_δ below which the prior distribution is triangular with a single mode decrease so that as p^* nears zero or one a respondent will not give an exact answer unless σ_δ is very small. Similarly, the lower boundary of the region in which 50-50 answers are given has an inverted U-shape. The reason is that as p^* deviates in either direction from 0.5, the critical value of σ_δ above which the prior distribution is U-shaped (and has approximately equal size modes near zero and one) increases. The regions between these two areas generate focal answers equal to zero if $p^* < 0.5$ or one if $p^* > 0.5$ because, as can be seen in Figure 2, the prior distributions in these two regions are either J-shaped with a single mode near zero or one or they are U-shaped with the larger mode near zero or one.

The modal choice hypothesis is consistent with the “reasonable” behavior of subjective probabilities that we discussed in Section 2 when responses are averaged across samples of respondents who give a mixture of exact and focal answers. That is, as $x\beta$ increases in a population of individuals with varying degrees of uncertainty, the average value of exact answers increases as does the fraction of heaped answers that reflect higher focal values.

In Figure 4, we present evidence that focal values of the subjective probability questions do contain such information. Specifically, we have estimated a set of regressions with subjective probability responses from the 1992 and 1998 waves of the HRS on the left hand side and a set of demographic characteristics (age, sex, race and education) on the right hand side. The regressions are estimated on two different samples: (a) the full sample of respondents who responded to a given question with either

an exact or focal answer and (b) the subsample of respondents who gave an “exact” (i.e., non-focal) answer. Predicted values for each regression are then sorted into centile groups. Within each group, the value of the responses of those who gave focal answers (0.0, 0.5 and 1.0) is regressed on the predicted probability. If the average value of the focal answers behaves in the same way as the exact answers, a plot of the predicted focal answer vs. the average predicted probability within the centile group should fall along a 45 degree line.

These plots are shown relative to a 45 degree line in the matrix of graphs in Figure 4 for nine questions from 1992 in Panels A and B and sixteen questions from 1998 in Panels C and D. The plotted lines are very close to the 45 degree line in Panels A and C, where the prediction equations are based on the full sample. While deviations from the 45 degree line are more prominent when the prediction equations are based only on the subsample of respondents who gave exact answers, all are positively sloped and there is no regular pattern to the deviations. We conclude that focal answers to subjective probability questions contain similar information to that contained in exact answers when averaged across respondents.

A more direct test of the modal choice hypothesis is to see whether the propensity to give particular kinds of answers follow the patterns depicted in Figure 3. In order to perform this test, we wish to estimate the relationship

$$(6) \quad \Pr(\text{Answer of Type } i) = f_i(p^*(x)) + u,$$

where i denotes “exact answer” or focal answers of “0”, “50-50” or “100,” x is a set of covariates that influence the level of p^* and u is an error term which is independent of x .

According to the modal choice hypothesis, as p^* increases we expect to find that the

probability of an exact answer or the probability of a “50-50” answer follows an inverted U-shaped curve, increasing to a maximum at $p^* = 0.5$ and then decreasing. We also expect to find that the probability of an answer of “0” is a monotonically decreasing function of p^* while the probability of an answer of “100” is monotonically increasing.

It is important to note that our theory suggests that we may have a serious identification problem in testing this model. Specifically, in order to obtain unbiased estimates of the $f_i()$ functions in (7) we want to vary p^* independently of σ_δ . However, as discussed earlier, for most of the probability questions in the HRS it is quite likely that variables such as age, race, sex, health and education which affect the level of a probability are also correlated with the precision of an individual’s probability beliefs. Fortunately, one of the questions – the warm-up question asking respondents to give the probability that tomorrow will be sunny¹⁵ – allows us to vary p^* by using information location, given by a primary sampling unit (psu) indicator, and month of interview to construct a measure of the average value of the probability of a sunny day within each month-psu cell. We use all cells with at least three responses. This measure produces a very wide range of average probabilities which presumably vary independently of average values of σ_δ .

The “sunny” question was asked in HRS 1994 of persons aged 53-63 and in AHEAD 1993 of persons age 70 and over and in AHEAD 1995 of persons age 72 and over. For each of these three questions, we estimate non-parametric lowess (robust locally weighted regression) functions to test for the hypothesized shapes of $f_{exact}()$, $f_{“50-50”}()$, $f_{“0”}()$, and $f_{“100”}()$. The estimated curves are shown graphically in four

panels in Figure 5. Panel A depicts the fraction of exact answers as a function of the mean subjective probability that tomorrow will be sunny in a given month-psu cell. Similarly, Panels B-D present estimated functions for the fraction of “50-“50,” “0” and “100” answers, respectively.

The estimated functions presented in the four panels of Figure 5 conform almost exactly to the patterns predicted by the modal choice hypothesis shown in Figure 3. In Panel A, the function describing the fraction of respondents giving an exact answer has inverted U-shape with a maximum near $p^* = 0.5$ for each of the three “sunny” questions asked in HRS-1994, AHEAD-1993 and AHEAD-1995. The curves for the two AHEAD questions are virtually coincident while the curve for the HRS respondents lies above those of the AHEAD respondents, indicating that the relatively younger HRS respondents have more precise probability beliefs. In Panel B, as predicted, the fraction giving “50-50” answers also has an inverted U-shape with a maximum near $p^* = 0.5$. The curves based on all three questions are virtually coincident, suggesting the absence of age variation in “epistemic uncertainty.” In Panel C, the fraction giving a focal answer of “0” is a monotonically decreasing function of p^* with a higher fraction of AHEAD respondents giving such answers. Finally, in Panel D we see that the fraction of focal answers of “100” are a monotonically increasing function of p^* with little age variation.

The evidence presented in this section provides strong support for our hypothesis that individuals who give focal answers to survey questions on probability beliefs have less precise beliefs about these probabilities than do individuals who give non-focal or “exact” answers. As a measure of the degree of precision of an individual’s probability

¹⁵ See Table 1 for the text of this question.

beliefs, we utilize his or her responses to a large number of probability questions asked in the HRS to construct an index of the propensity of individuals to give exact answers to such questions calculated as the fraction of questions answered by each individual to which a non-focal answer is given. This index of the propensity to give exact answers is assumed to provide a rough indication of the precision of the individual's probability beliefs. For brevity, we refer to this index as the "fraction of exact answers" or "FEA". In terms of the theory of survey response presented earlier in this section, we think of higher values of FEA as inversely related to the value of the individual's degree of uncertainty, σ_δ , averaged across all the questions he or she answers. According to the theory of uncertainty aversion presented above in Section 4, higher values of FEA are hypothesized to be associated with less risk averse behavior.

The distribution of FEA is shown by the histogram in Figure 6 for a sample of 12,339 individuals ranging in age from 51 to over 100 who were surveyed in the 1998 wave of the HRS. On average, only 31 percent of the answers given by these individuals were exact. Equivalently, more than two-thirds of responses were focal. Figure 6 shows that the fraction of exact answers has a wide dispersion, but it is noteworthy that 14 percent of individuals gave no exact answers.

The fraction of exact answers is very strongly and linearly negatively related to age and positively related to education, suggesting that the precision of probability beliefs is related to cognitive capacity. These relationships are shown in Figure 7 for respondents in HRS-1998 where the vertical axis measures the mean value of FEA by single year of age in Panel A and the mean value of FEA by single year of education in Panel B. As individuals age the propensity to give exact answers declines from about 45

percent at age 51 to less than 5 percent for respondents over the age of 100. As education increases, FEA increases from about 15 percent for persons with an elementary school education to about 40 percent for persons with a college degree or more. In our analysis of portfolio composition and growth in assets in the next section, we are careful to control for age, education and other cognitive measures in order to distinguish the effect of the precision of probabilistic thinking on economic behavior from the effects of general cognitive capacity, age and/or cohort.

6. Uncertainty and Wealth

In Section 5, we developed theory and evidence which link variations in the degree of uncertainty about personal probabilities across individuals to the propensity of individuals to give exact or focal answers to survey questions about their probability beliefs in the HRS. In this section, we address the empirical link between uncertainty and financial choices that we discussed theoretically in Section 4. In particular, we hypothesize that people who have more precise probabilistic beliefs, as measured by higher values of FEA, will tend to behave as if they are less risk averse than otherwise comparable individuals who are more prone to give focal answers.

To test our hypotheses about the impact of probabilistic thinking on savings and portfolio behavior, we use FEA and a similarly constructed index of the fraction of “50-50” answers in regression equations explaining the share of household assets held in risky assets in 1998 and the growth of assets between 1992 and 1998.¹⁶ According to the theory presented in Section 4, decreased uncertainty leads individuals to behave less risk aversely. Thus, we predict that individuals who have a higher propensity to give exact answers will hold a larger share of risky assets. We then examine the effect of FEA on

the growth of assets to determine whether increased uncertainty has a negative impact on economic status, either by leading individuals to choose portfolios with relatively low rates of return or because they have a lower propensity to save. (Unfortunately, we cannot distinguish between the effect on the rate of return, which is clearly related to the theory presented in this paper via risk aversion, and the savings propensity, whose relation to the theory is less clear.)

