

# OPTIMAL LIFE CYCLE PORTFOLIO CHOICE WITH VARIABLE ANNUITIES OFFERING LIQUIDITY AND INVESTMENT DOWNSIDE PROTECTION

VANYA HORNEFF, RAIMOND MAURER, OLIVIA S. MITCHELL AND RALPH ROGALLA \*

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Financially inexperienced consumers who must manage their self-directed retirement accounts may fail to understand investment and longevity risk. This problem can be rectified if defined contribution plans include variable deferred annuities with lifetime income guarantees and investment downside protection.

Our paper evaluates life-cycle consumption and portfolio allocation patterns resulting from access to Guaranteed Minimum Withdrawal Benefit (GMWB) variable annuities, one of the most rapidly-growing financial innovations over the last two decades. A key feature of these products is that they offer access to equity investments with downside protection, hedging of longevity risk, and partially-refundable premiums.

## Motivation

Defined contribution pensions are the most rapidly growing form of retirement saving around the world. Yet participants in such self-directed pension plans often fail to understand the risks associated with their investment and spending decisions, exposing them to the risk of potential retirement shortfalls. This suggests that, in addition to improving financial literacy, households would likely benefit from incorporating income and return guarantees into their defined contribution pension plans. Specifically, these products offer lifelong benefit payments during retirement, as well as protection of accumulated assets from downside market shocks. Such guarantees are offered by insurers in the form of investment-linked variable annuities, though relatively little is known about how to integrate these products into the pension context. This project shows how such variable annuities with guarantees can be used to enhance retirement security in the context of a life-cycle model.

## Prior Work

Previous research on variable annuities has taken two approaches. First, some authors have investigated how to price the complex option features embedded in these contracts. Second, the dynamic portfolio choice literature has examined household demand for life annuities and their welfare implications. Though some studies examine immediate and fixed annuities, few have considered variable annuities with deferred benefits in a realistically-calibrated life-cycle portfolio choice model. Nevertheless, no prior work incorporates the key guarantee features of variable annuities in line with those offered in the market.

## Methodology

Our new work extends prior research by incorporating fairly-priced variable annuities with guarantees into the investment opportunity set of a utility-maximizing investor who faces an uncertain lifetime, risky labor income, and

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\* *Vanya Horneff* is a research assistant at Goethe University. *Raimond Maurer* is chair of Investment, Portfolio Management and Pension Finance at the Finance Department of Goethe University, Frankfurt. *Olivia S. Mitchell* is the International Foundation of Employee Benefit Plans Professor, professor of Business Economics and Public Policy, professor of Insurance and Risk Management, and the executive director of the Pension Research Council at the Wharton School of the University of Pennsylvania. *Ralph Rogalla* is a professor of economics at Goethe University. This brief is based on MRRC Working Paper 2013-286.

stochastic equity returns. First we discuss the mechanics and pricing for the risk charges of the complex income/ investment guarantees inside the variable annuities, where there is also the possibility of periodic withdrawals (within some limits). To price these guarantees, in addition to using techniques from options pricing theory, it is important to determine whether the product provider can pool mortality risk (as with life insurers) as well as the interest rate environment. Building on and extending the researchers' prior work, next, we introduce a dynamic consumption and portfolio choice for a utility maximizing investor over the life cycle. We use this realistic calibrated life-cycle framework to generate the optimal consumption and portfolio allocation across risky stocks, bonds, and annuities of the sort of interest here.

The model incorporates individual risk-aversion, borrowing constraints, capital market volatility, and other background risks. In a base case, we assume that the variable annuities are purchased in non-qualified plans, that is, with contributions from after-tax income. Moreover, the base case eliminates labor income risk. Sensitivity analyses show how demand for such products differs in the context of a tax-qualified retirement plan, and with labor uninsurable income uncertainty. Additional robustness checks include alternative valuations for risk aversion, product costs, investment options allowed, bequest preferences, levels of Social Security benefits, and interest rate environments. Finally, we assess the welfare implications of having access to this innovative retirement financial product. Because the model structure and calibrated parameters are highly realistic, analytical solutions do not exist; accordingly, we solve this realistic life-cycle model with efficient numerical procedures using parallel-computing on a high-performance-cluster technology.

## Results

We find that investors will optimally purchase variable annuities prior to retirement because of their flexibility and access to the stock market. Moreover, many consumers will also adjust their portfolios and consumption streams along the way by taking cash withdrawals from the products. Overall, policyholders exercise the product's flexibility by taking withdrawals and dynamically adjusting their portfolios and consumption streams.

Whereas other studies have predicted that consumers will wait to buy deferred annuities late in life, here we show that investors optimally purchase measurable amounts of GMWBs well before retirement because of their flexibility and access to the stock market. This finding is consistent with empirical evidence of the growth in variable annuity demand over time. Moreover, and consistent with observed behavior, differences across cash out and annuitization patterns result from variations in realized equity market returns and labor income trajectories.

We also conduct three policy experiments. In the first, we find that people purchase more GMWBs and cash out less, when Social Security and private defined benefit pension benefits are lower. Second, we show that having GMWBs available in a tax-qualified retirement account enhances their popularity. Finally, if a GMWB requires deferring the payout until age 85 (in the spirit of pure longevity insurance), retirees will cash out more to finance consumption but they still enjoy a welfare gain from access to the product.

## Implications and Relevance

Variable annuities have been one of the most rapidly growing financial products over the last decades. They offer access to equity investments, downside protection against market risk, and the possibility of hedging longevity risk via annuitization. Our results will be of interest to financial advisers and plan sponsors seeking to enhance employees' retirement security, as well as to Social Security which is increasingly being asked to take on longevity risk due to the decline of defined benefit plans. Because of the withdrawal option embedded in these products, premiums are at least in part refundable, so such liquidity helps overcome consumer reluctance to voluntary annuitize their wealth. Finally, regulators may benefit from a clearer assessment of risks associated with individual retirement accounts, along with a possible role for regulation to protect individuals from the downside risk of fluctuating capital markets and the risk of running out of money.

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University of Michigan Retirement Research Center  
Institute for Social Research 426 Thompson Street Room 3026  
Ann Arbor, MI 48104-2321 Phone: (734) 615-0422 Fax: (734) 615-2180  
mrrcumich@umich.edu www.mrrc.isr.umich.edu

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