Preliminary Version

Transition Paths and Social Security Reform*

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According to the *Economic Report of the President 1999*, in 1929 there were 9.6% as many people over age 65 in the U.S. as between the ages of 20–64; in 1969 the percentage was 18.6; in 1998 it was 21.6. The rise reflects, of course, increasing longevity and declining birth rates. Evidently the U.S. economy is in the midst of period in which the consumption of the elderly will rise as a fraction of total output. Assuming past trends continue, questions arise as to how our society can make required resource reallocations without straining intergenerational amity or eroding the work and saving incentives of the young. The social security system provides a principal source of support for many elderly households. Thinking about social security, moving in the direction of a funded system, perhaps utilizing private accounts, is one possible avenue of reform. Funding the system might help to clarify for participants the relation between their taxes and benefits, and it might facilitate further reform which expands the latitude for individual choice. This paper examines funding changes from a macroeconomic perspective, paying particular attention to the possible transition process from an unfunded social security system to a funded one, and to interactions of social security and aggregate private wealth accumulation.

Section I shows the close connection between national debt and unfunded social security liabilities: within the context of the type of model economists often employ for macroeconomic analysis of public policy, it is possible to shift from an unfunded to a funded social security system, using changes in the national debt, in a way that leaves physical investment and interest rates and wage rates unaffected. The economy's total liabilities, explicit and implicit (see below), would remain the same, but the balance would shift to the "explicit" side. In principle, society could engineer such a shift rapidly.

Section II briefly outlines possible advantages and disadvantages of proceeding to a second stage of reform aimed at reducing the national debt. Analysts often combine the two stages, but this paper proposes thinking about them separately. One potential benefit of the latter strategy is that a society conceivably would favor the first stage of reform but not the second.

Section III considers implementation of the second stage of reform, reducing the (enlarged) national debt. Outcomes are seen to depend heavily upon one's framework of analysis.

I. Transition to a Funded Social Security System

Economists frequently rely on nonstochastic, dynamic general equilibrium models to study potential policy changes (i.e., Auerbach and Kotlikoff [1987], Kotlikoff [1998], Feldstein and Samwick [1997, 1998]). This section argues that in such a context, the transition from an unfunded to a funded social security system need not be too complicated or time consuming.

Let us work with an illustrative example. Suppose the economy has households of two types, young and old. For simplicity, let the two groups always be the same size as one another, there being no overall population growth. The young work, earning, say, E_t dollars at date t (per young household). Over time, assume technological progress, at rate g, lifts earnings so that

$$E_{t+1} = E_t \cdot (1+g), \quad g > 0. \tag{1}$$

The old are retired. Households young one year will be old the next. Young households pay part of their earnings, T_t per young household at time t, in social security taxes; at time t, each old household receives B_t dollars in social security benefits. Initially, the social security system is unfunded, taxes on the young immediately flowing to benefits to contemporaneous retirees. Thus,

$$B_t = T_t \quad \text{all} \quad t. \tag{2}$$

Suppose we begin with no government debt.[1] Suppose that social security taxes and benefits rise with living standards through time as in (1):

$$B_{t+1} = B_t \cdot (1+g) \quad \text{and} \quad T_{t+1} = T_t \cdot (1+g) \quad \text{all} \quad t. \tag{3}$$

Let the real interest rate be r.

Now consider a way in which the economy could fund the social security system. At time 0, it could print new government bonds which expire next period and pay in total at that moment exactly the aggregate amount of previously promised social security benefits. It could distribute the bonds to young households, say, setting up a private account for each. It would give each young household bonds with expiration value equaling that household's former social security benefits. The bonds, at their expiration, would replace the benefits. For simplicity, assume that regulations prohibit young households from modifying or drawing down their accounts prior to retirement, or borrowing against them. In subsequent generations, each young household would, under regulation, purchase bonds for its private account with next–period payoff equaling the household's (former) social security benefit. Bond payoffs would replace social security benefits, starting at time 1. At time 0, social security taxes on young households would pay the current benefits of retirees. There would be no traditional social security benefits after time 0. For cohorts born at time 0 and after, the present value of former social security benefits would tend to be less, in youth, than former social security taxes.[2] Government would collect the difference as a tax. The new tax would be needed to fund interest on bonds in the new private accounts. Name it the "national debt–service tax." Society could keep rolling over the principal on the new national debt.

How would the economy change?

Consider Tables 1–5. Old households at the date of reform are unaffected: they receive B_0 and play no role in the new system.

Households young at the reform date 0 pay social security tax T_0 — exactly as they would have under the old system. As Table 1 shows, although households lose their prospective social security benefit — with present value $B_1/(1+r)$ — government provides

[1] If there is an existing government debt, income taxes on the young and old pay the debt service.

[2] Assuming the economy's capital intensity was short of the so–called "golden–rule" level. Recall Abel *et al.* [1989].

them a transfer of new bonds with the same value — see Table 1′. On balance, each household's lifetime resources are the same as before. That means such households have no incentive to change their lifetime consumption pattern. Hence, their discretionary saving will remain unchanged. On the other hand, the new private–account contribution will count as national income and product account private saving, which will therefore increase, as shown in Table 4.

Young households started at time 1 formerly paid social security taxes T_1 and received benefits B_2, with present value $B_2/(1+r)$ — see Table 1. After reform, their mandatory contribution, in youth, to their private account is $B_2/(1+r)$, and their (new) debt service tax in youth is $T_1 - B_2/(1+r)$. Since in old age their private account is worth B_2, their lifetime resources are the same as before — see Tables 1′–2′. Again, they have no incentive to change their lifetime consumption pattern, and their discretionary saving will remain unchanged. However, their private–account contribution will count as national income and product account saving. And, private–account withdrawals of the previous cohort at the same time count as dissaving. See Table 4.

The lifetime resources of subsequent generations are also unaffected — compare Tables 1–2 with 1′–2′. At time 2, private saving rises from the mandatory private account contribution of currently young households, $B_2/(1+r)$ per household, and falls due to the dissaving of currently old households, $B_1/(1+r)$ per household — see Table 4. Similarly for later cohorts.

Tables 3–3′ show the government's budget constraint. Table 3 reflects the balance of (2) under the old system. With reform, government makes a one–time transfer at time 0 to set up private accounts, financing the transfer with new debt. Subsequently, the new debt remains a constant proportion of GPD; hence, its absolute size rises at rate g — see Table 5. The proportionate rise is a source of revenue to the government. The debt–service tax on households — see Tables 1–1′ — reflecting the residual from the old social security tax less the new mandatory private account contribution — is also a new source of government revenue. The two sources together are exactly sufficient to meet the interest obligations on the new government bonds in the private accounts — see Table 3′. Similarly for subsequent years.

The appearance and growth of the new national debt constitute deficit finance and therefore enter Table 4 as negative government saving. As the table shows, in each year this negative exactly offsets the increase in household saving due to funded private accounts. Hence, reform does not, in the end, affect total national income and product account saving. In a closed economy, saving equals investment. So, physical investment is unchanged.

