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## Abstract

We derive the optimal portfolio choice and consumption pattern over the lifecycle for households facing labor income, capital market, and mortality risk. In addition to stocks and bonds, households also have access to deferred annuities. Deferred annuities offer a hedge against mortality risk and provide similar benefits as Social Security. We show that a considerable fraction of wealth should be annuitized to skim the return enhancing mortality credit. The remaining liquid wealth (stocks and bonds) is used to hedge labor income risk during work life and to earn the equity premium. We find a marginal difference between a strategy involving deferred annuities and one where the investor can purchase immediate life annuities.

## Authors' Acknowledgements

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# 1 Introduction and Motivation

With the decline of traditional defined benefit pension plans, defined contribution plans gained considerable importance for companies around the world in the past two decades. Defined contribution plans and private saving plans are typically self-managed by households. Arguably, Social Security is said to experience major cuts in the scheduled benefit payments to guarantee the solvency of the traditional PAYGO system in aging societies.

The literature argues with a dominance criterion for purchasing annuities to finance consumption (e.g. Yaari 1965). Despite this theoretical dominance argument voluntary annuitization is still limited around the world. There are many advocates of embedding annuitization as a default mechanism in order to re-enforce withdrawal discipline in tax sheltered pension accounts that is naturally found in annuity payments, Social Security and defined benefit schedules. Many times the preferred instrument to implement the payout and savings discipline is the deferred annuity.

Similar to an immediate annuity, a deferred annuity promises life long payouts in exchange for an initial non-refundable premium paid to the insurance provider. Contrary to immediate annuities, the contract of a deferred annuity stipulates that the annuity does not provide any payments until a certain number of years  $u$  passed. If the annuitant perishes during the deferring period, the premium is lost for the annuitant's heirs. In fact, a deferred annuity is much cheaper compared to an immediate annuity with identical payouts because the benefits are deferred until  $t + u$ , where  $t$  is the age when the annuities are bought.

While immediate life annuities were recently studied to a large extent by Blake, Cairns, and Dowd (2003), Horneff, Maurer, and Stamos (2008a,b), Horneff, Maurer, Mitchell, and Dus (2008), Kingston and Thorp (2005), Kojen, Nijman, and Werker (2006), Milevsky and Young (2007), Milevsky, Moore, and Young (2006), Yogo (2008), deferred annuities have not been considered in the dynamic asset allocation of private

households so far.

Scott, Watson, and Hu (2008) recommend purchasing deferred annuities in case the extent to which annuities can be bought is limited. A key assumption of their analysis constitutes that the initial retirement budget is entirely spent on bonds and so called 'zero coupon annuities', i.e. contracts with a single payoff in a given year conditional on the annuitant's survival. Payoffs of both products are entirely consumed in each period. Milevsky (2005) analyzes the design as well as the pricing features of real (i.e. where benefits are adjusted for inflation) deferred annuities. Additionally, he argues that such products can overcome psychological factors frequently mentioned for explaining the empirically low levels of voluntary annuitization (also known as the annuity puzzle). Especially, he pointed out that "engaging in irreversible financial transactions - that is annuitization - involving large lump sums will never be appealing to individuals regardless of (whether they grasp) the importance of longevity insurance". The alternative would be to buy deferred annuities which are much cheaper than immediate annuities regardless how important longevity insurance perceived is. This is the alternative which is seemingly much cheaper than immediate annuities - over a long period of time. We argue that the household can construct a deferred annuity by rolling over payouts from immediate annuities (cf. to appendix A). We show that such a roll-over strategy provides the same benefits for the identical initial outlay as a deferred annuity. This is true whenever the discount factor and the mortality rates to price the annuity are certain. Yet, the household can use the annuity for different purposes than rolling them over into new immediate annuities. Insofar the annuity with immediate payouts gives the household greater flexibility to react to adverse developments in the labor income or in the capital markets, particularly if borrowing on human capital and illiquid life contingent assets is restricted. Therefore, it would be interesting to study the behavior as well as the utility loss if the household has only access to deferred life annuities.

Albeit, the lack of attention for deferred annuities in the literature, they appear to be ubiquitous. Intriguingly, the payment structure of deferred annuities is also hidden within Social Security. Typically employees repeatedly contribute a mandatory fraction of their current labor income to Social Security during their working life. In turn, contributions are used to fund the payments for beneficiaries currently in retirement. Beneficiaries receive a certain amount determined by their average past contributions and working years for as long as they are alive. In fact, Social Security can be considered as an instrument of purchasing deferred annuities repeatedly during working life because the benefit flow of Social Security closely resembles the payout structure of a deferred annuity. The same is true for defined benefit plans in which employers make contributions to fund the pension plan. Later in retirement, the household receives the payments from the defined pension benefit plan for as long as the head of the household stays alive.