The first set of regressions, presented in Table 5, examines the determinants of the share of risky assets in the portfolios of 12,339 households in the 1998 wave of the HRS. Descriptive statistics for this sample are presented in Table 4. These households include 6,954 couple households, 5,318 single female households and 1,927 single male households.¹⁷ The dependent variable is defined as a ratio of the value of risky assets to total household gross worth, excluding the value of housing, where risky assets are defined as the sum of the values of (non-housing) real estate, business assets, stocks plus the full value of IRA accounts if the respondent reported that these accounts contain mostly stocks, one half the value of the IRA if it was reported that the IRA contained about equal amounts of stocks and interest-earning assets, and zero if the entire account was in interest-earning assets.

The independent variables of main interest for this paper are measures of the propensity of respondents to give exact answers to subjective probability questions in the HRS. For the analysis in Table 5, FEA and the fraction of “50-50” answers are measured using responses in HRS-1998 for up to 17 questions that were answered by

¹⁶ See Table 4 for the descriptive statistics of these samples.

¹⁷ There are 14,209 household level observations in HRS-1998. Of these, our estimation sample of 12,339 was obtained by excluding 1231 cases in which the share of risky assets could not be defined because the household had either zero or missing gross worth and 630 had missing values on focal answers.

each respondent.¹⁸ In preliminary analysis, we found little difference in the effects of focal answers for husbands and wives in couple households so, for simplicity, we use the average fractions of exact and focal answers for couple households in the regressions reported in this paper. Only the fraction of exact answers is used in the regressions reported in columns (1a) and (1b) of Table 5. In columns (2a) and (2b), we add the fraction of “50-50” answers. From Table 4, we see that, on average, 31.3 percent of answers are exact and 14.8 percent are “50-50”.

The other independent variables are intended as controls. Since the composition of a portfolio is likely to vary with its magnitude, we control for the log of net worth and also enter a dummy variable for non-positive net worth. We also control for basic demographics including age of household head, marital status (single male or single female relative to married couple), and race.¹⁹ We estimate the equations with and without controls for education and immediate and delayed word recall to help control for cognitive capacity. We do so, in part, to determine how sensitive the estimated effect of the propensity to give focal answers is to cognitive capacity. The first three columns (marked “a”) do not control for education and word recall and the second three columns (marked “b”) include these variables.

The results reported in Table 5 show that households with a higher propensity to give exact answers have a significantly larger fraction of risky assets in their portfolio, as predicted by the theory. The magnitude of this effect is moderate. Using the estimates

¹⁸ Skip patterns caused variations in the number of probability questions that were asked of respondents and there were refusals to some questions.

¹⁹ Age, education and cognition are averaged across husband and wife in couple households. In unreported analyses, we found no statistically significant effects of within-couple differences in these variables.

reported in column (1a), an increase in the propensity to give exact answers from one standard deviation below the mean to one standard deviation above the mean would increase the fraction of risky assets from 25.6 percent to 27.7 percent. These results are only slightly affected by controls for education and word recall and there is not much change in the effect of the propensity to give exact answers when the propensity to give a “50-50” answer is added to the model. Both log net worth and the existence of zero or negative net worth have extremely large and significant coefficients. None of the other demographic or cognitive control variables are significant.

The second set of regression of results uses a sample of 4174 households who were respondents in the original HRS cohort in 1992 who also responded to the fourth wave of HRS in 1998. These regressions estimate the effects of the propensity to give exact answers in all four waves of the HRS (1992, 1994, 1996, and 1998) on the annual rate of growth of household net worth between 1992 and 1998 (calculated by dividing the difference in log net worth in 1998 and 1992 by six). Note that these households who contained at least one individual aged 51-61 in 1992 are much younger (56.7, on average) than the HRS-1998 sample (68.1, on average) used in the regressions reported in Table 5. Descriptive statistics are given in Table 4. The design of these regressions is identical to those reported in Table 5.

On average, HRS households experienced 4.88 percent annual growth in net worth between 1992 and 1998. During this time the CPI rose at an annual rate of 3.78 percent, implying that the real annual rate of increase was 1.10 percent. The regressions in Table 6 show that the propensity to give exact answers has a very large and highly significant effect on the rate of growth of assets. Using the coefficient of 0.2023 on FEA

reported in Column (1a), an increase in FEA from one standard deviation below to one standard deviation above the mean implies an increase in the nominal annual growth rate of net worth from 1.7 percent to 8.0 percent in nominal terms or, in real terms, from -2.1 percent to 4.1 percent. In contrast to the results for risky assets discussed earlier, the addition of a control for education has a large impact on the estimated effect of focal propensity, cutting the coefficient size in half in column (1b) compared to column (1a). (The word recall variables remain insignificant.) Assuming that the column (1b) estimates are more realistic, the implied range of growth rates implied by increasing FEA from one standard deviation below to one standard deviation above the mean is 3.3 to 6.4 percent in nominal terms or 0.1 to 2.6 percent in real terms.

The large variation in the estimated effect of FEA on the rate of asset growth calls depending on whether or not education is controlled calls for further investigation. One possibility is that education is proxying for the household's permanent income. To assess this possibility, we used data from Social Security covered earnings histories to construct nine dummy variables indicating the decile ranking of HRS households in average lifetime covered earnings similar to the variables constructed by Venti and Wise (1999).²⁰ These lifetime earnings variables are available only for the 75 percent of HRS-1992 respondents who signed a consent form allowing their social security data to be linked to HRS.

We first added the earnings decile variables to the models explaining the fraction of risky assets. These results are reported in Appendix Table 5a. Because Social

²⁰ Details concerning the construction of this variable will be contained in an Appendix that has not yet been written. Note that the analysis of the restricted Social Security covered earnings data were conducted in a secure data facility at the University of Michigan under an agreement approved by the HRS Confidentiality Committee.

Security earnings are available only for the original HRS cohort born in 1931-41, there are only 4,365 households in Table 5a compared to the 12,339 households from all HRS cohorts that were used in the regressions in Table 5. To determine the sensitivity of our results to this sample restriction, the first four columns of Table 5a replicate the specification in Table 5 with this smaller and younger sample. The coefficients on FEA and almost all other variables are only trivially affected by this sample restriction. The next four columns add the dummy indicators of decile rank in lifetime earnings. These variables are individually and jointly insignificant and the coefficients on FEA are not significantly affected by the control on lifetime income.

We then examined the effects of controlling for lifetime income on the growth rate of assets in the regressions reported in Table 6a.²¹ The lifetime income indicators have a positive and statistically significant effect on the rate of asset growth. However, they have no effect on coefficients of FEA nor do they affect the coefficients of other variables, including log 1992 net worth. This suggests that the large reduction in the coefficient of FEA when education is controlled is not due to education serving as a proxy for permanent income, but is probably the result of the correlation between education and unmeasured components of cognitive capacity.

7. Summary and Conclusions

This paper was motivated by the question of how well older Americans will be able to take advantage of trends in both the private and public sectors which expand the scope of individual choice in financial decision making. We have not attempted to address this question in general. Rather, we have focused attention on one aspect of

²¹ The restriction to persons with Social Security data reduced the sample in Table 6a to 3,133 households compared with 4,365 households in Table 6.

decision making –probabilistic thinking – which plays a crucial role in economic models of saving decisions and portfolio choice. To our knowledge, this is the first attempt to provide empirical evidence on the relationship between financial behavior and probabilistic thinking for a nationally representative sample of households. We are able to do so because the Health and Retirement Study asks a large number of subjective probability questions to its respondents which, in effect, creates a psychometric test of probabilistic thinking for the more than 20,000 HRS respondents.

One of the most prominent features of responses to these probabilistic questions is the large proportion –about 60 percent– of answers for which the probability that the event in question will occur is reported to be zero, fifty-fifty or one hundred percent rather than a more exact answer such as nine percent or seventy percent. While there are other possible interpretations of such heaping on focal answers, we emphasize the possibility that they reflect the respondent’s uncertainty about the true value of the probability. We then refer to a large literature originating with the Ellsberg paradox (Ellsberg 1961) in which it is hypothesized that individuals display “uncertainty aversion” that is inconsistent with conventional subjective expected utility theory. We suggest a new theoretical approach to this issue in which we are able to deal with uncertainty within the conventional SEU framework and show, within this framework, that uncertainty aversion is a consequence of risk aversion. This, in turn, implies that individuals who are more uncertain would tend to choose less risky portfolios and perhaps receive lower returns. We also note that our theory has clear implications for endogenous learning, which may reduce uncertainty, but we do not pursue this point in this paper.