Thus, we have outlined a way of funding the social security system in a single period. Doing so enlarges the national debt. However, promised future social security benefits to living households constitute, under the existing system, implicit liabilities. A comparison of the top and bottom of Table 5 shows the government's total balance sheet is, in economic terms, unaffected by our reform: explicit debt merely replaces implicit liabilities, the shift being dollar for dollar. The total balance reflects the history of the current system: at its start, the system awarded benefits to retirees who had paid practically no social security taxes. Table 4 shows this section's reform has no effect on physical investment; hence, there need be no general equilibrium effect on interest rates or wages.

Appendix 1 presents an alternative derivation of our equivalence result, using the context of the well–know Diamond [1965] overlapping generations model.

Although our illustrative example is highly stylized, the logic is much more general. It can accommodate, for instance, multiperiod life spans. With an individual with N periods of life, at time 0 government would transfer bonds equaling, in present value, the individual's vested social security benefits to date. Subsequently, the individual would make his own additional contributions, and pay debt service tax.

Changes in the interest rate over time would complicate but not invalidate the analysis (see Appendix 1, for instance). In fact, it is not even essential to our argument that government pay exactly the market rate of interest on bonds funding private accounts — assuming the accounts are heavily regulated (i.e., mandatory).[3] A high rate would imply the initial transfer of bonds could be small, though the subsequent interest payments would be great; a low rate would imply the initial transfer must be large, though future interest payments could be less.[4] Appendix 2 provides details of the argument. Presumably, government would, nevertheless, choose an interest rate close to market levels, say, the rate for inflation–protected government bonds.

What would be accomplished with such a program for funding social security?

First, the private accounts might ease young households' worries about the safety of their future benefits; future benefits would now presumably attain the same legal status as all national debt.[5]

Second, private accounts might form a convenient platform from which to implement further reform, perhaps reform allowing participants somewhat more latitude over their investment choices.[6]

Third, a system of private accounts might change participants' psychology enough to help arrest future growth of the sum of implicit and explicit government liabilities as a fraction of GDP. Suppose, for example, that SSA can predict at time t that young households, and future households, will live longer. And, suppose households determine they want to devote their extra years to retirement. Then young households at time t, and beyond, might vote for larger contributions in youth to their private accounts, taking the new money from their aftertax income. Such a program would raise future benefits; nevertheless, pre–funding would mean the expanded benefits would not increase the government debt, either explicit or implicit liabilities. In contrast, a standard course of action under the present system would be to do nothing until time $t+1$ and then increase benefits and

[3] There is a longstanding puzzle in economics of why the average rate of return on, say, government bonds is so much lower than the average return on common stock (e.g., Kocherlakota [1996]). The following argument shows that need not be a stumbling block for the analysis in this paper.

[4] Feldstein and Samwick [1997,tab. 8] provides an example related to this point.

[5] In the words of the President's Commission [2001,p.16], " ... retirement security for Social Security participants will be enhanced by ownership of assets accumulated through the Social Security system, relative to a claim to benefits that must remain subject to political negotiation.

[6] E.g., President's Commission [2001].

taxes simultaneously. Since old households at $t + 1$ would then receive enhanced benefits without themselves paying higher taxes, the government's implicit liability in Table 5 would rise.[7]

Fourth, this section's analysis shows clearly the close connection between unfunded social security liabilities and national debt: they are virtually two different sides of the same coin. Funding the system would simplify information problems for voters: instead of having to keep track of two types of government liability, there would be a single one.

II. Increasing Saving

The previous section suggests a way of converting an unfunded social security system into a funded one. The procedure leaves macroeconomic variables, such as the economy's physical capital stock, unchanged. On the other hand, reform advocates frequently suggest that an important goal is to increase the nation's capital stock. In fact, many suggest reforms combining steps to fund social security with others to generate new tax revenues to reduce Table 5's total government liability. Having considered funding the social security system in Section I, this paper turns in Section III to an analysis of paying down the national debt. This paper's strategy is to analyze the two steps separately, rather than simultaneously.

The present section pauses to examine the merit of reducing the national debt. What creates an issue is that debt reduction requires temporarily higher taxes and/or lower government spending and transfers. In other words, society must sacrifice in the near term for the benefit of a reduced debt in the future. Is such a program warranted?

An argument against sacrificing now to lower the national debt is as follows. Technological change tends to improve living standards over time. Mainly for this reason, each successive generation tends to live better than its predecessors. To the extent that government borrowing allows society to consume more today while postponing payment until the future, some might deem it "fair" because it, in effect, transfers resources to the present from future generations, who will presumably be better off.

On the other hand, several reasons society might decide to reduce its national debt are as follows. (i) Voters simply might not want to burden future generations with past debts. (ii) Though citizens within a closed economy own their country's national debt, so that the interest on that debt accrues to them, ultimately taxes must provide the revenue to pay the interest. And, income or social security taxes distort household incentives to work and save. Economists measure the effect of distortions with deadweight loss. National debt not only requires future populations to pay for the consumption of their predecessors, but also it burdens the future with deadweight losses from taxes needed to pay the debt service.[8] (iii) In so-called overlapping generations models, an economy with a larger national debt

[7] In the same vein, the President's Commission [2001,p.31] writes, "It is impossible to know with precision the degree to which the federal government would otherwise save Social Security revenues that are to be deposited in personal accounts. The most that can be said is that as a matter of historical record, the government has not tended to save this money.

[8] Deadweight loss tends to rise with the square of marginal tax rates. The explicit

has a smaller physical capital stock: households are willing to do a certain amount of saving relative to their lifetime earnings; to the extent that government bonds fill their portfolios, there is less space available to hold private stocks and bonds, which finance physical capital. Thus, an economy with a larger national debt, *cet par.*, may carry a smaller physical capital stock to the future. A smaller physical capital stock means a lower potential output.

This paper does not attempt to settle this debate. Rather, the next section analyzes possible debt–reduction scenarios, assuming society has decided to proceed in that direction.

III. Reducing Public Debt

Economists employ two basic frameworks in their dynamic simulations (cited above): the life–cycle, or overlapping generations, model; and, the dynastic, or altruistic, model. The first assumes that households care exclusively about their own lives.[9] Since a household's earnings typically rise with age until retirement and then decline or disappear, the model predicts that each household's wealth holdings will follow a lifetime cycle as well, rising in youth and middle age as the household saves in anticipation of retirement, and declining thereafter as it dissaves to pay for its retirement. The second is the so–called altruistic model.[10] In it, households care about their descendants as well as themselves. Such households may receive inheritances and may save to build estates. There is a longstanding debate within the economics profession about the quantitative importance of bequest–motivated relative to life–cycle saving (e.g., Kotlikoff and Summers [1981], Modigliani [1988]). A second empirical issue concerns the distribution of private wealth: in the U.S., the latter is extremely concentrated (e.g., Wolff [1995]) — seemingly much more so than is the case for earnings — and the life–cycle model may not be able to explain the unevenness (e.g., Huggett [1996]). A third issue is that while life spans increased substantially in the U.S. during the twentieth century and taking a period of retirement at the end of life became much more popular, national wealth accumulation (relative to output) changed very little. This does not seem consistent with the life–cycle model, according to which saving should have increased (e.g., Darby [1979]). This section suggests a third framework for analysis, combining the other two, and it examines the qualitative implications of debt reduction in each of the three models.