Social Security and defined benefit plans have a deferral period reaching up to the beginning of retirement. Therefore, we augment the strategic asset allocation by deferred annuities where payments also start at the beginning of the retirement period in order to compare the interaction among Social Security, defined benefit plans, and deferred annuities. Apart from the consumption strategy, we also derive the optimal equity-bond-annuity portfolio for a *CRRA* utility maximizing household facing unspanned labor income. In order to better understand the deferral strategy, we augment the portfolio by immediate annuities as done in Horneff, Maurer, and Stamos (2008). Then we analyze how closely the underlying annuitization strategy including immediate annuities resembles a deferral strategy. Both annuitization strategies are compared in depth to highlight the similarities and differences between them.

In the first section, we discuss the model we apply to find the optimal deferred annuity demand over life. Here, we discuss preferences, labor income, annuity as well as

capital market specifications, wealth accumulation, and our numerical analysis. In the following chapter, we conduct a Monte Carlo analysis, in order to analyze the expected life cycle profile as well as the expected asset allocation. We try to identify the difference between an annuitization strategy when only deferred annuities are involved before retirement and a strategy with immediate annuities only. A final chapter concludes.

## 2 The Model

### 2.1 Preferences

In our study, we employ a time discrete model with  $t \in \{0, \dots, T + 1\}$ , where  $t$  constitutes the investor's adult age. The adult age is the actual age less 19. We denote  $p_t^s$  as the investor's subjective probability to survive from  $t$  until  $t + 1$ . Furthermore, we assume that the investor's preferences are given by the *CRRA* utility function defined over a single non-durable consumption good. Let  $C_t$  be the consumption level at time  $t$ . Then the *CRRA* preferences can be put in a Bellman equation as:

$$V_t = \frac{C_t^{1-\rho}}{1-\rho} + \beta p_t^s E_t [V_{t+1}] \quad (1)$$

where  $\rho$  is the level of relative risk aversion and  $\beta$  is the personal discount factor. Here we assume that the household does not derive any utility from bequeathing potential heirs.<sup>1</sup> Today's utility is given as the utility from consumption and tomorrow's discounted utility from future consumption. We have  $p_T^s = 0$  (1) for the final period. In  $T$  equation (1) boils down to:

$$V_T = \frac{C_T^{1-\rho}}{1-\rho}, \quad (2)$$

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<sup>1</sup>This can be justified that the household put only that part of financial wealth into the annuity which is not intended for a bequest, i.e. we only consider that part of wealth which is required for consumption purchases. Also see Stamos (2008) on this point.

which gives us the terminal condition for  $V_T$ . From the final value, we can work backwards to find the optimal strategies how to consume, invest in bonds, stocks, and how to purchase deferred annuities.

## 2.2 Labor Income Process

In order to understand how the illiquidity of deferred annuities affects the overall asset allocation, we model transitory and permanent income shocks. Previous literature on strategic asset allocation such as Bodie, Merton, and Samuelson (1992), Cocco, Gomes and Maenhout (2005), Heaton and Lucas (1997), and Viceira (2001) highlighted the relevance of considering unspanned labor income when analyzing the strategic asset allocation decisions of households. The labor income  $Y_t$  is given by:

$$Y_t = \exp(f(t))P_tU_t, \quad (3)$$

$$P_t = P_{t-1}N_t, \quad (4)$$

where  $f(t)$  is used to recover the hump shape of the empirically observed income profile over time  $t$ . Here,  $P_t$  is a permanent component with innovation  $N_t$ .  $U_t$  is a transitory shock. The logarithms of both  $N_t$  and  $U_t$  are normally distributed with means zero and with volatilities  $\sigma_N$ ,  $\sigma_U$ , respectively. The shocks are assumed to be uncorrelated. In retirement ( $t \geq K$ ), we assume for the sake of simplicity that the individual receives constant pension payments  $Y_t = \zeta \exp(f(K))P_K$ , where  $\zeta$  is the constant replacement ratio.

## 2.3 Annuity and Capital Markets

The household can directly invest in two financial assets: riskless bonds and risky stocks. The real bond gross return is given by  $R_f$ , while the real risky stock return in  $t$  is  $R_t$ .

The risky return is log-normally distributed with an expected return  $\mu$  and volatility  $\sigma_s$ . Let  $\phi_n(\phi_u)$  denote the correlation between the stock returns and the permanent (transitory) income shocks.

In our model, the household can also purchase deferred constant real payout life annuities before retirement and immediate annuities with constant payouts during the retirement period. In our analysis, a life annuity is a financial contract between an individual and an insurer "that pays out a periodic amount for as long as the annuitant is alive, in exchange for an initial premium" (Brown et al., 2001). During the working life, the life annuity does not pay out until the investor reaches the retirement age  $K$ , even though the premium is possibly exchanged years before the end of the investor's working life.

The illiquidity related to deferred annuities adversely affects the investor's ability to react to either adverse developments of labor income or sudden declines in the stock market. In return for the illiquidity, the household gains a spread over the typical bond investment. The spread comes about because the funds of those who die in the annuity pool are distributed among the living members of a cohort. The literature refers to this attribute of annuities as the mortality credit. Therefore, a deferred annuity simply constitutes a separate asset class with distinctive risk and return characteristics. We treat the purchase of deferred annuities as a portfolio choice problem by putting them on an equal footing with equity and bond investments. In the remainder, we model the annuitization decision essentially in a dynamic portfolio choice framework akin to Horneff, Maurer, and Stamos (2008).