We then turn to an attempt to provide an explicit link between the theoretical concept of uncertainty and the characteristics of survey response to subjective probability questions. The first step in building this link is to propose a simple formal model of the determinants of an individual's subjective prior distribution of probabilities. This model is formulated as a heterogeneous probit model in which the heterogeneity term reflects an individual's uncertainty about the true probability. A graphical representation of these prior density functions for varying levels of the true probability and degrees of uncertainty is presented. After examining these prior density functions, we propose an empirical hypothesis to describe the determinants of whether a respondent gives a focal or exact answer to a probability question. We call this the "modal choice" hypothesis. According to this hypothesis, when asked to state a probability the respondent chooses the most likely probability, given his prior density, which is the mode of the distribution. This hypothesis has two testable empirical implications. First, it implies that when averaged across individuals the mean value of focal responses should vary in the same way as the mean value of exact responses. We find this to be broadly true in the HRS data. Second, the hypothesis implies that the probability of giving an exact (i.e., non-focal) answer to a probability question is a nonlinear, inverted U-shaped function of the level of the true probability. Using data from questions in three different waves of HRS about the likelihood that tomorrow will be sunny, the empirical relationship conforms to this prediction.

In the final section of the paper, we construct a measure of the propensity of an individual to give exact answers by calculating the fraction of all probability questions that he or she was asked that received a non-focal response. We then estimate the effect

of this propensity on the fraction of risky assets in a household's portfolio in 1998 and its effect on the rate of growth of assets from 1992 to 1998. The propensity to give exact answers is found to have a highly significant negative effect on both the fraction of risky assets and the rate of growth of net worth. The magnitude of the effect is modest on risky asset holding but very large on the rate of growth of assets.

We believe that this paper provides clear evidence that there is considerable heterogeneity in the precision of probabilistic thinking in the population and that more precise probabilistic thinking leads individuals to be willing to take more risks and to enjoy higher growth in wealth. These results provide some justification for fears that have been expressed about expanding the scope for choice through individual accounts because significant portions of the population will be unable to exploit the benefits of choice. However, it would be a mistake in our view to jump to policy conclusions too quickly. In particular, we believe that it is important to explore the degree to which individuals reduce their uncertainty through experience with financial management and, as a consequence, become better able to manage their own affairs for their own benefit. We hope to pursue this notion in future research by exploiting the longitudinal information in the HRS.

References

- Bassett, William F. and Robin L. Lumsdaine. 1999a. "Outlook, Outcomes and Optimism." Unpublished manuscript, Brown University.
- Bassett, William F. and Robin L. Lumsdaine. 1999b. "Probability Limits: Are Subjective Assessments Adequately Accurate?" Unpublished manuscript, Brown University.
- Camerer, Colin F. 1995. "Individual Decision Making." *The Handbook of Experimental Economics*. Princeton: Princeton University Press: pp. 587-703.
- Dominitz, Jeff and Charles F. Manski. 1999. "The Several Cultures of Research on Subjective Expectations," in J.P. Smith and R.J. Willis, eds. *Wealth, Work and Health: Innovations in Measurement in the Social Sciences. Essays in honor of F. Thomas Juster*. Ann Arbor: University of Michigan Press.
- Ellsberg, Daniel. 1961. "Risk, Ambiguity, and the Savage Axioms." *Quarterly Journal of Economics*, 75, pp. 643-669.
- Gilboa, Itshak and David Schmeidler. 1989. "Maxmin Expected Utility with a Non-unique Prior." *Journal of Mathematical Economics*, 18, pp. 141-153.
- Heckman, James J., and Robert J. Willis. "A Beta-Logistic Model for the Analysis of Sequential Labor Force Participation by Married Women." *Journal of Political Economy*, 85 (February 1977), pp. 27-58.
- Hurd, Michael and Kathleen McGarry. 1995. "Evaluation of the Subjective Probabilities of Survival in the Health and Retirement Study." *Journal of Human Resources*, Supplemental, 30, pp. S268-S292.
- Hurd, Michael, Daniel McFadden, and Li Gan. 1998. "Subjective Survival Curves and Life Cycle Behavior," in David A. Wise, ed., *Inquiries in the Economics of Aging*. Chicago: University of Chicago Press, pp. 259-305.
- Johnson, William R. 1978. "A Theory of Job Shopping." *Quarterly Journal of Economics*, 92, pp. 261-277.
- Juster, F.T. and Richard Suzman. 1995. "An Overview of the Health and Retirement Study," *The Journal of Human Resources*, Supplemental, 30, S7-S56.
- Knight, Frank H. 1921. *Risk, Uncertainty and Profit*. Boston: Houghton Mifflin.
- Lillard, Lee and Robert J. Willis. "Dynamic Aspects of Earnings Mobility." *Econometrica*, 46 (September 1978), pp. 985-1012.

Mehra, R. and Prescott, E. 1985. "The Equity Premium: A Puzzle." *Journal of Monetary Economics*, 15:145-61.

Poterba, James M. and David Wise. 1998. "Individual Financial Decisions in Retirement Savings Plans and the Provision of Resources for Retirement," in M. Feldstein, ed., *Privatizing Social Security*. Chicago: University of Chicago Press, pp. 363-402.

Ramsey, Frank P. 1926. "Truth and Probability." In R.B. Braithwaite, *The Foundations of Mathematics and Other Logical Essays*. New York: Harcourt Brace, 1931.

Rothschild, Michael and Joseph Stiglitz. "Increasing Risk I: A Definition." *Journal of Economic Theory*, 2 (September 1970), pp. 225-243.

Savage, Leonard J. 1954. *The Foundations of Statistics*. New York: Wiley.

Scheeweiss, Hans. 1999. "Resolving the Ellsberg Paradox by Assuming that People Evaluate Repetitive Sampling," in U. Leopold-Wildburger, G. Feichtinger, and K. P. Kistner, eds., *Modelling and Decisions in Economics: Essays in Honor of Franz Ferschl*. Heidelberg: Physica-Verlag, pp.83-95.

Smith, James P. 1999a. "Inheritances and Bequests," in J.P. Smith and R.J. Willis eds., *Wealth, Work and Health: Innovation in Measurement in the Social Sciences*. Ann Arbor: University of Michigan Press, pp. 121-149.

Smith, James P. 1999b. "Healthy Bodies and Thick Wallets: The Dual Relation Between Health and Economic Status," *Journal of Economic Perspectives*, 13 (2), 145-166

Soldo, Beth J., Michael D. Hurd, Willard L. Rodgers, and Robert B. Wallace. 1997. "Asset and Health Dynamics Among the Oldest Old: An Overview of the AHEAD Study," *The Journals of Gerontology: Social Sciences*, Vol. 52B (Special Issue), 1-20.

Starmer, Chris. "Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk." *Journal of Economic Literature*, 38 (June 2000), pp. 332-382.

Willis, Robert J. "Theory Confronts Data: How the HRS is Shaped by the Economics of Aging and How the Economics of Aging Will be shaped by the HRS," *Labour Economics*, 6 (June 1999), pp. 119-45.

Table 1. Illustration of Probability Questions in HRS

1. Basic Introduction

Next I have some questions about how likely you think various events might be. When I ask a question I'd like for you to give me a number from 0 to 100, where "0" means that you think there is absolutely no chance, and "100" means that you think the event is absolutely sure to happen.

2. Sunny Day Warm Up Question

Let's try an example and start with the weather. What do you think are the chances that it will be sunny tomorrow?("0" means 'a 0 percent chance of sunny weather.' "100" means 'a 100 percent chance of sunny weather.' And you can say any number from 0 to 100.)

3. Types of Probability Questions

A. General Events

Social Security to Be Less Generous

How about the chances that Congress will change Social Security so that it becomes less generous than now?

Double Digit Inflation

And how about the chances that the U.S. economy will experience double-digit inflation sometime during the next 10 years or so?

B. Events with Personal Information

Survival Probability

(What is the percent chance) that you will live to be 75 or more?

Income Will Keep Up with Inflation

What do you think are the chances that your income will keep up with inflation for the next five years?

C. Events with Personal Control

Will leave inheritance

And what are the chances that you (and your (husband/wife/partner)) will leave an inheritance totaling \$10,000 or more?

Will Work at Age 62

IF R IS WORKING FOR SOMEONE ELSE (NOT SELF-EMPLOYED): Thinking about work in general and not just your present job, what do you think the chances are that you will be working OTHERWISE: What do you think the chances are that you will be working full-time after you reach age 62?