Suppose we have a closed economy with a Cobb–Douglas aggregate production function summarizing the business sector:

$$Q = K^{\alpha} \cdot L^{1-\alpha}, \quad \alpha \in (0,1), \tag{4}$$

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national debt is already sizable (in 1998, for example, the three largest outlays of the U.S. Federal government were social security benefits, \$379 bil.; national defense, \$268 bil.; and, net interest on the (explicit) national debt, \$243 bil.); if it expands relative to GDP in the future, the distortion–cost eventually could become very great.

[9] See Modigliani [1986] and Diamond [1965].

[10] See Becker [1974] and Barro [1975].

where Q is real GDP, K is the economy's physical capital stock, and L is the labor supply. For simplicity, omit depreciation of physical capital, population growth, and technological progress. Normalize the price of output to one. If W is the steady–state wage rate and r the steady–state interest rate, with competitive factor pricing we have

$$W \cdot L = (1 - \alpha) \cdot Q \quad \text{and} \quad r \cdot K = \alpha \cdot Q. \tag{5}$$

Thus,

$$\frac{K}{W \cdot L} = \frac{\alpha}{1 - \alpha} \cdot \frac{1}{r}. \tag{6}$$

Figures 1–3 graph this relation as the production sector's "demand for capital" curve.

Suppose the economy has only life–cycle saving. In a steady–state equilibrium with both r and W constant, one can derive aggregate desired life–cycle net worth holdings at each r. Plotting the latter in units of earnings, one has Figure 1's "supply of net worth" curve S. In the frequently used simple case with two period lives and logarithmic utility functions, the supply curve is vertical. In more realistic cases, it could have a negative or positive slope — with the latter being the most typical in existing work. (See, for instance, Tobin [1967].) The initial long–run equilibrium is at e^0.

Let the national debt be D. Assume society rolls the debt over so that D/Q remains constant through time. The debt might have originated from funding the social security system, as in Section I, or it might have arisen in other ways. Private net worth accumulations must finance both the physical capital stock, K, and the national debt. Furthermore, taxes to pay interest on D reduce household lifetime resources, tending to shift S west to S'. Thus, the steady–state equilibrium interest rate rises from r^0 to to r^1, where private net worth is exactly large enough to cover the national debt and the physical capital stock. Combining (4)–(5),

$$W = (1 - \alpha) \cdot [\frac{K}{L}]^{\alpha} \quad \text{and} \quad r = \alpha \cdot [\frac{K}{L}]^{\alpha - 1}. \tag{7}$$

Equation (7) shows that the higher steady–state interest rate associated with $D > 0$ implies a lower capital–to–labor ratio, K/L; a lower steady–state wage; and, a higher output at each date.

A well–know study of debt in this type of framework is Auerbach and Kotlikoff [1987, ch.6]. A reduction in debt can lead to a large increase in the physical capital stock, raising output in turn. The latter could expedite the debt–reduction program: initial debt reduction would raise output, providing additional resources for future payments.

In the simplest dynastic model — see Barro [1974] — the supply–of–household–wealth curve is horizontal — see S in Figure 2. The idea is that households care about their descendants into the distant future and that first–order conditions for utility–maximization therefore connect marginal utilities over very long time spans. This implies that even small changes in the steady–state interest rate have a highly leveraged effect on behavior, leading to a very interest elastic supply curve. The initial long–run equilibrium is at e^0.

It is easy to see that Figure 2's horizontal S curve makes the steady–state equilibrium interest rate invariant to changes in D: higher taxes, which shift S to the west, do not affect the supply curve's position; households willingly finance debt $D > 0$, as opposed to debt 0, with only an infinitesimal increase in the interest rate. This is one manifestation of Barro's famous "Ricardian equivalence" result. In fact, in the simplest dynastic model debt reduction accomplishes virtually nothing, even in the short run. The equilibrium interest rate remains at r^0.

Laitner [2001b,c] presents a combined model with four basic elements. First, each household has a life–cycle of earnings and mortality. Second, all households care about their descendants as well as themselves (though in their calculations they may weigh the utility of their descendants less heavily than their own). Third, there is an exogenous distribution of earning abilities in every birth cohort. Fourth, financial institutions do not allow households to have negative net worth, nor can households choose to make negative intergenerational transfers to their descendants. In equilibrium, all households do life–cycle saving and dissaving. On the other hand, low earners, and those without large inheritances, tend to fall at a zero–bequest corner solution, whereas high earners, and/or households with large inheritances, save to build estates as well as for life–cycle purposes. The idea is that low earners expect their descendants to do at least as well as themselves, whereas high earners have more doubts.

Consider Figure 3. At low interest rates, overall bequest activity is relatively modest, so that S may closely resemble Figure 1. At higher prospective interest rates, on the other hand, bequests become more important, and rich families' time horizons become longer — thus, the supply of net worth expands and it becomes more interest elastic. In fact, Laitner [2001c] shows the combined model's supply curve must asymptotically approach Barro's curve.

In Laitner's [2001c] calibration, the hybrid model's long–run equilibrium occurs at a point resembling e in Figure 3, in the supply curve's flat section.

In terms of empirical evidence, Figure 3 hints that the hybrid model can be consistent with the U.S. economy's large stock of private net worth without requiring unrealistically high interest rates. In Laitner's [2001c] calibration, estate building accounts for about 30 percent of private wealth. Other recent work studying different motives for bequests seems to arrive at similar fractions (e.g., Altig *et al.* [2001]).

The calibrated hybrid model is consistent with the high concentration of U.S. wealth as well — see Laitner [2001c, tab.6]. In the model, all households accumulate net worth to finance their retirement, but high earners save extra to share with their descendants. The latter saving tends to raise the concentration of the cross–sectional distribution of private wealth holdings.[11]

Laitner [2001a] shows that a combination model can also be broadly consistent with U.S. twentieth century history. Consider Figure 4. Early in the century, a long retirement was rare. Thus, the contribution of life–cycle saving — i.e., the relatively steep part of the supply curve — might have been small. When retirement played a greater role later, the supply curve may have been at S'. The long–run interest rate need not have changed

[11] Gokhale *et al.* [2001] and Nishiyama [2001] present alternative hybrid models which can also explain a high degree of wealth inequality. See also the discussion of different models in Laitner [2002].

appreciably, however, as the figure illustrates: the old long–run equilibrium was at e_{1900}, and the new one is at e_{2000}.