The actuarial premium  $A_t$  of a deferred life annuity with payments  $L$  starting in  $K$  is different from the life annuity payments within the retirement period. During working life, the household can only purchase deferred annuities where payments commence only at retirement. In retirement, the household can buy immediate annuities, where

payments start from the next period onwards.  $L_t$  denotes the payouts from immediate annuities because we allow the household to also purchase immediate life annuities in retirement (i.e. from age  $K$  onwards).

$$A_t = \begin{cases} L_K h_t, & t < K, \\ L_t h_t, & t \geq K \end{cases} \quad (5)$$

where  $h_t$  is the annuity factor for an individual with adult age  $t$ . The annuity factor during the working life is essentially the pricing equation for deferred annuities with payments starting at the beginning of the retirement period.

$$h_t = (1 + \delta) \left( \prod_{u=t}^{K-2} p_u^a \right) R_f^{-(K-1-t)} \sum_{s=1}^{T-K} \left( \prod_{u=K-1}^{K-1+s} p_u^a \right) R_f^{-s} \quad t < K \quad (6)$$

where  $p_u^a$  are the survival probabilities used by the life annuity provider and  $\delta$  is the loading factor. In turn, the annuity factor is the loading factor times the deferral discount factor times the sum of the discounted expected payouts. In retirement, the household can also purchase immediate annuities. The annuity factor reduces to:

$$h_t = (1 + \delta) \sum_{s=1}^{T-t} \left( \prod_{u=t}^{t+s} p_u^a \right) R_f^{-s}, \quad t \geq K \quad (7)$$

In general, insurers use survival probabilities  $p_u^a$  that are higher than the average population survival probabilities  $p_u^s$ . The additional price increment is thought of as a compensation for both the adverse selection going on in annuity markets (Brugiavini, 1993, Finkelstein and Poterba, 2004) and the macro longevity risk (Cairns, Blake, and Dowd, 2006b). Adverse selection in annuity markets arises because heads of households who believe themselves to be healthier than average are more likely to buy annuities. Macro longevity risk refers to the risk of changing mortality probabilities. Administra-

tion costs to organize the pool are covered by the loading factor  $\delta$ . We assume zero loads and no asymmetries in mortality beliefs except for the welfare analysis where we introduce a loading factor.

We only consider highly incomplete annuity markets inasmuch only deferred annuities with life long payouts are available during working life and funds underlying the annuity are totally invested in bonds. We do not account for annuities which payout at only one specific age and state (as in the complete markets case in Davidoff, Brown, and Diamond, 2005). Due to adverse selection issues and market incompleteness such Arrow Debreu annuities do not exist. (see Yagi and Nishigaki (1993))

## 2.4 Wealth Accumulation

At the beginning of every period, the utility maximizing household under consideration can decide on how to spread wealth on hand  $W_t$  across bonds  $M_t$ , stocks  $S_t$ , new annuities purchases  $A_t$ , and consumption  $C_t$ . Therefore, the budget constraint is

$$W_t = M_t + S_t + A_t + C_t, \quad (8)$$

where we refer to  $M_t + S_t$  as the value of financial wealth. The individual's disposable wealth on hand in  $t + 1$  is given by

$$W_{t+1} \begin{cases} M_t R_f + S_t R_{t+1} + Y_{t+1} & t < K \\ M_t R_f + S_t R_{t+1} + L_{t+1} + Y_{t+1} & t \geq K, \end{cases} \quad (9)$$

where  $M_t R_f + S_t R_{t+1}$  denotes the next period value of financial wealth,  $L_{t+1}$  is the sum of annuity income which the investor gets from all previously purchased annuities and  $Y_{t+1}$  is the labor income. The state variable  $L_K$  records the claims of accessing annuity payouts at adult age  $K$  inasmuch deferred payments start at this age. Whereas the

state variable  $L_t$  after age  $K$  denotes the sum of payouts from previously purchased immediate and deferred annuities. Note that the sum of claims to annuity payouts and the sum of annuity payments follow the processes:

$$\begin{aligned} L_K^{\{t+1\}} &= L_K^{\{t\}} + A_t/h_t \quad t < K \\ L_{t+1} &= L_t + A_t/h_t \quad t \geq K \end{aligned} \tag{10}$$

where  $L_K$  is the sum of all annuity payments from annuities purchased before  $K$  and  $A_t/h_t$  is the annuity payment purchased in  $t$ . In  $t + 1$  the investor has to make a new decision on how to spread wealth on hand  $W_{t+1}$  across bonds, stocks, annuities, and consumption. We prevent households from borrowing against human capital and from selling annuities. Both restrictions are binding because otherwise households would engage in highly leveraged stock positions financed by short positions in bonds and/or annuities in order to compensate the over-investment in human capital at young ages. Thus each year the optimal policy has to satisfy:

$$M_t, S_t, A_t \geq 0. \tag{11}$$

## 2.5 Numerical Method and Calibration

Optimization problems of this type cannot be solved analytically due to the untradeable labor income, the irreversibility of annuity purchases, and the shortselling restrictions. Therefore we adopt the standard approach of dynamic stochastic programming to solve the household's optimization problem. The household maximizes (1) under budget and short-selling restrictions (8),(9), and (11), whereby the choice variables in each year the household is alive are the demand for stocks  $S_t$ , bonds  $M_t$ , new life annuities  $A_t$ , and consumption  $C_t$ . The optimal policy depends on four state variables: the permanent income level  $P_t$ , wealth on hand  $W_t$ , annuity payouts from previously purchased an-

nuities  $L_t$ , and age  $t$ . First of all, the curse of dimensionality (Bellman, 1961) can be partly mitigated by reducing the state space by one state variable as we exploit the scale independence of the optimal policy if we rewrite all variables using lower-case letters as ratios of the permanent income component  $P_t$  (see for example Cocco, Gomes, and Maenhout, 2005). We solve the problem in a three-dimensional state space by backward induction. The continuous state variables normalized wealth  $w$  and normalized annuity payouts  $l$  have to be discretized and the only discrete state variable is age  $t$ . The size of the grid is  $40(w) \times 40(l) \times 81(t)$ . The grid we use is equally spaced for the logarithms of  $w$  and  $l$  since the policy functions and value function are especially sensitive in the area with low  $w$  or  $l$ . For each grid point we calculate the optimal policy and the size of the value function.

To provide numerical insight into our setup, we calibrate the stylized case as follows: The starting age is set to 20, the retirement age to 65 ( $K = 46$ ), and the maximum age to 100 ( $T = 81$ ). In addition, we also study the case when annuity payouts start only at age 85. The preference parameters are set to standard values found in the life-cycle literature (e.g. Gomes and Michaelides, 2005): coefficient of relative risk aversion  $\rho = 5$  and the personal discount factor  $\beta = 0.96$ . Applying nonlinear least squares we fit the Gompertz force of mortality to the 2000 Population Basic mortality table for US females. We use them for pricing the annuities and for evaluating the utility from consumption. The deterministic age-dependent labor income function  $f(t)$  for individuals with high school education excluding college education and the replacement ratio  $\zeta = 0.68$  are taken from Cocco, Gomes, and Maenhout (2005). Our volatility parameters  $\sigma_u = 0.15$  and  $\sigma_n = 0.1$  are in line with the estimates found by Gourinchas and Parker (2002). Furthermore, we select a real interest rate  $R_f$  of 2 percent, an equity premium  $\mu - R_f$  of 4 percent, and a stock volatility  $\sigma$  of 18 percent. We choose correlations between the stock returns and the transitory (permanent) income shocks

of  $\phi_n = 0$  ( $\phi_u = 0$ ).

## 3 Optimal Annuity Demand

### 3.1 Deferred Life Annuities

In this section, we analyze the simulated distributions of the relevant choice and state variables by conducting an extensive Monte Carlo analysis based on the optimal feedback controls we obtained from solving the Bellman equation under the shortselling restrictions (see appendix B). Drawing 50,000 independent stochastic scenarios, we first compute the expected life cycle profile for our stylized case with risky labor income as the inclined reader can infer from figure (1). Graph A shows that the household starts purchasing deferred annuities from age 38 on in expectation and continues until the retirement before turning to immediate annuities in our analysis. If deferred annuities are purchased as early as 38, the household is willing to wait at least 27 years before the annuity starts paying off. After entering retirement, the household further buys immediate life annuities. The liquid savings of the household peak at 55 when savings are 5.5 times the average labor income. When the household turns 77, all liquid savings are exhausted. After the age 77, the household uses both the current pension and annuity income to bet on survival by purchasing more immediate annuities. Expected consumption increases because the household is not able to borrow against both the annuity and future pension income. Turning to the expected asset allocation, we find the typical life cycle profile as the overall equity exposure is successively reduced in favor of deferred annuities. Initially, the equity exposure is high to counterbalance the implicit holdings in human capital. Over time the value of human capital shrinks and the mortality credit of deferred annuities surges. In turn, annuities crowd out the equity exposure of the household. Bonds only play a miniscule role in the asset allocation