**Table 2. Fraction of Probability Questions Heaped and Refused
HRS 1998, Full Sample**

| Variable | Fraction of Heaped Answers | | | | Refusal Rate | Number of Cases |
|--|----------------------------|-------------|-------------|-------------|-----------------|--------------------|
| | to 0 | to 100 | 0 or 100 | 50-50 | | |
| <i>General Events</i> | | | | | | |
| Soc.Security To Be Less Genrous | 0.08 | 0.17 | 0.25 | 0.22 | 0.05 | 4,608 |
| Major Economic Depression | 0.14 | 0.05 | 0.19 | 0.26 | 0.09 | 4,605 |
| 2-Digit Inflation in the U.S. | 0.10 | 0.07 | 0.17 | 0.32 | 0.14 | 18,884 |
| Subtotal | 0.10 | 0.08 | 0.19 | 0.29 | 0.12 | 28,097 |
| <i>Events With Personal Information</i> | | | | | | |
| Live To Be 75 | 0.05 | 0.21 | 0.26 | 0.25 | 0.05 | 9,905 |
| Live To Be 85 | 0.10 | 0.10 | 0.19 | 0.22 | 0.07 | 9,441 |
| Receive Financial Help | 0.74 | 0.02 | 0.75 | 0.04 | 0.02 | 18,884 |
| Receive Inheritance | 0.75 | 0.06 | 0.81 | 0.05 | 0.01 | 18,884 |
| Income To Keep Up with Inflation | 0.17 | 0.11 | 0.28 | 0.24 | 0.06 | 18,884 |
| Lose Job | 0.57 | 0.02 | 0.60 | 0.12 | 0.02 | 6,014 |
| Find Job (In Case Losing One) | 0.20 | 0.19 | 0.40 | 0.15 | 0.02 | 6,014 |
| Find Job (If Looking For One) | 0.11 | 0.14 | 0.25 | 0.20 | 0.00 | 844 |
| Health to Limit Work | 0.16 | 0.04 | 0.21 | 0.34 | 0.05 | 7,595 |
| Subtotal | 0.40 | 0.09 | 0.49 | 0.16 | 0.04 | 96,465 |

Table 2 (continued)

| Variable | Fraction of Heaped Answers | | | | Refusal Rate | Number of Cases |
|--|----------------------------|-------------|-------------|-------------|--------------|-----------------|
| | to 0 | to 100 | 0 or 100 | 50-50 | | |
| <i>Events With Personal Control</i> | | | | | | |
| Go To Nursing Home | 0.56 | 0.01 | 0.57 | 0.15 | 0.07 | 11,744 |
| Move in 2 Years | 0.66 | 0.04 | 0.70 | 0.11 | 0.02 | 11,951 |
| Give Financial Help | 0.43 | 0.14 | 0.57 | 0.13 | 0.02 | 18,884 |
| Leave Any Inheritance | 0.65 | 0.10 | 0.76 | 0.08 | 0.11 | 3,907 |
| Leave Inheritance >\$10,000 | 0.18 | 0.47 | 0.65 | 0.09 | 0.03 | 18,884 |
| Leave Inheritance >\$100,000 | 0.31 | 0.29 | 0.60 | 0.10 | 0.05 | 15,541 |
| Work Sometimes in the Future | 0.77 | 0.02 | 0.80 | 0.05 | 0.01 | 11,237 |
| Work at Age 62 | 0.27 | 0.20 | 0.46 | 0.14 | 0.01 | 5,433 |
| Work at Age 65 | 0.25 | 0.11 | 0.35 | 0.16 | 0.02 | 4,042 |
| Subtotal | 0.44 | 0.18 | 0.62 | 0.11 | 0.03 | 101,623 |
| Total | 0.39 | 0.13 | 0.52 | 0.15 | 0.05 | 226,185 |

**Table 3. Effect of Uncertainty on the Distribution of Returns
when n Balls are Drawn from Urn I or Urn II.**

| Returns from n draws | Probability if n balls are drawn from Urn I | Probability if n balls are drawn from Urn II | Difference in probability between Urn II and Urn I ($n > 1$) |
|---|---|--|---|
| <i>$\\$n$</i> | \bar{p}^n | $E(p^n)$ | <i>positive</i> |
| <i>$\\$1$ to $\\$n-1$</i> | $1 - \bar{p}^n - \bar{q}^n$ | $E(1 - p^n - q^n)$ | <i>negative</i> |
| <i>$\\$0$</i> | \bar{q}^n | $E(q^n)$ | <i>positive</i> |
| Expected Value | $n\bar{p}$ | $n\bar{p}$ | <i>zero</i> |

Table 4. Descriptive Statistics for the Samples used in Tables 5 and 6

| | Table 4. Determinants of Fraction of Total Assets which are Risky | | Table 5. Determinants of Average Yearly Percentage Change in Net Worth | |
|--|--|-------------------------------|---|-------------------------------|
| | 12339 | | 4174 | |
| Number of Observations | Mean | Standard Deviation | Mean | Standard Deviation |
| Dependent Variable | 0.2674 | 0.3371 | 0.0488 | 0.2373 |
| Fraction of Exact Answers | 0.3127 | 0.2183 | 0.4185 | 0.1552 |
| Fraction of Answers at .5 | 0.1475 | 0.1218 | 0.1884 | 0.0827 |
| Log of Net Worth | 10.2178 | 3.2532 | 10.8156 | 1.8332 |
| 1998 Zero or Negative Net Worth | 0.0541 | 0.2263 | | |
| Single Female | 0.3279 | 0.4695 | 0.1770 | 0.3818 |
| Single Male | 0.1275 | 0.3335 | 0.0745 | 0.2626 |
| Household Age | 68.0715 | 10.4612 | 56.7053 | 4.7905 |
| Hispanic | 0.0512 | 0.2205 | 0.0630 | 0.2430 |
| Black | 0.1282 | 0.3343 | 0.1222 | 0.3275 |
| Education | 12.1850 | 2.9737 | 12.6174 | 2.6673 |
| Immediate Word Recall | 5.3768 | 1.8125 | 7.7522 | 2.2281 |
| Delayed Word Recall | 4.3212 | 2.0670 | 5.7183 | 2.4321 |

Note: Variables used in Table 4 use 1998 data, those used in Table 5 use 1992 data.

Table 5. Determinants of Share of Risky Assets in Portfolio in 1998*

| | Share of Total Assets** which are Risky | | | |
|-------------------------------------|---|-----------------------------|-----------------------------|-----------------------------|
| | (1a) | (2a) | (1b) | (2b) |
| Fraction of Exact Answers | 0.0536 (4.215) | 0.0498 (3.848) | 0.0470 (3.622) | 0.0441 (3.334) |
| Fraction of Answers at .5 | | -0.0216 (-0.997) | | -0.0244 (-1.125) |
| Log of 1998 Net Worth** | 0.0817 (63.614) | 0.0818 (63.472) | 0.0804 (58.117) | 0.0805 (58.078) |
| Zero or Negative Net Worth** | 0.6330 (36.508) | 0.6336 (36.521) | 0.6227 (34.871) | 0.6232 (34.888) |
| Single Female | 0.1264 (2.131) | 0.0126 (2.117) | 0.0092 (1.512) | 0.0090 (1.486) |
| Single Male | 0.0072 (0.908) | 0.0069 (0.0875) | 0.0064 (0.808) | 0.0061 (0.769) |
| Household Age, 1998 | -0.0034 (-12.826) | -0.0035 (-12.622) | -0.0032 (-10.999) | -0.0033 (-10.955) |
| Hispanic | 0.0043 (0.367) | 0.0038 (0.319) | 0.0097 (0.806) | 0.0092 (0.763) |
| Black | 0.0027 (0.346) | 0.0023 (0.290) | 0.0045 (0.570) | 0.0041 (0.512) |
| Education | | | 0.0016 (1.586) | 0.0017 (1.623) |
| Immediate Word Recall | | | 0.0026 (1.056) | 0.0026 (1.060) |
| Delayed Word Recall | | | 0.0003 (0.135) | 0.0003 (0.153) |
| Constant | -0.3896 (-16.181) | -0.3815 (-15.028) | -0.4256 (-15.417) | -0.4172 (-14.593) |
| Number of Observations | 12,339 | 12,339 | 12,339 | 12,339 |
| Adjusted R-Squared | 0.3119 | 0.3119 | 0.3121 | 0.3121 |