Returning to Figure 3, an equilibrium at f would imply policy implications for national debt similar to the life–cycle model; an equilibrium at e would suggest outcomes more closely resembling Barro's [1974]. In fact, as stated, Laitner's calibration points to an intersection at e.

With an equilibrium at e, a program of debt reduction would not ultimately lead to a sizable reduction in interest rates or a substantial increase in the physical capital stock.[12] Unlike Barro, there would be short–run adjustments, however. And, for the same reason, the long–run distribution of wealth would change: households planning substantial bequests give Figure 3's supply curve its high interest elasticity; following a reduction in D, the same households (i.e., the elastic ones) would be the ones most likely to reduce their portfolio sizes. Since the bequeathers are the high accumulators, that would tend to make the distribution of wealth more equal (see Laitner [2001a]).

IV. Conclusion

Section I of this paper suggests that the U.S. social security system could be reformed from an unfunded to a funded system almost instantaneously through the use of government debt. Such a reform would have almost no direct economic consequences. It might nevertheless be significant: it might change society's psychology with regard to coping with future demographic trends, it might help to clarify for voters the full extent of the burden of the economy's indebtedness, and it might facilitate future additional reforms.

While proposed reforms usually include provisions for new tax revenues, this paper suggests splitting the task into two parts: funding the system through national debt, and then paying down the national debt. Section II examines possible rationales for proceeding to the second step.

Section III catalogs different macroeconomic implications of debt reduction for different modeling frameworks. In all cases, paying down the debt reduces the tax burden on future generations. In some cases, it leads as well to a substantial long–run increase in the economy's stock of physical capital and, hence, potential output. Other models predict more modest changes in economic variables, perhaps with reductions in the inequality of private wealth holdings.

[12] Note that this analysis assumes an inelastic labor supply. An elastic supply would produce more dramatic results, even with an inelastic interest rate.

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Appendix 1

Unfunded Social Security.

Each household lives two periods, and a household born at t has utility function

$$\theta \cdot \ln(c_t^1) + (1-\theta) \cdot \ln(c_t^2), \tag{A1}$$

where c^i is consumption in period of life $i = 1, 2$. A household supplies one unit of labor in youth and none in old age. The gross–of–tax wage is W_t. The social security tax falls on wages and has rate τ^{ss}. There is a proportional income tax of rate τ on wages and return to saving. The net–of–tax rate of return on saving is r_t.

There is an aggregate production function

$$Q_t = [K_t]^{\alpha} \cdot [L_t \cdot (1+g)^t]^{1-\alpha}, \quad \alpha \in (0,1). \tag{A2}$$

Technological progress, with rate $g > 0$, augments the effectiveness of labor. The GDP is Q_t, and the physical capital stock is K_t. The labor supply is L_t. For simplicity, assume

$$L_t = (1+n)^t. \tag{A3}$$

Set the price of units of GDP to 1 every period. Competitive factor pricing leads to

$$W_t = (1-\alpha) \cdot [K_t]^{\alpha} \cdot [L_t \cdot (1+g)^t]^{-\alpha} \cdot (1+g)^t, \tag{A4}$$

$$r_t = \alpha \cdot [K_t]^{\alpha-1} \cdot [L_t \cdot (1+g)^t]^{1-\alpha} \cdot (1-\tau) - \delta, \tag{A5}$$

where δ is the rate of physical depreciation on capital and where income taxes fall on the marginal physical product of capital. To simplify the formulas below, we set

$$\delta = 1. \tag{A6}$$

Government spends income tax revenues on goods and services (e.g., defense). The social security system is unfunded: if a household born at t receives benefit b_{t+1} in old age,

$$\tau^{ss} \cdot W_{t+1} \cdot L_{t+1} = b_{t+1} \cdot L_t. \tag{A7}$$

"Equilibrium" requires that households maximize their individual well–being given factor prices and their endowment, and that household net worth finances the physical capital stock. (The economy is closed to international trade and capital flows.)

In equilibrium, a household born at t solves

$$\max_{c_t^1,\, c_t^2} \{\theta \cdot \ln(c_t^1) + (1\theta) \cdot \ln(c_t^2)\} \tag{A8}$$

$$\text{subject to:} \quad c_t^1 + \frac{c_t^2}{1+r_{t+1}} \leq W_t \cdot (1-\tau-\tau^{ss}) + \frac{b_{t+1}}{1+r_{t+1}} \ .$$

Maximization yields

$$c_t^1 = \theta \cdot [W_t \cdot (1 - \tau - \tau^{ss}) + \frac{b_{t+1}}{1 + r_{t+1}}], \tag{A9}$$

$$C_t^2 = (1 - \theta) \cdot [W_t \cdot (1 - \tau - \tau^{ss}) + \frac{b_{t+1}}{1 + r_{t+1}}] \cdot (1 + r_{t+1}). \tag{A10}$$

The net worth such a household chooses to carry into its second period of life equals its first–period aftertax income less its first–period consumption:

$$a_t \equiv (1 - \theta) \cdot W_t \cdot (1 - \tau - \tau^{ss}) - \theta \cdot \frac{b_{t+1}}{1 + r_{t+1}}. \tag{A11}$$

The second part of the definition of equilibrium requires

$$K_{t+1} = L_t \cdot a_t. \tag{A12}$$

Define

$$E_t \equiv L_t \cdot (1 + g)^t \quad \text{and} \quad k_t \equiv \frac{K_t}{E_t}. \tag{A13}$$

Then

$$k_{t+1} = \frac{L_t \cdot a_t}{E_{t+1}} = \frac{(1 + n)^t \cdot a_t}{(1 + n)^{t+1} \cdot (1 + g)^{t+1}} =$$
$$\frac{1}{1 + n} \cdot (\frac{1}{1 + g})^{t+1} \cdot [(1 - \theta) \cdot (1 - \alpha) \cdot [K_t]^{\alpha} \cdot [E_t]^{-\alpha} \cdot (1 + g)^t \cdot (1 - \tau - \tau^{ss}) -$$
$$\frac{\theta \cdot (1 + n) \cdot (1 - \alpha) \cdot [K_{t+1}]^{\alpha} \cdot [E_{t+1}]^{-\alpha} \cdot (1 + g)^{t+1} \cdot \tau^{ss}}{1 + (1 - \tau) \cdot \alpha \cdot [K_{t+1}]^{\alpha - 1} \cdot [E_{t+1}]^{1 - \alpha} - \delta}] \,. \tag{A14}$$

So,

$$k_{t+1} = \frac{1}{1 + n} \cdot \frac{1}{1 + g} \cdot (1 - \alpha) \cdot [(1 - \theta) \cdot [k_t]^{\alpha} \cdot (1 - \tau - \tau^{ss}) -$$
$$\frac{\theta \cdot (1 + n) \cdot [k_{t+1}]^{\alpha} \cdot (1 + g) \cdot \tau^{ss}}{(1 - \tau) \cdot \alpha \cdot [k_{t+1}]^{\alpha - 1}}] \,. \tag{A15}$$

So,

$$k_{t+1} \cdot [1 + \frac{\theta \cdot (1 - \alpha) \cdot \tau^{ss}}{\alpha \cdot (1 - \tau)}] = \frac{1}{1 + n} \cdot \frac{1}{1 + g} \cdot (1 - \alpha) \cdot (1 - \theta) \cdot (1 - \tau - \tau^{ss}) \cdot [k_t]^{\alpha}. \tag{A16}$$

If the analysis begins at time 0, history must provide our starting value k_0. Then we can iterate A(16) to determine the equilibrium path k_t all t.