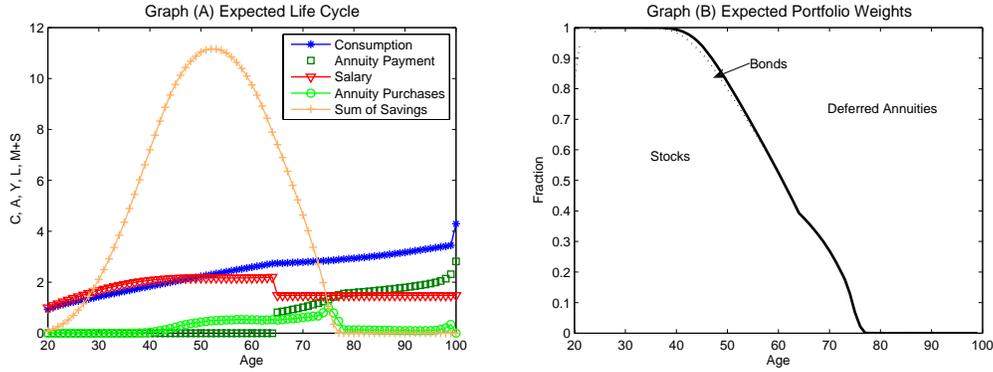


Figure 1: Expected Life Cycle and Asset Allocation: Graph (A) shows the expected Life Cycle profile in terms of consumption, new annuity purchases, liquid savings, sum of annuity payouts, and labor as well as retirement income. Graph (B) depicts the expected asset allocation of the household over the life-cycle. In our analysis, we assume a female with maximum life-span age 20 - 100, no loads for annuities, no mortality asymmetries,  $\rho = 5$ , income is stochastic. We calculate the expectations by resorting to 50,000 Monte-Carlo simulations using the optimal policies derived by solving the Bellman equation. When computing the expected portfolio weights we determine the value of annuity wealth as the actuarial present value of payouts from all previously purchased annuities.

because they are dominated by the mortality-credit enhanced deferred annuities early on. The only reason for the household to have some bond exposure is to rebalance the portfolio quickly after a sharp drop within the investment portfolio.

### 3.2 Roll-over Features of Immediate Life Annuities

In this section, we analyze to what extent immediate annuity purchases resemble a deferring strategy over the life cycle. To do so, we have to modify two transition equations to account for immediate payout annuities during working life. The state variables develop as follows:

$$L_{t+1} = L_t + A_t/h_t \quad (12)$$

where the annuity factor is simply computed as:

$$h_t = (1 + \delta) \sum_{s=1}^{T-t} \left( \prod_{u=t}^{t+s} p_u^a \right) R_f^{-s}, \quad (13)$$

The annuity factor is the expected sum of discounted annuity payouts. We also have to make sure that the household receives immediate payouts in the next period, therefore we have to change the cash on hand during the working life period.

$$W_{t+1} = M_t R_f + S_t R_{t+1} + L_{t+1} + Y_{t+1} \quad (14)$$

In order to analyze how frequently a deferred annuity is constructed or partly resembled through the ongoing purchase of immediate life annuities, we first derive the optimal feedback controls for purchasing immediate annuities over the life cycle. After obtaining the new annuity dynamics, we conduct a Monte Carlo analysis with 50,000 iterations. Now we consider on each of the life cycle paths how many times a household rolls the annuity payout over into a new annuity purchase. To do so, we compute the payouts  $L_t$  from previously purchased annuities and compare them to the amount of new annuity purchases  $A_t$ . Whenever  $L_t < A_t$  we know that all payouts are rolled over to a new annuity in that given year. Using this specification, we compute for each household the maximum number of successive roll-over years. Figure (2) provides the distribution of the maximum roll-over years. Hereby, graph (A) reports the result for the case when labor income is certain. The average time of a continued roll-over strategy is 7.6 years while the most frequent maximum roll-over is seven years. Therefore, a typical household with certain labor income buys immediate annuities for seven successive years by spending at least as much cash on hand as available from annuity payouts.

As graph (B) in figure (2) shows: if the household faces uninsurable stochastic labor

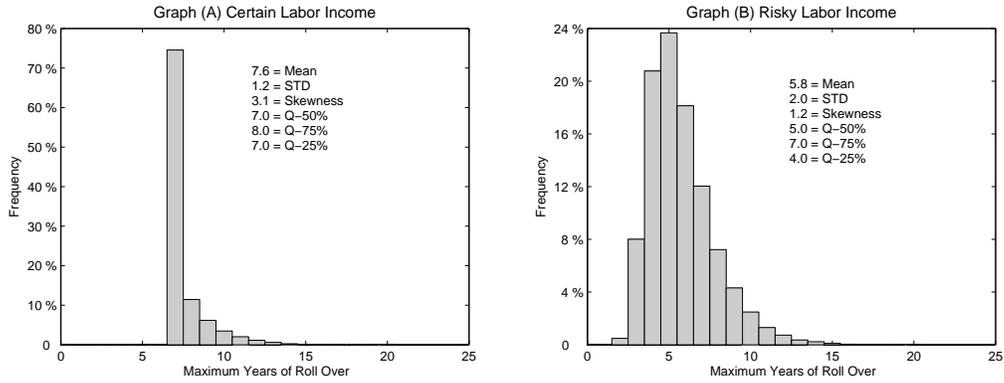


Figure 2: Maximum Years of Roll-Over: Graph (A) shows the case with certain labor income. Graph (B) depicts the case with risky labor income. Maximum years of roll-over means the number of successive years in which the payouts from annuities cannot fully cover the new annuity purchases. In our analysis, we assume a female with maximum life-span age 20 - 100, no loads for annuities, no mortality asymmetries,  $\rho = 5$ . We calculate 50.000 different life-cycles by using the optimal policies derived by the maximization of the Bellman equation under constraints.

income, the distribution is moved to the left hand side (mean 5.8) and shows a much higher variability (2.0). This can be explained by the fact that the household requires more flexibility to use the payouts from annuities in a different way.