**t-values in parentheses.

**Without housing equity.

Table 6. Determinants of Growth of Assets from 1992 to 1998*

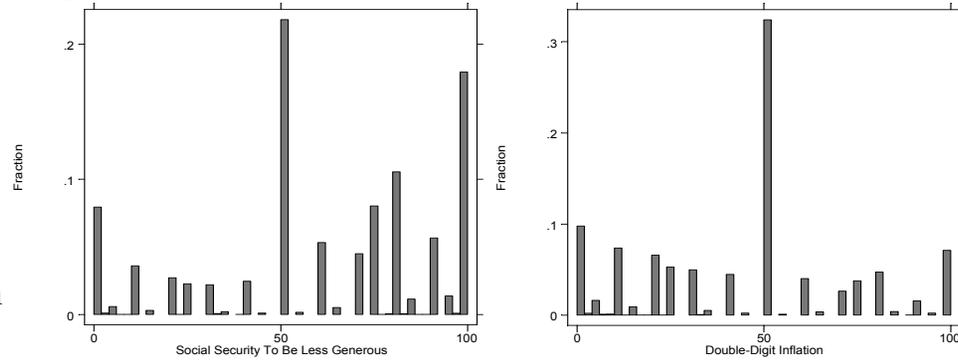
| | Yearly Percent Change in Net Worth** | | | |
|----------------------------------|---|-----------------------------|-----------------------------|-----------------------------|
| | (1a) | (2a) | (1b) | (2b) |
| Fraction of Exact Answers | 0.2023 (8.828) | 0.2111 (8.847) | 0.1000 (4.199) | 0.1117 (4.527) |
| Fraction of Answers at .5 | | 0.0572 (1.327) | | 0.0766 (1.804) |
| Log of 1992 Net Worth** | -0.0479 (-23.692) | -0.0480 (-23.723) | -0.0584 (-27.454) | -0.0585 (-27.508) |
| Single Female | -0.0749 (-7.921) | -0.0747 (-7.899) | -0.0960 (-10.094) | -0.0958 (-10.078) |
| Single Male | -0.0343 (-2.603) | -0.0334 (-2.525) | -0.0534 (-4.038) | -0.0521 (-3.932) |
| Household Age, 1992 | -0.0012 (-1.695) | -0.0011 (-1.564) | -0.0003 (-0.378) | -0.0002 (-0.210) |
| Hispanic | -0.1274 (-8.809) | -0.1262 (-8.710) | -0.0696 (-4.702) | -0.0678 (-4.575) |
| Black | -0.0856 (-7.864) | -0.0841 (-7.693) | -0.0775 (-7.179) | -0.0756 (-6.971) |
| Education | | | 0.0208 (13.316) | 0.0209 (13.346) |
| Immediate Word Recall | | | 0.0004 (0.185) | 0.0005 (0.231) |
| Delayed Word Recall | | | 0.0016 (0.784) | 0.0016 (0.744) |
| Constant | 0.5868 (12.355) | 0.5676 (11.435) | 0.4132 (8.412) | 0.3871 (7.562) |
| Number of Observations | 4,174 | 4,174 | 4,131 | 4,131 |
| Adjusted R-Squared | 0.1374 | 0.1376 | 0.1751 | 0.1756 |

* t-values in parentheses. ** Without housing equity.

Figure 1. Histograms of Probability Responses in HRS

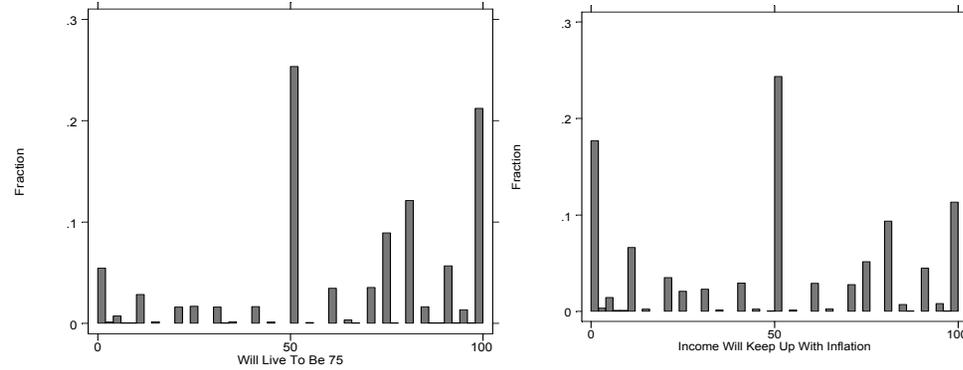
A. General Events

Social Security less generous
Double digit inflation



B. Events with Personal Information

Survival to 75
Income increase faster than inflation



C. Events with Personal Control

Leave inheritance
Work at age 62

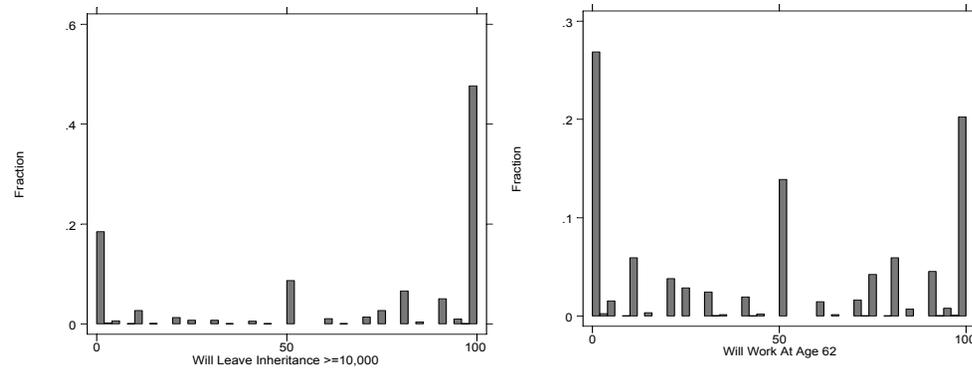


Figure 2. Distribution of Prior Probabilities with Varying Expected Values and Precision.

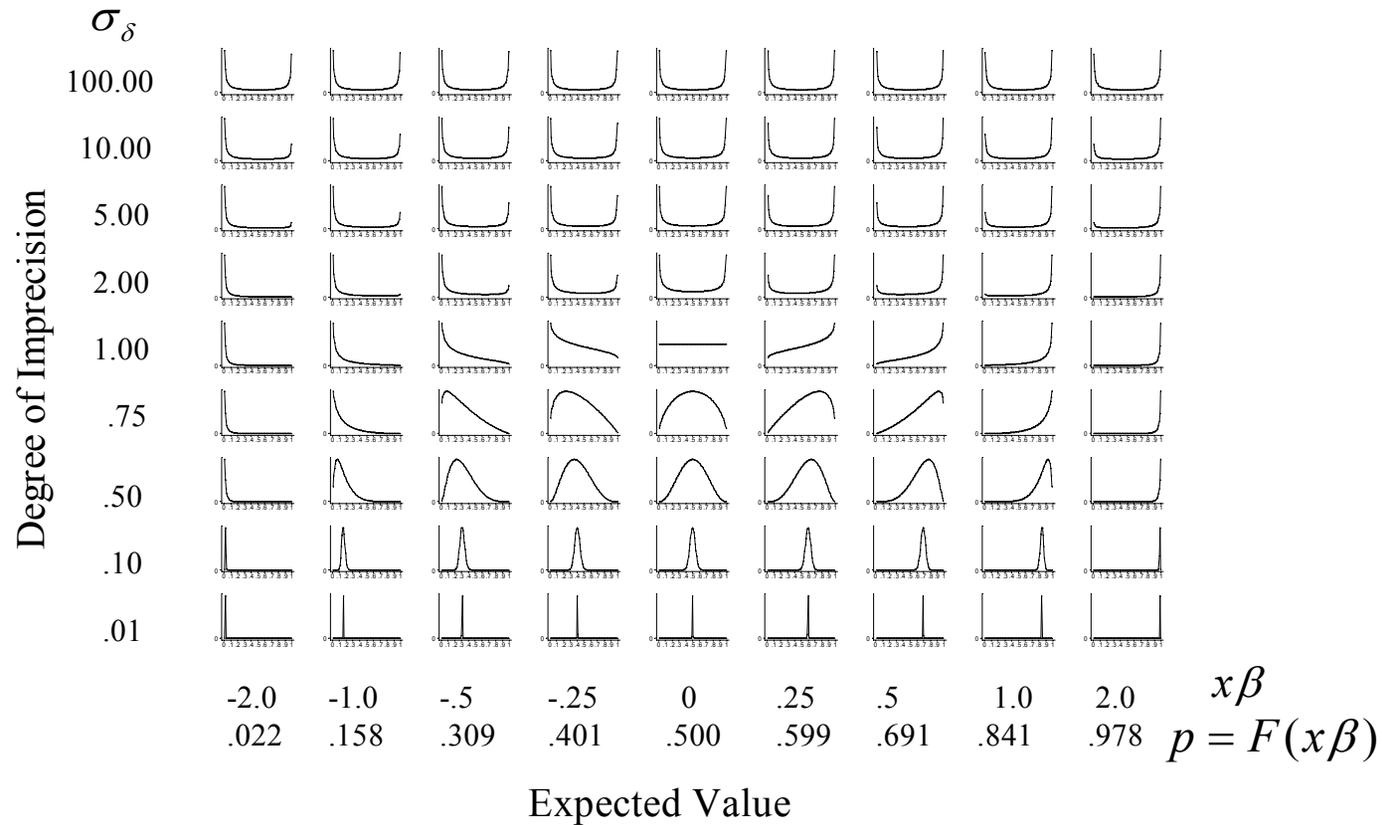


Figure 3. Relation Between Prior Distribution and Heaping under Modal Choice Hypothesis.