Funded Social Security.

The model is as before, except that we now fund social security, using national debt as described in the text. Make the reform to a funded social security system at time $t = 0$.

The generation old at time 0 is unaffected.

Consider the generation young at time 0. Setting $t = 0$, its lifetime budget constraint is still

$$c_t^1 + \frac{c_t^2}{1 + r_{t+1}} \leq W_t \cdot (1 - \tau - \tau^{ss}) + \frac{b_{t+1}}{1 + r_{t+1}} \ .$$

However, the last term on the right now represents government transfers at time 0, rather than the present value of future social security benefits. Utility–maximizing consumption remains as before — since lifetime resources are as before. The contemporaneous nature of the last term in the budget changes a household's net worth carried to period 1 to

$$a_t \equiv (1 - \theta) \cdot [W_t \cdot (1 - \tau - \tau^{ss}) + \frac{b_{t+1}}{1 + r_{t+1}}] \qquad (A17)$$

(where $t = 0$ in the formula).

Since there is now government debt, say, D_t carried into period t, the second part of the definition of equilibrium changes: household net worth must finance both the physical capital stock and the government debt. Hence, we need

$$D_{t+1} + K_{t+1} = L_t \cdot a_t \quad \text{all} \quad t \qquad (A18)$$

in place of (A12).

The equation for the evolution of k_t remains unchanged at time $t = 0$. To see this, note that the nature of our social security reform implies

$$D_{t+1} = L_t \cdot \frac{b_{t+1}}{1 + r_{t+1}} \quad \text{all} \quad t. \qquad (A19)$$

Subtracting the left–hand side of (A19) from the left of (A18), and the right–hand side of (A19) from the right of (A18), we recover (A11)–(A12) — ie, (A16).

Consider a generation young at $t > 1$. Such a household's lifetime budget constraint is

$$c_t^1 + \frac{c_t^2}{1 + r_{t+1}} \leq W_t \cdot (1 - \tau) - [\tau^{ss} \cdot W_t - \frac{b_{t+1}}{1 + r_{t+1}}] \ .$$

In terms of lifetime resources, this is equivalent to the original constraint in (A8). However, the [.] term on the right–hand side is a tax due in youth. With identical lifetime resources, the household chooses the same consumption as before — i.e., (A9)–(A10). The contemporaneous nature of the new tax means the household's net worth carried into old age is as in (A17). But, A(18) and (A19) continue to hold. Thus, as at time $t = 0$, we continue to recover (A11)–(A12), hence (A16).

Thus, the evolution of k_t remains as before the reform. Hence, factor prices do not change either.

Appendix 2

Suppose we fund social security at time $t = 0$, as in the second part of Appendix 1. Suppose, however, that government places bonds paying interest rate R in the new private accounts, with R not necessarily equal to the current market interest rate.[13]

As before, the generation old at time 0 is unaffected.

A household in the generation young at time 0 now has lifetime budget constraint

$$c_t^1 + \frac{c_t^2}{1 + r_{t+1}} \leq W_t \cdot (1 - \tau - \tau^{ss}) + \frac{b_{t+1}}{1 + R} + [\frac{b_{t+1}}{1 + r_{t+1}} - \frac{b_{t+1}}{1 + R}] \ . \qquad (A20)$$

The middle term on the right side equals government transfers when the system is funded. The sum arrives at $t = 0$. The last term on the right reflects the (possible) discrepancy between the present value at $t = 0$ of the government transfer and the actual value at expiration of the bonds the government transfers into the household's private account. The household realizes this sum at $t = 1$. Notice that the value of the household's lifetime resources are identical to (A8). Thus, its consumption remains as in (A9)–(A10).

A household young at time $t > 0$ has lifetime budget constraint

$$c_t^1 + \frac{c_t^2}{1 + r_{t+1}} \leq W_t \cdot (1 - \tau) - [\tau^{ss} \cdot W_t - \frac{b_{t+1}}{1 + R}] + [\frac{b_{t+1}}{1 + r_{t+1}} - \frac{b_{t+1}}{1 + R}] \ . \qquad (A21)$$

Such a household no longer faces traditional social security taxes. However, the middle term on the right reflects its new debt–service tax. This tax is due at time t. The last term on the right is as in (A20). It falls at $t+1$. Notice that lifetime resources remain the same as (A8). Thus, consumption will remain as in (A9)–(A10).

With either (A21) or (A22), we have

$$a_t = (1 - \theta) \cdot \left[W_t \cdot (1 - \tau - \tau^{ss}) + \frac{b_{t+1}}{1 + R}\right] - \theta \cdot \left[\frac{b_{t+1}}{1 + r_{t+1}} - \frac{b_{t+1}}{1 + R}\right] \ . \qquad (A22)$$

Equation (A18) remains valid, but we need

$$D_{t+1} = L_t \cdot \frac{b_{t+1}}{1 + R} \quad \text{all} \quad t \qquad (A23)$$

in place of (A19).

As in the second part of Appendix 1, we subtract the right side of (A23) from the right of (A18), and the left side of (A23) from the left of (A18), then we substitute from (A22). We end up with

$$K_{t+1} = L_t \cdot \{(1 - \theta) \cdot W_t \cdot (1 - \tau - \tau^{ss}) - \theta \cdot \frac{b_{t+1}}{1 + R} - \theta \cdot [\frac{b_{t+1}}{1 + r_{t+1}} - \frac{b_{t+1}}{1 + R}]\} \ .$$

[13] We assume R is constant with respect to time. The reader will be able to see that is not essential, however.

Canceling like terms on the right–hand side yields

$$K_{t+1} = L_t \cdot \{(1-\theta) \cdot W_t \cdot (1-\tau-\tau^{ss}) - \theta \cdot \frac{b_{t+1}}{1+r_{t+1}}\} \ , \tag{24}$$

which is identical to (A11)–(A12). Hence, (A16) remains valid.

In other words, if government sets a non–market interest rate on the bonds in private social security accounts, and if those accounts are mandatory, the time path for K_t is unchanged from the reform with market interest rates.