While the previous analysis considered all life-cycles pathwise, the following breakdown looks at percentage of a roll-over strategy per year across all households. Again, we consider the simulated paths based on the optimal strategy with immediate annuities. Contrary to the previous analysis, we compare the probability of rolling the annuity payouts over into new life annuity purchases in a cross section analysis by considering all households at a certain point in time. The results are displayed in figure (3). As one can infer from graph (A), the roll-over strategy is done in most cases early on. However, only few households voluntarily annuitize at this stage of their life cycle, especially if labor income is certain. So the percentage term might not be as representative as other figures in our analysis. Here, the percentage of roll-over strategies seldom drops below 50 percent before retirement starts. In the case of risky labor income, the

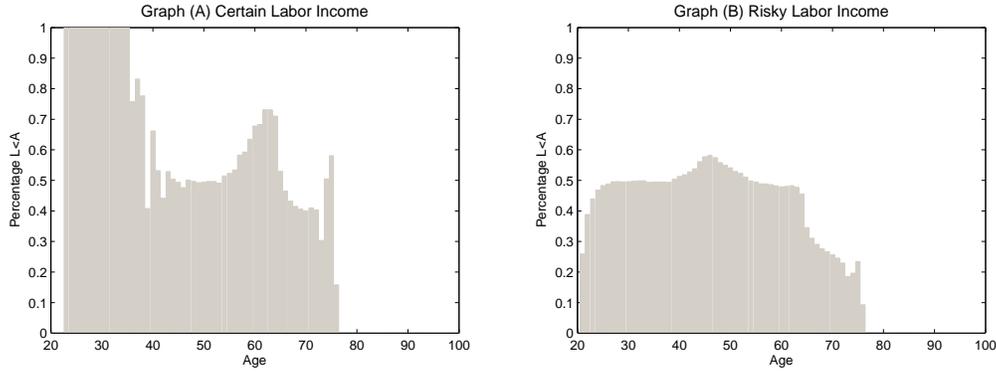


Figure 3: Percentage of Roll-Over: Graph (A) shows the case with certain labor income. Graph (B) depicts the case with risky labor income. In our analysis, we assume a female with maximum life-span age 20 - 100, no loads for annuities, no mortality asymmetries,  $\rho = 5$ . We calculate 50.000 different life-cycles by using the optimal policies derived by the maximization of the Bellman equation under constraints.

roll-over strategies are less likely since the household has to rebalance the portfolio and buffer adverse career developments.

### 3.3 Deferred vs. Immediate Life Annuities

In this section, we compare the purchasing behavior of households as far as deferred and immediate life annuities are concerned. First, we analyze the expected new annuity purchases over the life cycle for the case with certain labor income. The results are reported in Graph A of figure (4). The household uses more cash on hand to purchase immediate than deferred life annuities. The discrepancy among all three strategies reaches its largest point at age 60. While the difference for all annuity strategies is moderate in case of certain labor income, the difference becomes even more pronounced in case the labor income is risky. Again, the greatest discrepancy between the deferring and the annuitization strategy peaks at age 60. The following analysis considers the welfare loss of having only access to deferred annuities as compared to immediate ones.

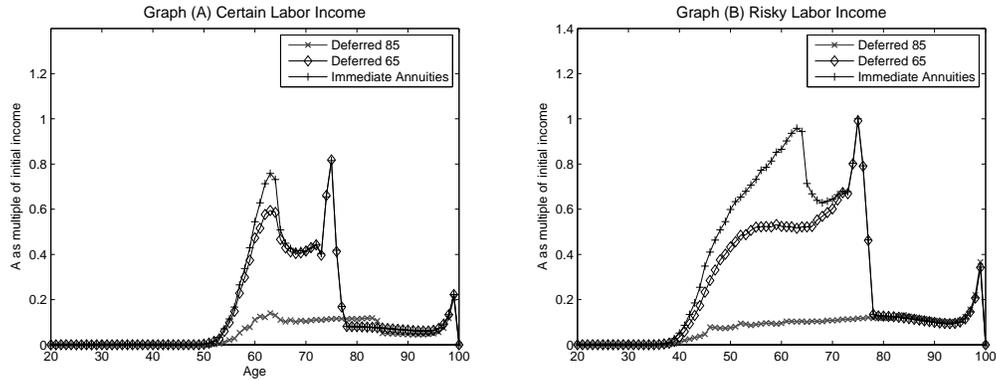


Figure 4: Expected Spending on New Annuities: Graph (A) shows the case with certain labor income. Graph (B) depicts the case with risky labor income. In our analysis, we assume a female with maximum life-span age 20 - 100, no loads for annuities, no mortality asymmetries,  $\rho = 5$ . We calculate 50,000 different life-cycles by using the optimal policies derived by the maximization of the Bellman equation under constraints.

In our welfare analysis, we also consider cases where we include explicit loads of  $\delta = 2.38$ . In addition, we assume a difference in mortality beliefs by using an annuity mortality table for pricing the life annuities. Welfare gains are computed as the additional constant life long income (as a fraction of average labor income) an individual without access to deferred and immediate annuity markets would need in order to attain the same expected utility as in the case with annuity markets. The numerical computations are done for age 20. Then we compute the marginal welfare loss of having only access to deferred annuities as compared to immediate annuities by calculating the difference in the additional consumption stream between the deferred and the immediate annuitization strategy. The actual difference in the additional life long income stream is displayed in the following summarizing table.