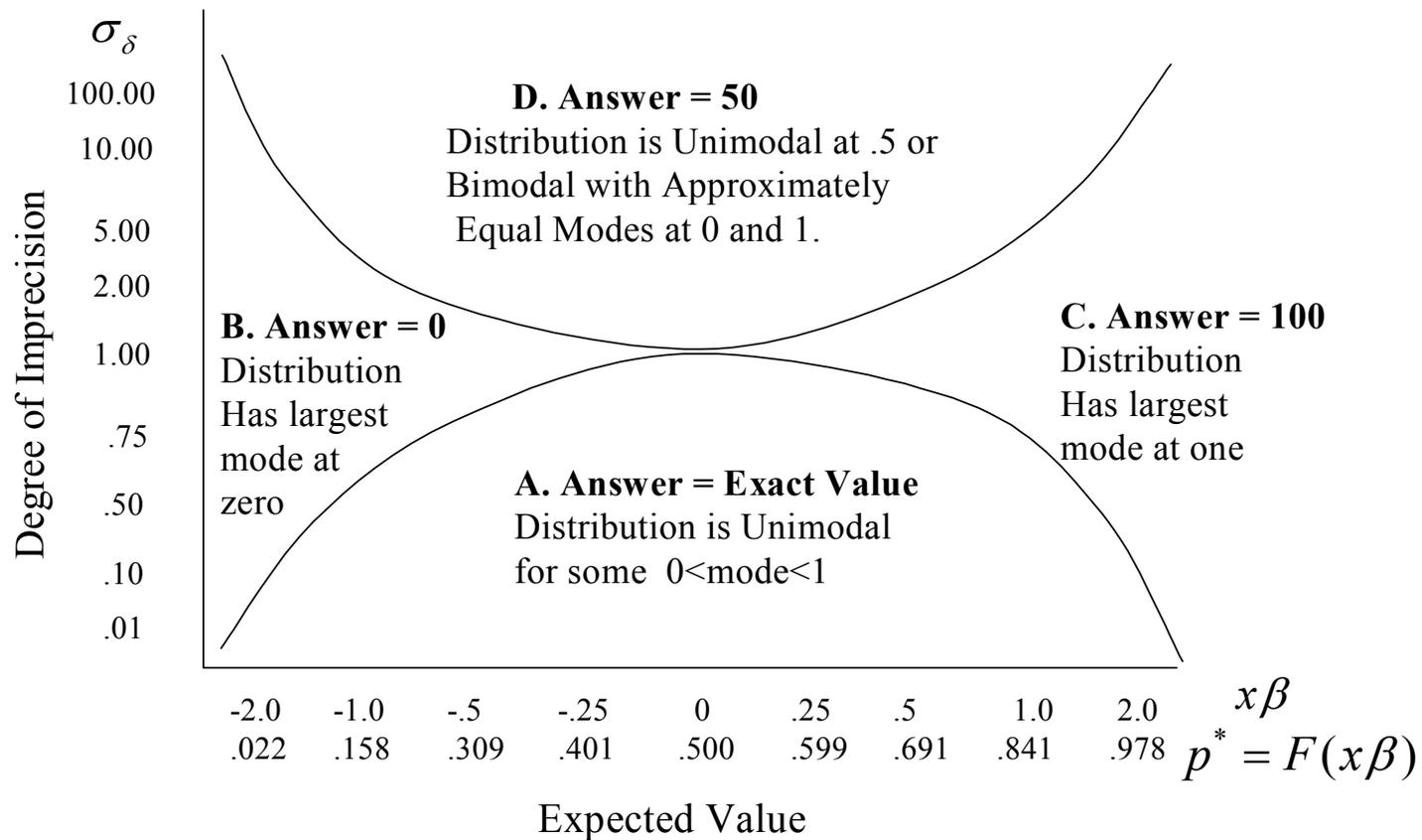
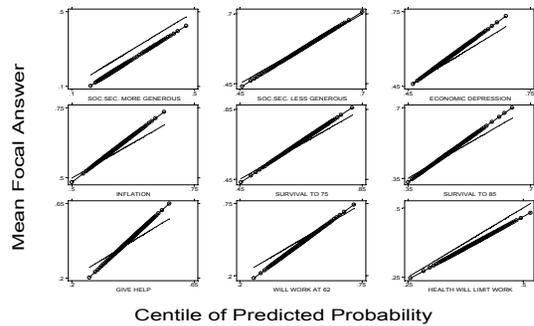
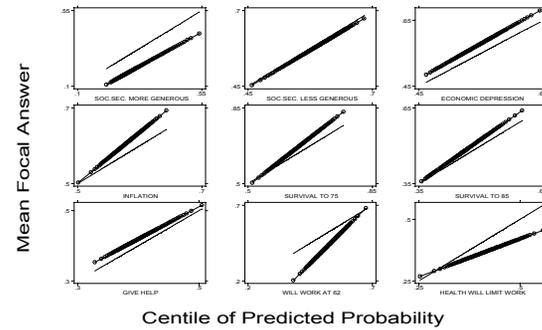


Figure 4. Mean Focal Answers vs. Predicted Probabilities

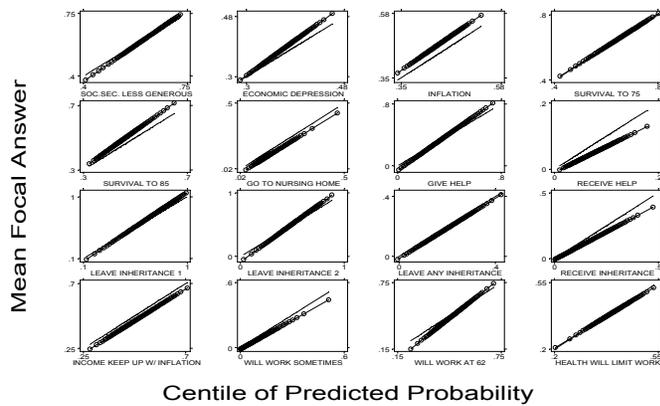
A. Prediction Based on Full Sample, 1992



B. Prediction Based on Sample with Exact Answers, 1992



C. Prediction Based on Full Sample, 1998



D. Prediction Based on Sample with Exact Answers, 1998

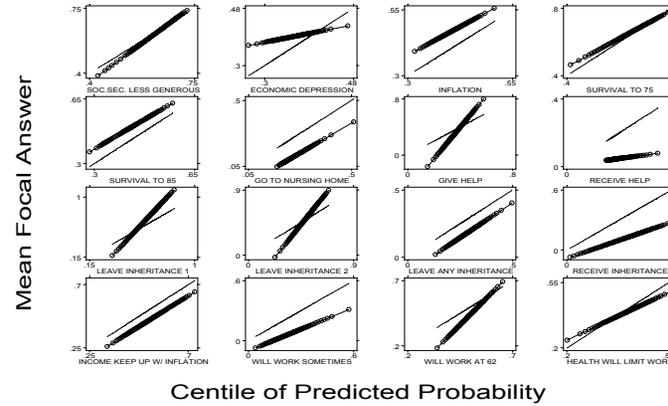


Figure 5. Analysis of Focal and Exact Answers to Sunny Question Using Interview Responses Grouped by Month and PSU.