Table 1. Old Regime: Household Taxes and Government Transfers
(per young household)

Cohort Birth YR	Taxes (all in youth)		Bond Transfer	SSB (present value)
	SST	Debt Service		
0	T_0	0	0	$\frac{B_1}{1+r} = B_0 \cdot \frac{1+g}{1+r}$
1	$T_1 = T_0 \cdot (1+g)$	0	0	$\frac{B_2}{1+r} = B_0 \cdot \frac{(1+g)^2}{1+r}$
2	$T_2 = T_0 \cdot (1+g)^2$	0	0	$\frac{B_3}{1+r} = B_0 \cdot \frac{(1+g)^3}{1+r}$
3	$T_3 = T_0 \cdot (1+g)^3$	0	0	$\frac{B_4}{1+r} = B_0 \cdot \frac{(1+g)^4}{1+r}$

Table 1′. New Regime: Household Taxes and Government Transfers (per young household)				
Cohort Birth YR	Taxes (all in youth)		Bond Transfer	SSB (present value)
	SST	Debt Service		
0	T_0	0	$\frac{B_1}{1+r} = B_0 \cdot \frac{1+g}{1+r}$	0
1	0	$T_1 - \frac{B_2}{1+r} =$ $B_0 \cdot (1+g) - B_0 \cdot \frac{(1+g)^2}{1+r}$	0	0
2	0	$T_2 - \frac{B_3}{1+r} =$ $B_0 \cdot (1+g)^2 - B_0 \cdot \frac{(1+g)^3}{1+r}$	0	0
3	0	$T_3 - \frac{B_4}{1+r} =$ $B_0 \cdot (1+g)^3 - B_0 \cdot \frac{(1+g)^4}{1+r}$	0	0

Table 2. Old Regime: Household Private Account Transactions
(per young household)

YR (t)	Account Deposit at Time t by Young Household	Account Withdrawal at Time $t+1$ by Old Household (present value time t)
0	0	0
1	0	0
2	0	0
3	0	0

Table 2′. New Regime: Household Private Account Transactions (per young household)		
YR (t)	Account Deposit at Time t by Young Household	Account Withdrawal at Time $t+1$ by Old Household (present value time t)
0	$\frac{B_1}{1+r} = B_0 \cdot \frac{1+g}{1+r}$	$\frac{B_1}{1+r} = B_0 \cdot \frac{1+g}{1+r}$
1	$\frac{B_2}{1+r} = B_0 \cdot \frac{(1+g)^2}{1+r}$	$\frac{B_2}{1+r} = B_0 \cdot \frac{(1+g)^2}{1+r}$
2	$\frac{B_3}{1+r} = B_0 \cdot \frac{(1+g)^3}{1+r}$	$\frac{B_3}{1+r} = B_0 \cdot \frac{(1+g)^3}{1+r}$
3	$\frac{B_4}{1+r} = B_0 \cdot \frac{(1+g)^4}{1+r}$	$\frac{B_4}{1+r} = B_0 \cdot \frac{(1+g)^4}{1+r}$

Table 3. Old Regime: Government Flows

YR	Transfers			Taxes		Debt Change
	SSB	Bonds	Interest on Debt	SST	Debt Service	
0	B_0	0	0	T_0	0	0
1	$B_1 = B_0 \cdot (1+g)$	0	0	$T_1 = T_0 \cdot (1+g)$	0	0
2	$B_2 = B_0 \cdot (1+g)^2$	0	0	$T_2 = T_0 \cdot (1+g)^2$	0	0
3	$B_3 = B_0 \cdot (1+g)^3$	0	0	$T_3 = T_0 \cdot (1+g)^3$	0	0

Table 3′. New Regime: Government Flows

	Transfers			Taxes		
YR	SSB	Bonds	Interest on Debt	SST	Debt Service	Debt Change
0	B_0	$\frac{B_1}{1+r} = B_0 \cdot \frac{1+g}{1+r}$	0	T_0	0	$\frac{B_1}{1+r} = B_0 \cdot \frac{1+g}{1+r}$
1	0	0	$\frac{r \cdot B_1}{1+r} = r \cdot B_0 \cdot \frac{1+g}{1+r}$	0	$T_1 - \frac{B_2}{1+r} = B_0 \cdot [(1+g) - \frac{(1+g)^2}{1+r}]$	$\frac{B_2}{1+r} - \frac{B_1}{1+r} = B_0 \cdot [\frac{(1+g)^2}{1+r} - \frac{1+g}{1+r}]$
2	0	0	$\frac{r \cdot B_2}{1+r} = r \cdot B_0 \cdot \frac{(1+g)^2}{1+r}$	0	$T_2 - \frac{B_3}{1+r} = B_0 \cdot [(1+g)^2 - \frac{(1+g)^3}{1+r}]$	$\frac{B_3}{1+r} - \frac{B_2}{1+r} = B_0 \cdot [\frac{(1+g)^3}{1+r} - \frac{(1+g)^2}{1+r}]$
3	0	0	$\frac{r \cdot B_3}{1+r} = r \cdot B_0 \cdot \frac{(1+g)^3}{1+r}$	0	$T_3 - \frac{B_4}{1+r} = B_0 \cdot [(1+g)^3 - \frac{(1+g)^4}{1+r}]$	$\frac{B_4}{1+r} - \frac{B_3}{1+r} = B_0 \cdot [\frac{(1+g)^4}{1+r} - \frac{(1+g)^3}{1+r}]$

Table 4. Incremental Investment and Saving Following Regime Change (per young household)

YR	Investment	National Income and Product Saving	
		Household Saving	Government Budget Surplus
0	0	$\frac{B_1}{1+r}$	$-\frac{B_1}{1+r}$
1	0	$\frac{B_2}{1+r} - \frac{B_1}{1+r}$	$-[\frac{B_2}{1+r} - \frac{B_1}{1+r}]$
2	0	$\frac{B_3}{1+r} - \frac{B_2}{1+r}$	$-[\frac{B_3}{1+r} - \frac{B_2}{1+r}]$
3	0	$\frac{B_4}{1+r} - \frac{B_3}{1+r}$	$-[\frac{B_4}{1+r} - \frac{B_3}{1+r}]$