The welfare loss of having only access to deferred annuities starting at age 65 as compared to immediate annuities is small at 1 bp difference in the case without any loadings. The welfare difference for an annuity starting paying off at age 85 is bigger with 51 bp. If we add initial loadings to our analysis, deferred annuities become

## Welfare Gains at Age 20

	Deferred 65	Deferred 85
<i>Risky Labor Income</i>		
Without Loads	-1 bp	-51 bp
With Loads	+1 bp	-18 bp
<i>Riskless Labor Income</i>		
Without Loads	> -1 bp	-28 bp
With Loads	+1 bp	-3 bp

Table 1: This table reports welfare gains in the presence of annuity markets for all cases considered previously. Welfare gains are computed as the additional constant life-long income (as a fraction of average labor income) at age 20 an individual without access to annuity markets would need in order to attain the same expected utility as in the case with annuity markets. Then we compute the additional welfare loss of having only access to deferred annuities as compared to immediate annuities. This is done by computing the difference in the additional consumption stream between the deferred and the immediate annuitization strategy. The numerical computations are done for deferred annuities where payouts start either at age 65 or at age 85.

preferable to a small extent in as much the household has to bear the costs of rolling the annuity payouts over in each period. When dropping the riskiness of labor income, the difference to immediate annuities becomes even smaller. While in the case with risky labor income, immediate payouts are an important cash flow whenever the labor income and the stock market decline, the utility loss in the certain labor income case is smaller due to the lack of uncertainty generated by income fluctuations. As we have seen in the previous section, the household rolls the payouts over more often if income is certain.

## 4 Conclusion and Discussion

In this study, we derive the optimal deferred-annuity-bond-equity portfolio of a household facing unspanned labor income. We compare the resulting annuitization to the

one with immediate annuities. We find an astonishing similarity between the two different annuitization strategies. The purchasing behavior of deferred annuities is very much in line with the one observed in the previous literature, which analyzes the annuitization strategy with immediate annuities. Even though the household purchases less deferred annuities than immediate annuities in expectation, the life cycle profiles and the expected asset allocations are virtually the same. Furthermore, we find that households having only access to immediate annuities engage in a roll-over strategy mimicking deferred annuities to a considerable extent. The roll-over strategy is less pronounced if the household faces risky labor income as compared to certain labor income. A welfare comparison indicates a small difference between immediate and deferred annuitization, particularly for the case where annuity payouts commence at age 65. Deferred annuities might become more appealing in relative terms to immediate, if loading is considerably high. Since deferred annuities are less capital intensive than immediate annuities, they might be a good instrument to overcome the reluctance to engage in irreversible financial transactions such as annuitization early in life.

## Appendix A: Rollover vs. Deferred Strategy

We argue that a roll-over strategy using annuities with immediate payouts leads - with identical cash flows - to the same payout as the purchase of a deferred annuity. Here we consider two intervals: period 1 and period 2. The investor can decide between a deferred annuity and a roll-over strategy using immediate annuities. Furthermore,  $p_1$  denotes the probability of surviving the first period from time 0 to 1, while  $p_2$  is the probability of surviving the second period from time 1 to 2. The discount rate  $r$  is constant, the loading factor is  $\lambda$ , and the resulting price for the annuity is  $X$ . The payout  $L_2^D$  in the second period for a deferred annuity is:

$$\begin{aligned} X &= \frac{L_2^D p_1 p_2}{r^2} \lambda \\ L_2^D &= \frac{X r^2}{p_1 p_2 \lambda} \end{aligned} \tag{15}$$

In the case of an immediate annuity with payments  $L_1^I$  in period 1 and of  $gL_1^I$  in period 2, the premium is given by ( $g$  is the escalating factor  $g > 0$ ):

$$\begin{aligned} X &= \left( \frac{L_1^I p_1}{r} + \frac{L_1^I g p_1 p_2}{r^2} \right) \lambda \\ L_1^I &= \frac{X r}{(p_1 + \frac{g p_1 p_2}{r}) \lambda} \end{aligned} \tag{16}$$

Now the investor uses the proceeds  $L_1^I$  from the immediate annuity to purchase another annuity. The additional payouts in period 2 from this annuity is given by  $L_2^I$ :

$$L_2^I = \frac{X r}{\lambda^2 (p_1 + \frac{g p_1 p_2}{r})} \frac{r}{p_2} \tag{17}$$

The resulting sum of the payouts from the previous annuity and the current annuity purchased in period 2 is  $L_{cum}^I = gL_1^I + L_2^I$  and can be rewritten as:

$$\begin{aligned} L_{cum}^I &= \frac{Xrg}{\lambda(p_1 + \frac{gp_1p_2}{r})} + \frac{Xr}{\lambda^2(p_1 + \frac{gp_1p_2}{r})} \frac{r}{p_2} \\ &= \frac{Xr^2}{\lambda(p_1r + gp_1p_2)} \left( g + \frac{r}{\lambda p_2} \right) \end{aligned} \quad (18)$$