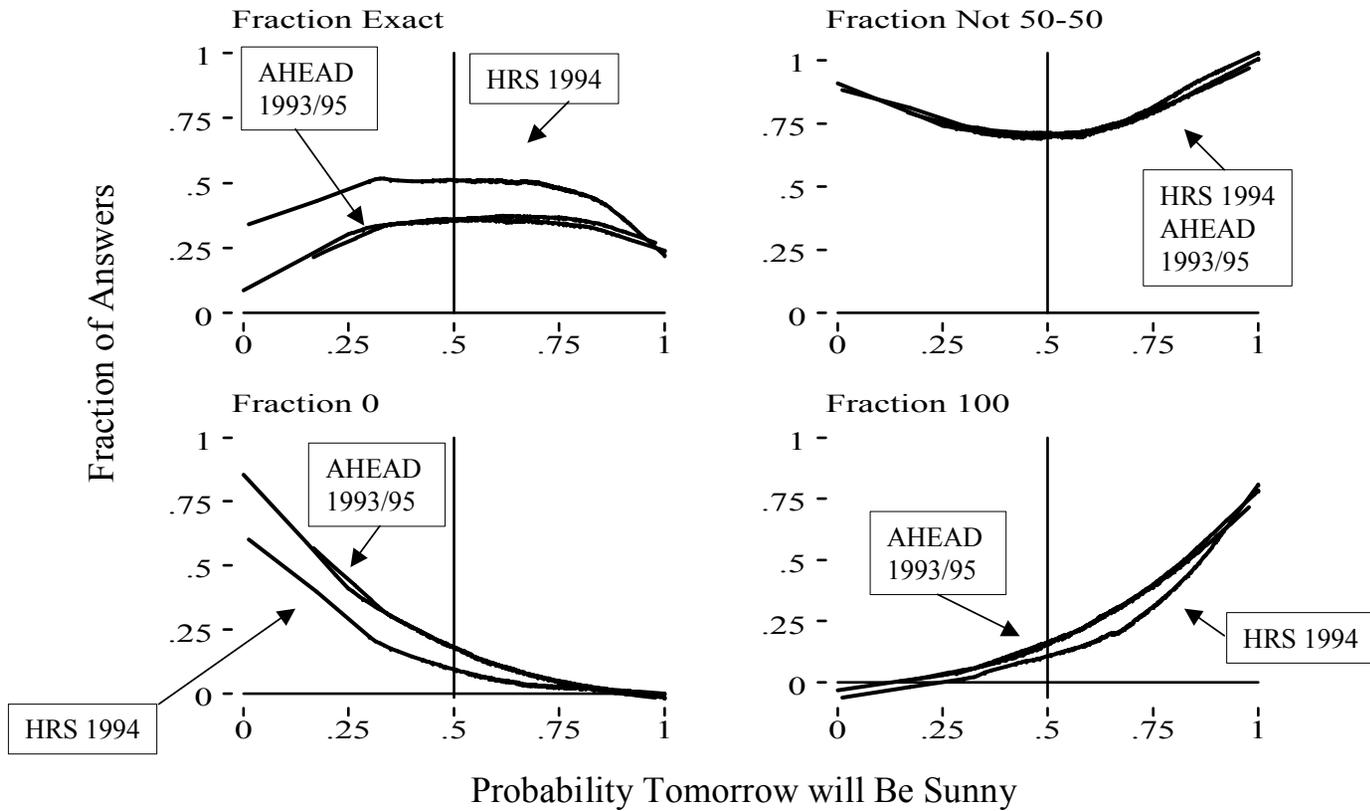


Figure 6. Distribution of Fraction of Exact Answers to Subjective Probability Questions in HRS-1998.

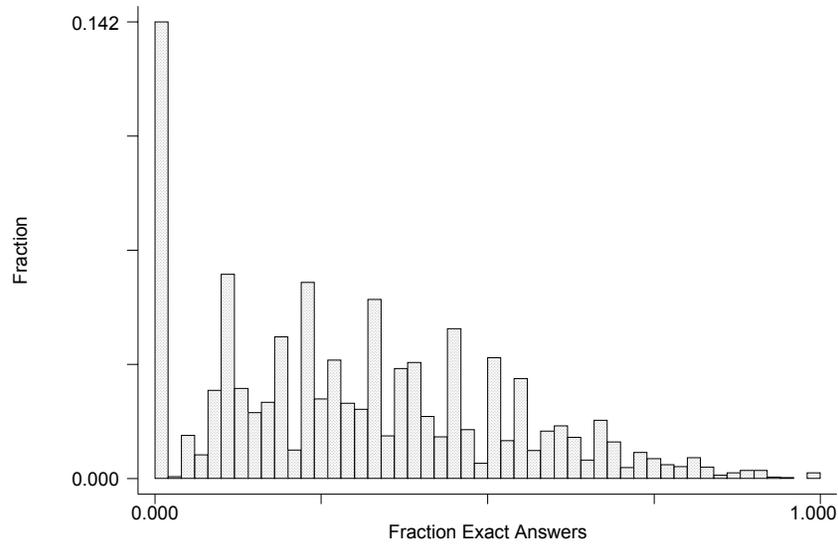


Figure 7. Propensity to Give Exact Answers to Subjective Probability Questions by Age and Education

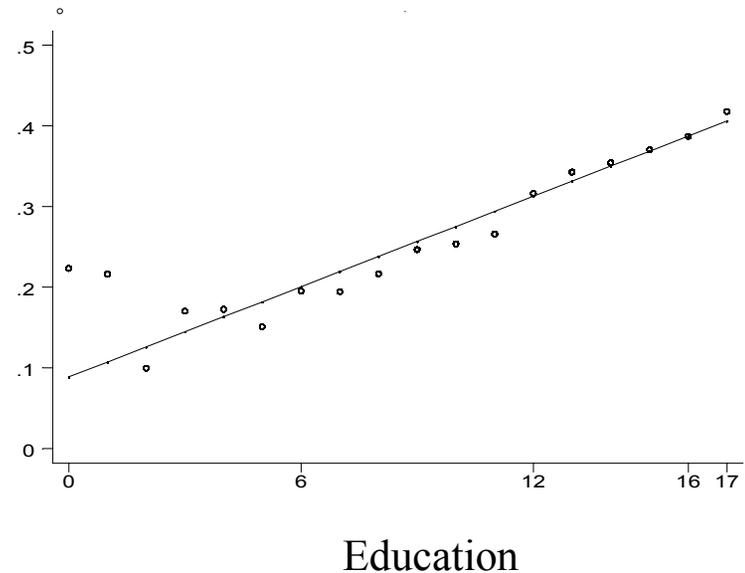
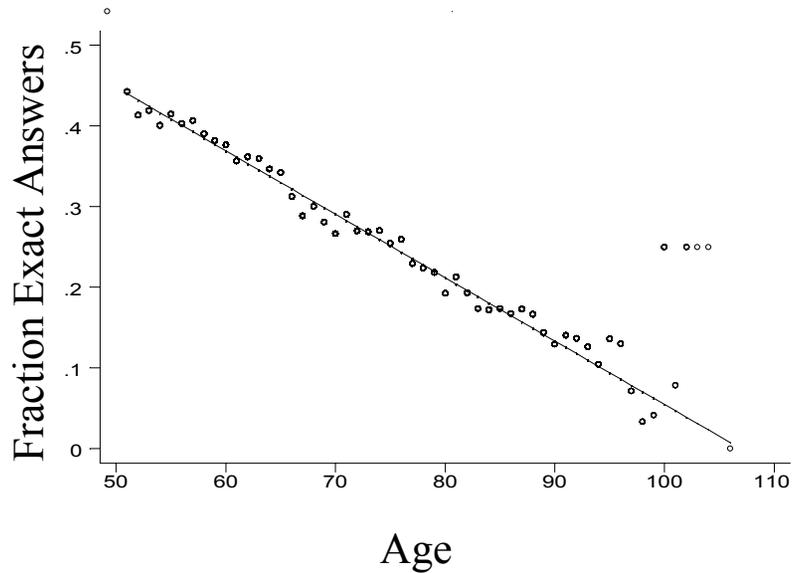


Table 5A. Determinants of Share of Risky Assets in Portfolio in 1998*
Controlling for SSA Lifetime Earnings