Table 5. Government Debt (end of year, per young household)			
YR	Implicit Debt	Explicit Debt	Total
Prior to Regime Change			
0	$\frac{B_1}{1+r} = B_0 \cdot \frac{1+g}{1+r}$	0	$\frac{B_1}{1+r} = B_0 \cdot \frac{1+g}{1+r}$
1	$\frac{B_2}{1+r} = B_0 \cdot \frac{(1+g)^2}{1+r}$	0	$\frac{B_2}{1+r} = B_0 \cdot \frac{(1+g)^2}{1+r}$
2	$\frac{B_3}{1+r} = B_0 \cdot \frac{(1+g)^3}{1+r}$	0	$\frac{B_3}{1+r} = B_0 \cdot \frac{(1+g)^3}{1+r}$
3	$\frac{B_4}{1+r} = B_0 \cdot \frac{(1+g)^4}{1+r}$	0	$\frac{B_4}{1+r} = B_0 \cdot \frac{(1+g)^4}{1+r}$
After Regime Change			
0	0	$\frac{B_1}{1+r} = B_0 \cdot \frac{1+g}{1+r}$	$\frac{B_1}{1+r} = B_0 \cdot \frac{1+g}{1+r}$
1	0	$\frac{B_2}{1+r} = B_0 \cdot \frac{(1+g)^2}{1+r}$	$\frac{B_2}{1+r} = B_0 \cdot \frac{(1+g)^2}{1+r}$
2	0	$\frac{B_3}{1+r} = B_0 \cdot \frac{(1+g)^3}{1+r}$	$\frac{B_3}{1+r} = B_0 \cdot \frac{(1+g)^3}{1+r}$
3	0	$\frac{B_4}{1+r} = B_0 \cdot \frac{(1+g)^4}{1+r}$	$\frac{B_4}{1+r} = B_0 \cdot \frac{(1+g)^4}{1+r}$