Now, we can check the condition that the cumulative payouts from the roll-over strategy with immediate annuities are identical to the payments of the deferred annuity:

$$L_{cum}^I = L_2^D \Leftrightarrow \frac{Xr^2}{\lambda(p_1r + gp_1p_2)} \left( g + \frac{r}{\lambda p_2} \right) = \frac{Xr^2}{\lambda p_1 p_2} \Leftrightarrow \frac{gp_2 + \frac{r}{\lambda}}{gp_2 + r} = 1 \quad (19)$$

This condition is fulfilled in the case of a loading factor equal to one,  $\lambda = 1$ , i.e. if the annuities are actuarially fairly priced. In case  $\lambda > 1$  the resulting payoffs from the roll-over strategy is lower compared to the deferred annuity strategy.

## Appendix B: Numerical Integration and Optimal Annuitization Policy

For computing the value function, we have to solve the multiple integral. Thereby, the (multiple) integrals of the expectation term in:

$$\begin{aligned} \iiint (p_t^s v_{t+1}^{1-\rho} + (1 - p_t^s) k b_{t+1}^{1-\rho}) N_{t+1}^{1-\rho} \cdot \varphi(N_{t+1}, U_{t+1}, R_{t+1}) dN_{t+1} dU_{t+1} dR_{t+1} & \quad t < K \\ \int (p_t^s v_{t+1}^{1-\rho} + (1 - p_t^s) k b_{t+1}^{1-\rho}) \varphi(R_{t+1}) dR_{t+1} & \quad t \geq K, \end{aligned} \tag{20}$$

with  $\varphi(\cdot)$  denoting the (multivariate) probability density function of the log-normal distribution is computed by resorting to gaussian quadrature integration and the optimization is done by numerical constrained maximization routines. The values of the policy functions and value function lying between the grid points are computed by cubic-splines interpolation. The optimal annuitization policy is displayed in the graph below.

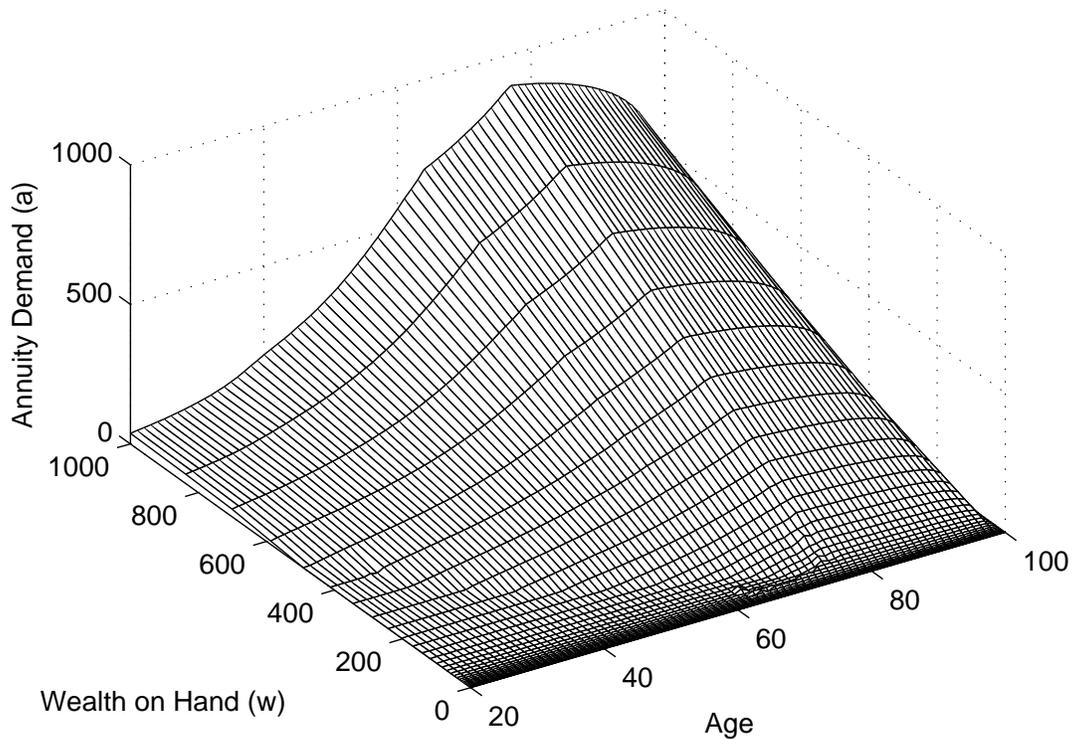


Figure 5: Annuity Policy: Graph shows annuity policy with risky labor income. Until age 65 with household purchases deferred annuities with payouts starting at age 65. Later, the household purchases immediate annuities. In our analysis, we assume a female with maximum life-span age 20 - 100, no loads for annuities, no mortality asymmetries,  $\rho = 5$ , where  $L$  is set to zero.

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