| | Share of Total Assets** which are Risky | | | | | | | |
|----------------------------|---|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|----------------------------|
| | (3a) | (4a) | (3b) | (4b) | (3c) | (4c) | (3d) | (4d) |
| Fraction of Exact Answers | 0.0696 (3.486) | 0.0678 (3.322) | 0.0644 (3.133) | 0.0622 (2.957) | 0.0706 (3.524) | 0.0686 (3.349) | 0.0654 (3.175) | 0.0630 (2.988) |
| Fraction of Answers at .5 | | -0.0150 (-0.420) | | -0.0177 (-0.493) | | -0.0163 (-0.452) | | -0.0189 (-0.527) |
| Log of 1998 Net Worth** | 0.0909 (41.659) | 0.0909 (41.584) | 0.0902 (38.391) | 0.0902 (38.366) | 0.0907 (39.517) | 0.0908 (39.480) | 0.0901 (37.044) | 0.0901 (37.034) |
| Zero or Negative N.Worth** | 0.7249 (25.740) | 0.7252 (25.740) | 0.7194 (24.765) | 0.7196 (24.767) | 0.7239 (25.124) | 0.7241 (25.126) | 0.7191 (24.336) | 0.7192 (24.337) |
| Single Female | 0.0157 (1.154) | 0.0156 (1.503) | 0.0125 (1.167) | 0.0123 (1.149) | 0.0169 (1.510) | 0.0169 (1.509) | 0.0129 (1.114) | 0.0128 (1.107) |
| Single Male | 0.0305 (2.239) | 0.0303 (2.221) | 0.02831 (2.071) | 0.0280 (2.048) | 0.0304 (2.202) | 0.0302 (2.183) | 0.0278 (2.001) | 0.0275 (1.978) |
| Household Age, 1998 | -0.0034 (-4.138) | -0.0034 (-4.150) | -0.0033 (-4.024) | -0.0034 (-4.051) | -0.0033 (-4.091) | -0.0034 (-4.109) | -0.0033 (-3.963) | -0.0033 (-3.996) |
| Hispanic | 0.0202 (1.204) | 0.0197 (1.174) | 0.0260 (1.501) | 0.0256 (1.473) | 0.0213 (1.254) | 0.0209 (1.225) | 0.0268 (1.532) | 0.0264 (1.505) |
| Black | 0.0031 (0.242) | 0.0027 (0.209) | 0.0036 (0.277) | 0.0031 (0.240) | 0.0043 (0.338) | 0.0039 (0.303) | 0.0047 (0.368) | 0.0043 (0.329) |
| Education | | | 0.0012 (0.641) | 0.0012 (0.655) | | | 0.0012 (0.660) | 0.0012 (0.675) |
| Immediate Word Recall | | | 0.0106 (2.491) | 0.0106 (2.499) | | | 0.0105 (2.469) | 0.0105 (2.477) |
| Delayed Word Recall | | | -0.0077 (-2.048) | -0.0077 (-2.047) | | | -0.0075 (-1.995) | -0.0075 (-1.995) |
| 2nd Earnings Decile | | | | | -0.0194 (-0.833) | -0.0189 (-0.858) | -0.0180 (-0.818) | -0.0174 (-0.789) |
| 3rd Earnings Decile | | | | | -0.0284 (-1.323) | -0.0280 (-1.301) | -0.0278 (-1.295) | -0.0273 (-1.270) |
| 4th Earnings Decile | | | | | -0.0030 (-0.139) | -0.0022 (-0.103) | -0.0030 (-0.139) | -0.0021 (-0.098) |
| 5th Earnings Decile | | | | | -0.0161 (-0.750) | -0.0157 (-0.730) | -0.0165 (-0.766) | -0.0160 (-0.743) |
| 6th Earnings Decile | | | | | -0.0119 (-0.552) | -0.0112 (-0.518) | -0.0120 (-0.558) | -0.0112 (-0.520) |
| 7th Earnings Decile | | | | | 0.0036 (0.167) | 0.0044 (0.201) | 0.0037 (0.171) | 0.0046 (0.211) |
| 8th Earnings Decile | | | | | -0.0085 (-0.386) | -0.0076 (-0.353) | -0.0101 (-0.460) | -0.0093 (-0.422) |
| 9th Earnings Decile | | | | | -0.0042 (-0.191) | -0.0037 (-0.168) | -0.0061 (-0.274) | -0.0055 (-0.247) |
| 10th Earnings Decile | | | | | -0.0206 (-0.922) | -0.0199 (-0.890) | -0.0223 (-0.999) | -0.0216 (-0.963) |
| Constant | -0.4978 (-8.625) | -0.4916 (-8.249) | -0.5305 (-8.759) | -0.5237 (-8.425) | -0.4877 (-8.058) | -0.4816 (-7.761) | -0.5223 (-8.233) | -0.5156 (-7.965) |
| Number of Observations | 4,365 | 4,365 | 4,365 | 4,365 | 4,365 | 4,365 | 4,365 | 4,365 |
| Adjusted R-Squared | 0.3509 | 0.3508 | 0.3515 | 0.3514 | 0.3504 | 0.3503 | 0.3510 | 0.3509 |

** t-values in parentheses.

**Without housing equity.

Table 6A. Determinants of Growth of Assets from 1992 to 1998*
Controlling for SSA Lifetime Earnings

| | Yearly Percent Change in Net Worth** | | | | | | | |
|---------------------------|--------------------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| | (3a) | (4a) | (3b) | (4b) | (3c) | (4c) | (3d) | (4d) |
| Fraction of Exact Answers | 0.2162 (8.206) | 0.2288 (8.362) | 0.1059 (3.849) | 0.1199 (4.220) | 0.1936 (7.404) | 0.2039 (7.500) | 0.0965 (3.532) | 0.1083 (3.833) |
| Fraction of Answers at .5 | | 0.0850 (1.696) | | 0.0961 (1.955) | | 0.0684 (1.379) | | 0.0799 (1.635) |
| Log of 1992 Net Worth** | -0.0474 (-20.465) | -0.0475 (-20.506) | -0.0575 (-23.706) | -0.0576 (-23.757) | -0.0531 (-22.469) | -0.0532 (-22.491) | -0.0615 (-25.068) | -0.0616 (-25.097) |
| Single Female | -0.0815 (-7.574) | -0.0814 (-7.563) | -0.1011 (-9.369) | -0.1010 (-9.358) | -0.0238 (-1.904) | -0.0243 (-1.940) | -0.0491 (-3.892) | -0.0497 (-3.938) |
| Single Male | -0.0512 (-3.427) | -0.0499 (-3.336) | -0.0680 (-4.561) | -0.0663 (-4.445) | -0.0167 (-1.099) | -0.0158 (-1.037) | -0.0372 (-2.439) | -0.0360 (-2.358) |
| Household Age, 1998 | -0.0011 (-1.376) | -0.0010 (-1.212) | -0.0002 (-0.290) | -0.0001 (-0.109) | -0.0014 (-1.754) | -0.0013 (-1.620) | -0.0006 (-0.718) | -0.0005 (-0.556) |
| Hispanic | -0.1293 (-7.489) | -0.1269 (-7.332) | -0.0727 (-4.148) | -0.0699 (-3.974) | -0.1005 (-5.796) | -0.0988 (-5.684) | -0.0528 (-3.005) | -0.0506 (-2.875) |
| Black | -0.0956 (-7.256) | -0.0932 (-7.031) | -0.0875 (-6.717) | -0.0847 (-6.469) | -0.0925 (-7.100) | -0.0905 (-6.909) | -0.0854 (-6.611) | -0.0831 (-6.397) |
| Education | | | 0.0211 (11.758) | 0.0211 (11.764) | | | 0.0193 (10.799) | 0.0194 (10.806) |
| Immediate Word Recall | | | -0.0004 (-0.162) | -0.0002 (-0.087) | | | -0.0011 (-0.396) | -0.0009 (-0.332) |
| Delayed Word Recall | | | 0.0019 (0.815) | 0.0018 (0.763) | | | 0.0023 (0.957) | 0.0022 (0.913) |
| 2nd Earnings Decile | | | | | 0.0064 (0.282) | 0.0052 (0.227) | 0.0086 (0.384) | 0.0070 (0.312) |
| 3rd Earnings Decile | | | | | 0.0212 (0.963) | 0.0201 (0.909) | 0.0232 (1.066) | 0.0217 (0.994) |
| 4th Earnings Decile | | | | | 0.0433 (1.961) | 0.0453 (1.877) | 0.0439 (2.012) | 0.0416 (1.906) |
| 5th Earnings Decile | | | | | 0.0743 (3.343) | 0.0727 (3.268) | 0.0710 (3.240) | 0.0690 (3.144) |
| 6th Earnings Decile | | | | | 0.0584 (2.643) | 0.0563 (2.554) | 0.0536 (2.460) | 0.0510 (2.336) |
| 7th Earnings Decile | | | | | 0.0868 (3.937) | 0.0847 (3.834) | 0.0790 (3.631) | 0.0764 (3.500) |
| 8th Earnings Decile | | | | | 0.1079 (4.808) | 0.1056 (4.694) | 0.0985 (4.450) | 0.0956 (4.308) |
| 9th Earnings Decile | | | | | 0.1315 (5.815) | 0.1300 (5.734) | 0.1183 (5.299) | 0.1161 (5.195) |
| 10th Earnings Decile | | | | | 0.1557 (6.898) | 0.1539 (6.806) | 0.1363 (6.113) | 0.1340 (5.997) |
| Constant | 0.5720 (10.558) | 0.5438 (9.599) | 0.4033 (7.224) | 0.3707 (6.366) | 0.5635 (9.982) | 0.5423 (9.274) | 0.4076 (7.000) | 0.3825 (6.351) |
| Number of Observations | 3,133 | 3,133 | 3,111 | 3,111 | 3,133 | 3,133 | 3,111 | 3,111 |
| Adjusted R-Squared | 0.1407 | 0.1412 | 0.1792 | 0.1799 | 0.1648 | 0.1650 | 0.1963 | 0.1968 |

** t-values in parentheses.
**Without housing equity.