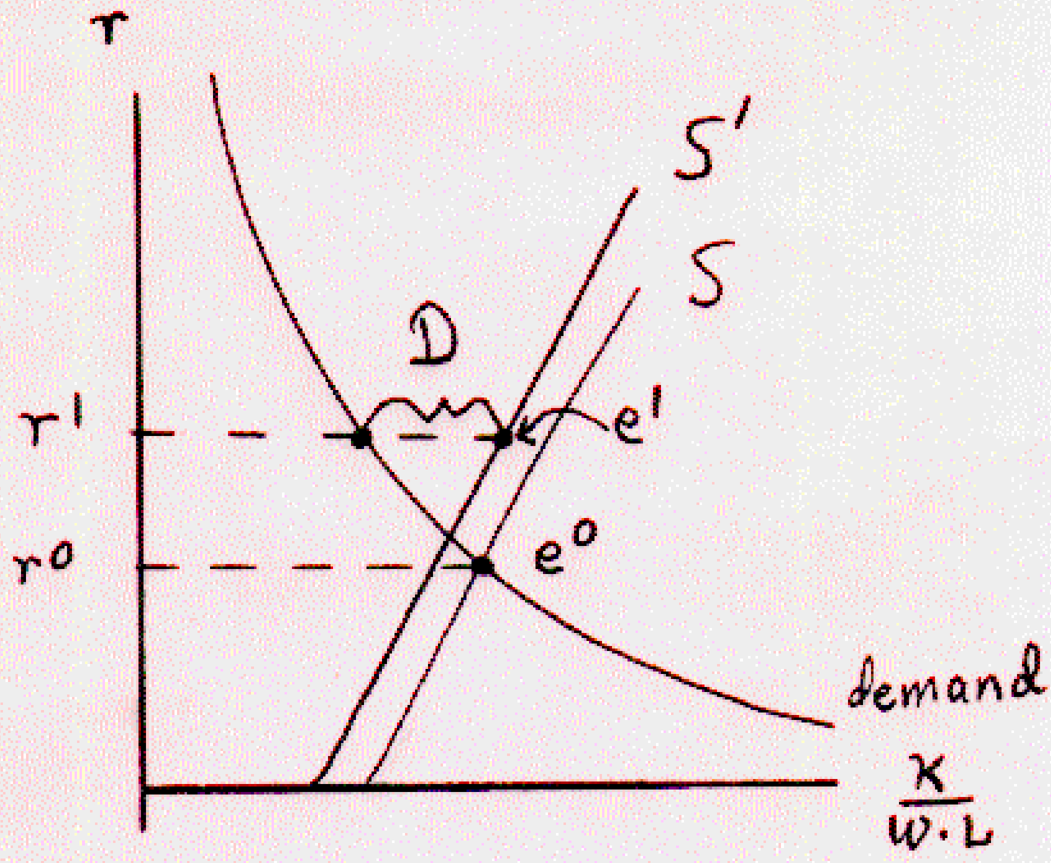

Figure 1. The life–cycle, or overlapping generations, model

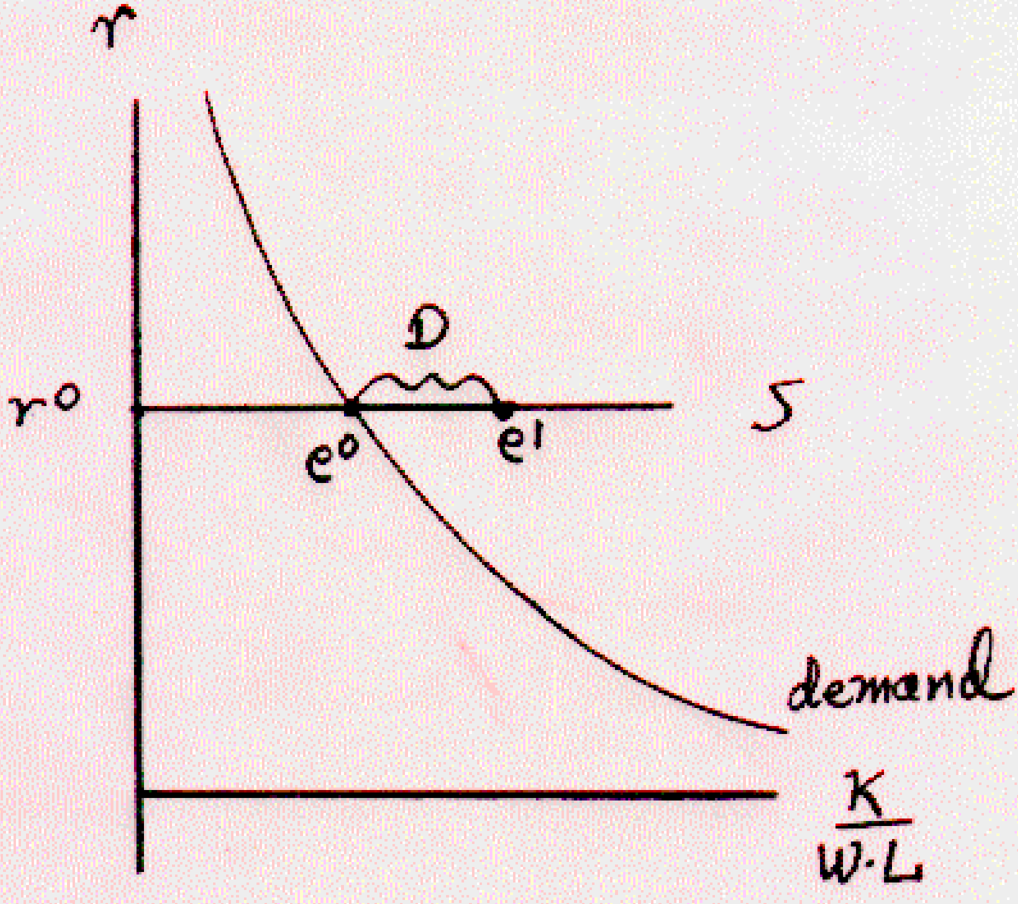

Figure 2. The simplest dynastic model

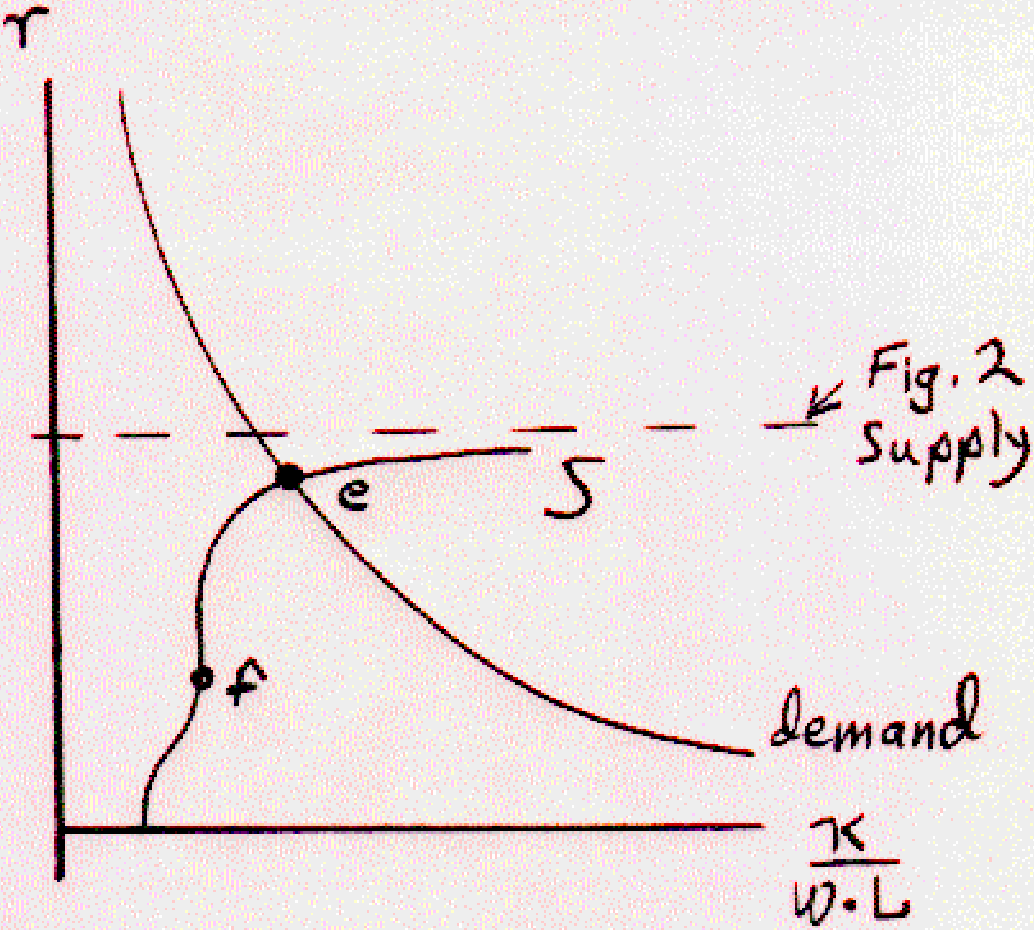

Figure 3. The hybrid dynastic, life–cycle model

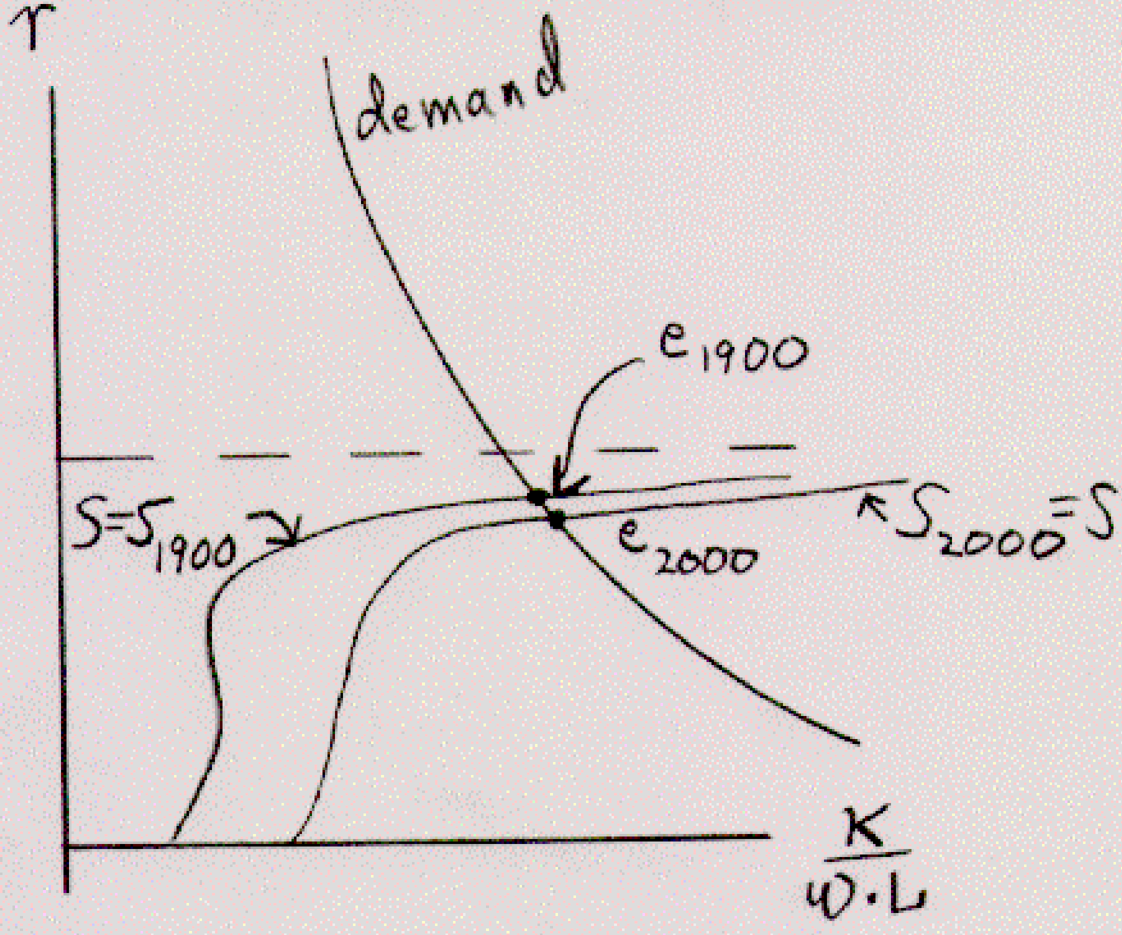

Figure 4. An interpretation of changes over time